SDNV105 - Swinging of a block on a table

Summary:

This CAS-test is used to validate the capacity of `DYNA_NON_LINE` and of the methods of contact in *Code_Aster* available to deal with problems of nonregular dynamics in the presence of collisions with implicit resolution. The problem consists in analyzing the dynamic response of a heavy homogeneous rectangular block posed on a table, subjected to various loadings (to release, harmonic excitation): the swinging led to successive collisions with the table.

One evaluates various manners of treating the contact, like various choices of temporal integration:

1. treatment of the contact by method by penalized shock absorbers and temporal integration by implicit scheme Newmark-HHT in displacement (modeling B, in 2D);
2. treatment of the contact by continuous method and temporal integration by implicit scheme Newmark-HHT in displacement (modeling C, in 2D);

The got results are in relatively good agreement with the results of reference. These results of reference are of three natures:

1. benches analytically with the rigid assumption of body without rebound, while simulation *Code_Aster* is made with elastic bodies, which induces a little distant results;
2. benches analytically starting from the drainage efficiency of average shock recorded on simulation *Code_Aster*, which makes it possible to ensure the accuracy of several dynamic variables: speeds, moments of collision, energies kinetic, reactions, percussions;
3. obtained numerically with software LMGC90 of laboratory LMGC (University of Montpellier), with one θ - diagram of speed.

However, it will be noted that this dynamic problem is very sensitive and thus that the precision can be degraded quickly during the transient; moreover, the choice of the algorithm (method of contact, temporal integration) has a great influence. Also the tolerances on the values tested grow during the transient. One advises the use of θ - diagram which is at the same time more robust and cheaper.

1 Problem of reference

1.1 Geometry

One considers a heavy homogeneous parallelepipedic block resting initially on a rigid table (null game).

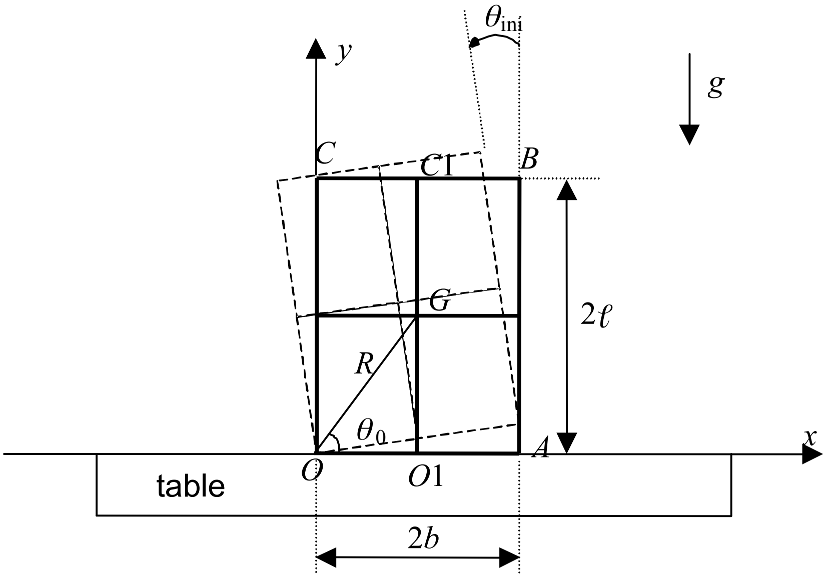

Figure 1.1-a: Front view of the block on the table.

Geometrical data:

Face of the block: $2b \times 2l$: $0{,}360\ m \times 0{,}800\ m$. Thickness: $e=1\ m$. One notes $R=\sqrt{b^2+l^2}$ and θ_0 the angle $(\vec{OQ}, \vec{OG})$. The table has a vertical thickness of $0{,}08\ m$.

1.2 Material properties

Characteristic	block	table
Young modulus	$6{,}0\,10^{11}\ Pa$	$1{,}0\,10^{14}\ Pa$
Poisson's ratio	0.2	0.3
Density	$1450\ kg/m^3$	$2500\ kg/m^3$
AMOR_ALPHA	$0.0001\ s$	$0.001\ s$
AMOR_BETA	0.0	0.0

The table is selected "hard". The coefficient of dry friction of Coulomb between the block and the table is: $\mu=0{,}9$. This high coefficient makes privilege situations where rotation around the corners of the block is done without slip. However, during the installation of the heavy block on the table, one admits that there is free slip: $\mu=0{,}0$. The mass of the block is $417{,}600\ kg$.

