SDNV104 - Dynamic response of a rigid shoe rubbing subjected to a pressure and a back pulling force

Summary

One considers a mass in contact rubbing with a rigid plan. It is retained by a spring and one imposes a side pressure to him. Friction is modelled by the law of Coulomb. Calculation is a direct dynamic calculation.

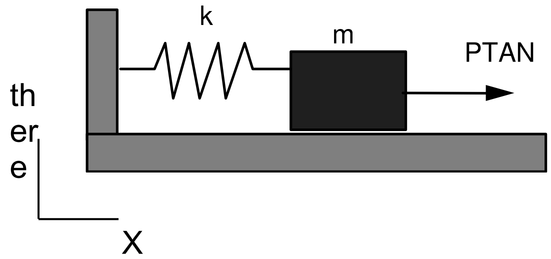
The reference solution is analytical.

Modelings suggested use `DYNA_NON_LINE` with an elastic law of behavior in 2D, for two solveurs. The contact is managed by various methods available in `AFFE_CHAR_MECA`.

1 Problem of reference

1.1 Geometry

The system considered consists of a shoe: square of 1m on 1m, posed on a support. It is subjected to its weight, by the strength of recall of a spring of stiffness K and with a side pressure. The contact is a rubbing contact.



1.2 Properties of the model

Mass:	$7.10^3 kg$
Stiffness of the spring:	$24.10^3 N/m$
Coefficient of Coulomb:	$0,3$
Gravity:	$70\,000\,Pa$
Side pressure:	$200\,000\,Pa$
Young modulus of the shoe:	$2,1 . 10^{11} Pa$
Young modulus of the solid mass:	$1,0 . 10^{11} Pa$
Poisson's ratio:	0

1.3 Boundary conditions, conditions initial and loadings

The mass rests on the rigid level with the dimension $x=0$.

The loadings of weight and side pressure are applied with a slope which reaches its maximum into 0.07 second.

The support is embedded in x and in y.

2 Reference solution

The reference solution is analytical.

If the shoe is considered as sufficiently rigid one can compare it to a specific mass, there then exists an analytical solution with this problem of mass-arises rubbing. The assumption is made that the application of the loading is done immediately (not slope).

Like $\frac{F_t}{F_n} > \mu$ there is never phase of adherence but only of the slip. The force of friction is thus worth $f = \mu F_n$

One can write the equation of the movement as follows:

$$m\ddot{x} + kx = F_t \pm f$$

One notes: $\omega = \sqrt{\frac{k}{m}}$

Stage 1: The force of friction is opposed to the movement which is carried out initially according to x positive

$m\ddot{x} + kx = F - f$

with a X-coordinate and a worthless initial speed. There is then the following solution after taking into account of these initial conditions:

$$x(t) = \frac{F - f}{k}(1 - \cos(\omega t))$$

this result is valid as long as speed remains positive is $\dot{x} \geq 0$, i.e. until $\omega . t = \pi$.

The first extremum of the curve $x(t)$ is $x_1 = 2.\frac{F - f}{k}$.

Stage 2: The force of friction changes sign to be opposed to the movement which is done now according to x negative

$m\ddot{x} + kx = F + f$

the initial X-coordinate is worth x_1 , and initial speed is worthless. One has then, by posing the new X-coordinate of times like origin with π / ω :

$x(t) = \frac{F + f}{k} + \frac{F - 3\mathrm{f}}{k}\cos(\omega t)$, until $\omega . t = \pi$.

The second extremum of the curve $x(t)$ is $x_2 = \frac{4\mathrm{f}}{k}$.

Stage $2\mathrm{n} - 1$ and $2\mathrm{n}$:

One separates the movement according to the sign speed. For an odd stage the movement is done according to x positive. For an even stage, the movement is done according to x negative.

One shows by recurrence the following result:

$$x_{2p-1}=x((2p-1)T)=\frac{2(F-(2p-1)f)}{k}$$
$$x_{2p}=x(2pT)=\frac{4pf}{k}$$

with $T=\frac{\pi}{\omega}$ and $\omega=\sqrt{\frac{k}{m}}$.