One adds a damping of Rayleigh to introduce a certain dissipation high frequency material using the keyword AMOR_ALPHA (see above), from where forces: $\mathbf{C}\,\dot{\mathbf{U}}=\alpha\,\mathbf{K}\,\dot{\mathbf{U}}$.

1.3 Boundary conditions and loadings

Boundary conditions

Translations of the table according to the axes y and z are blocked on the lower face of this one.
Translation of the table according to the axis x is blocked at the point $S1$ in with respect to $O1$.
Translations of the block according to the axis z are blocked.
The point $O1$ block, medium of its base is constrained by: $dx=0$ only for the initial phase of phased introduction of the loading of gravity.
The acceleration of gravity is worth: $g=9.81\ m/s^2$.
A condition of contact-friction is ensured between the base of the block and the higher face of the table.

To release after slope of the block, constant gravity remaining

The condition of contact-friction is ensured between lower face of the block and higher face of the table. The acceleration of gravity is worth: $g=9.81\ m/s^2$.

Harmonic loading of excitation of the table

Under investigation, not restored.

1.4 Initial conditions

Loading of gravity

The block and the table are initially at rest: with $t=0$, $dx(0)=0$, $dx/dt(0)=0$ in any point.

Loading to release

The point $C1$, medium on the higher side of the block is constrained: $dx=-0.008\ m$, that is to say an angle of 10^{-2} . whereas the point $O1$ is fixed in x . One expects then the stabilization of the vibrations, to carry out to release it under gravity, since this position drawn aside at worthless initial speed in any point.

Harmonic loading of excitation of the table

Under investigation.

2 Reference solution

The results of reference are of two natures:

- benches analytically with the rigid assumption of body, without rebound, while simulation *Code_Aster* is made with elastic bodies, which induces a little distant results;
- benches analytically starting from the average ratio of kinetic loss of energy raised during the collisions on simulation *Code_Aster*, which makes it possible to ensure the accuracy of several dynamic variables: speeds, moments of collision, energies kinetic, reactions, percussions;
- obtained numerically with software LMGC90 of laboratory LMGC (University of Montpellier), with one θ - diagram of speed, cf [bib3, bib4].

2.1 Method of calculating used for the reference solution

For more details on the solution with the rigid assumption of body, to refer to [bib1]. The mass of the block is: $M=\rho V=4\rho b l e$. A possible acceleration of training of the table is considered: $(\ddot{u}_{ent}, \ddot{v}_{ent})$. The parameter of configuration is noted θ. The inertia of rotation in the centre of gravity is $J_G=\frac{1}{3}M\left(b^2+l^2\right)$, that with the corner O is $J_O=J_G+MR^2$. The equations of the movement of the block, within the framework of modeling in rigid body are:

$$\begin{cases} \textit{rocking about } O \ : \ J_O\ddot{\theta}+MR\left(\ddot{v}_{ent}+g\right)\cos\left(\theta_0+\theta\right)-MR\,\ddot{u}_{ent}\sin\left(\theta_0+\theta\right)=0 \quad \textit{with} \quad \theta\in\mathbb{R}^+ \\ \textit{rocking about } A \ : \ J_A\ddot{\theta}-MR\left(\ddot{v}_{ent}+g\right)\cos\left(\theta_0-\theta\right)-MR\,\ddot{u}_{ent}\sin\left(\theta_0-\theta\right)=0 \quad \textit{with} \quad \theta\in\mathbb{R}^- \end{cases}$$

The pulsation of swinging is: $\omega_{rO}=\sqrt{MgR/J_O}$, that is to say $\omega_{rO}=\sqrt{3g/4R}$; with the geometrical data: $\omega_{rO}=4.0956\,s^{-1}$.

If one analyzes the head-on collision of the base of the block on a rigid wall, total duration of contact τ (time of return ticket of the elastic wave) is given by:

$$\tau=4l\sqrt{\frac{\rho}{E}}$$

that is to say here: $\tau\approx1.5731\ 10^{-4}\ s$.

What is directly associated with the frequency with the mode with vertical extension with the block, which is: $f_1=6357\,Hz$. One can make the same analysis for the table: $f_1=625\,kHz$. The frequency of the first mode of inflection transverse of the block, considered as a beam of Euler, rotulée at his base and free at its top, is: $f_f=8100\,Hz$. This value gives an idea of the type of dynamic response of the block likely to occur at the time of the collision after releasing.