The stop of the movement occurs when x_n is understood enters $\frac{F_t-f}{k}$ and $\frac{F_t+f}{k}$.

2.1 Results of reference

Modeling proposed Ci below correspond to the analytical solution until the stop of the shoe. By preoccupations with a time-saving of calculation, one tests only the two first extremum.

Time (s)	**Displacement in x (m)**
1.697	14.917
3.393	3.500

2.2 Uncertainty on the solution

The analytical solution gives an exact result under the assumption of infinitely rigid bodies. It is also supposed that the loading is applied directly (it does not depend on time).

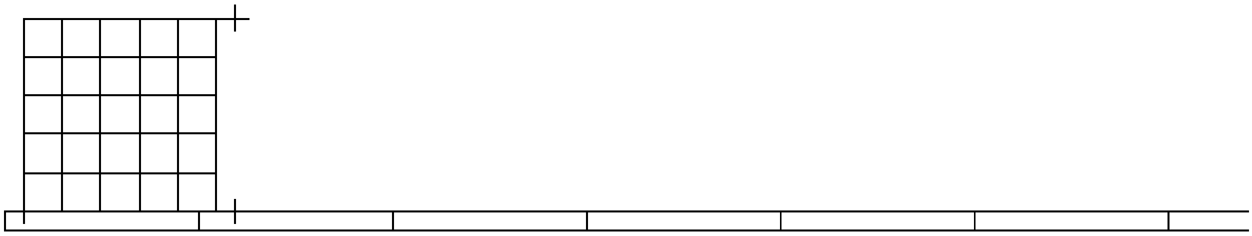
3 Modeling B

3.1 Characteristics of modeling

The problem is D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD4. An element 2D_DIS_T represent the spring, its component nonworthless is in the direction x .

The operator is used DYNA_NON_LINE to carry out dynamic calculation. The efforts of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method PENALIZATION.

3.2 Characteristics of the grid



Many nodes:	42	many meshs and types:	26 QUAD4 26 SEG2

3.3 Values tested for the method **PENALIZATION**, **MULT_FRONT**

t	Reference	*Aster*	% difference
1.697	14.91	14.84	-0.5%
3.393	3.50	3.62	3.6%

3.4 Values tested for the method **PENALIZATION**, **LDLT**

t	Reference	*Aster*	% difference
1.697	14.91	14.83	-0.5%
3.393	3.50	3.62	3.6%

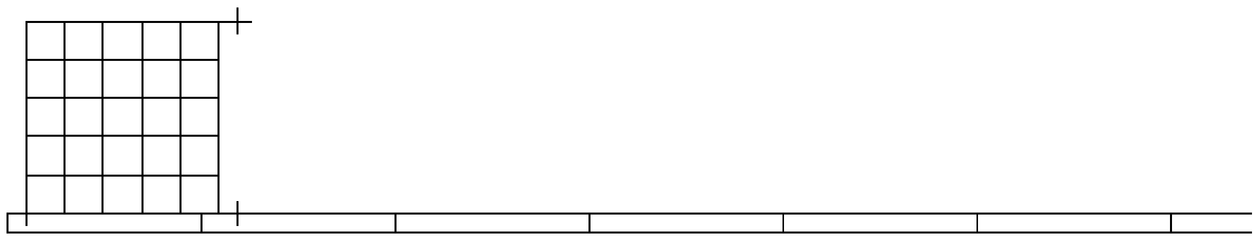
4 Modeling C

4.1 Characteristics of modeling

The problem is D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD4. An element 2D_DIS_T represent the spring, its component nonworthless is in the direction x .

The operator is used DYNA_NON_LINE to carry out dynamic calculation. The efforts of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method CONTINUOUS.

4.2 Characteristics of the grid



Many nodes: 42 many meshs and types: 26 QUAD4
26 SEG2

4.3 Values tested for the method CONTINUOUS, MULT_FRONT

t	Reference	*Aster*	% difference
1.697	14.91	14.84	-0.5%
3.393	3.50	3.62	3.6%

4.4 Values tests for the method CONTINUOUS, LDLT

t	Reference	*Aster*	% difference
1.697	14.91	14.83	-0.5%
3.393	3.50	3.62	3.6%

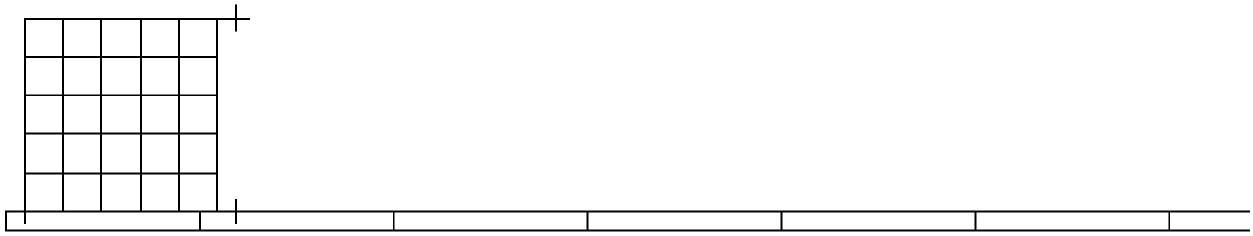
5 Modeling D

5.1 Characteristics of modeling

The problem is D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD8. An element 2D_DIS_T represent the spring, its component nonworthless is in the direction x .

The operator is used DYNA_NON_LINE to carry out dynamic calculation. The efforts of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method CONTINUOUS.

5.2 Characteristics of the grid



Many nodes:	110	many meshs and types:	26 QUAD 8 24 SEG3 2 SEG 2

5.3 Values tested for the method LAGRANGIAN, MULT_FRONT

t	Reference	Aster	% difference
1.697	14.91	14.83	-0. , 5%
3.393	3.50	3.62	3.6%

5.4 Values tested for the method LAGRANGIAN, LDLT

t	Reference	Aster	% difference
1.697	14.91	14.83	-0. , 5%
3.393	3.50	3.62	3.6%

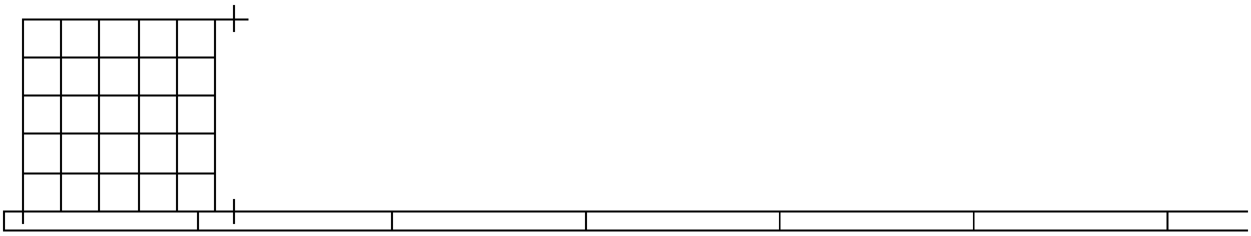
6 Modeling E

6.1 Characteristics of modeling

The problem is D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD8. An element 2D_DIS_T represent the spring, its component nonworthless is in direction X.

The operator is used DYNA_NON_LINE to carry out dynamic calculation. The efforts of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method PENALIZATION.

6.2 Characteristics of the grid



Many nodes:	110	many meshs and types:	26 QUAD 8 24 SEG3 2 SEG 2

6.3 Values tested for the method PENALIZATION, MULT_FRONT

t	Reference	*Aster*	% difference
1.697	14.91	14.83	-0,50%
3.393	3.50	3.62	3.6%

6.4 Values tested for the method PENALIZATION, LDLT

t	Reference	*Aster*	% difference
1.697	14.91	14.83	-0,50%
3.393	3.50	3.62	3.6%

7 Summary of the results

The results got on the whole of this case test are satisfactory, as well into linear as into quadratic. The values obtained are with less than 1% from/to each other; and less than 4% of the reference solution.

It is noted that the value of reference of the second point is lower than the two others, which increases the percentage of error artificially.

The choice of the coefficients of the penalized method is delicate. But it is noted that once the coefficients chosen, the result is stable with respect to the choice of the finite elements and the solvor.