Total duration of contact τ, at the time of a frontal shock, allows to estimate the stiffness of shock absorbers to be placed at the lower corners of the block, when one considers a technique of treatment of the contact by shock absorbers: it is the case of modeling B. One has as follows:

$$K_{res}=M_{bloc}\frac{\pi^2}{\tau^2}=\frac{\pi^2 ES}{16\,\ell}$$

This value corresponds to a longitudinal wave propagation in a continuous medium. If one considers the answer of a finite element linear rubber band into unidimensional, with matrix of consistent mass, length $2l$, one a:

$$K_{res}=M_{bloc}\frac{3E}{\rho l^2}=\frac{3ES}{l}$$

For this configuration one finds respectively: $9.253\,10^{11}\,N/m$ and $4.5\,10^{12}\,N/m$.

2.1.1 Case to release starting from a position inclined at rest: rigid body

Like $\cos^2\theta_0>\frac{2}{3}\Leftrightarrow l/b<\sqrt{2}/2$, the block balances alternatively corner on the other, cf [bib1]. One notes θ_{in} the initial slope, presumedly weak. The equilibrium equation for the phases of "coasting flight" (valid for small angles, to the 1er order) is reduced to $\frac{4}{3}MR^2\ddot{\theta}+Mbg-M\ell g\,\theta=0$. Duration of the coasting flight t_{coll} around O before collision on the other corner A is thus solution of:

$$0=\frac{b}{l}+\left(\theta_{in}-\frac{b}{l}\right)\cosh\left(t_{coll}\sqrt{\frac{3gl}{4R^2}}\right)$$

For $\theta_{in}=10^{-2}$, and dimensions of the block considered, one a: $t_{coll}\approx 0{,}05440978\ s$. This moment corresponds to the first impact with the corner A .

The kinetic energy right before the collision is: $E^{-}_{kin}=\frac{1}{2}J_O\dot{\theta}^2=Mbg\,\theta_{in}$. It is worth: $7.37398\,J$.

One from of also deduced the angular velocity before the first collision:

$$\dot{\theta}(t_{coll})=\frac{1}{R}\sqrt{\frac{3}{2}bg\,\theta_{in}}\approx-\frac{3}{4}bgt_{coll}/R^2$$

Vertical balance gives the vertical reaction in O :

$$F_O=Mg+Mb\,\ddot{\theta}\approx Mg\left(1-\frac{3b^2}{4R^2}\right)$$

Horizontal balance gives the horizontal reaction:

$$H_O=Ml\,\ddot{\theta}\approx-\frac{3Mgbl}{4R^2}$$

Their values are: $F_O\approx 3579{,}25\ N$ and $|H_O|\approx 1149{,}79\ N$.

Their report is thus $\frac{3bl}{4R^2-3b^2}\approx 0.3212$.

This report is weaker than the coefficient of selected friction: one thus does not expect slip.

The equilibrium equations in percussion of shock at the times of the impacts provide for their part:

$$I_y = -\frac{4R^2 - 3b^2}{2R^2} Mb\,\omega_- \quad \text{while} \quad \frac{|I_x|}{|I_y|} = \frac{3b\,l}{4R^2 - 3b^2} \approx 0.3212$$

This report is weaker than the coefficient of selected friction: one thus does not expect slip.

To each impact a share of the kinetic energy is transmitted block to the table; since, at the time of the collision, it is admitted that there is no rebound (i.e. just at the moment of the collision, the impacted point has a fixed position), one identifies an equivalent "drainage efficiency then" connecting the ratio angular velocities before and after the collision (one points out that in phase "coasting flight", accelerations are quasi constant, therefore speeds quasi closely connected):

$$\omega_+ = \frac{2R^2 - 3b^2}{2R^2}\,\omega_- \approx 0.7474\ \omega_-$$

This same ratio corresponds to that of the time intervals between two successive collisions (to the 1 er order).

One of deducted the ratio enters the kinetic energies after and before each collision: 0.5586086 (because of the square). This same ratio corresponds to that of the swing angles of the block after and before each collision, because it is admitted that there is no dissipation of energy during free rotation between two collisions, which provides the values of maximum vertical displacements of the corners of the block.

The time interval between two successive collisions $k \to k+1$ is given by the maximum swing angle θ_k reached at the time of the phase right after the collision k (or θ_{k-1} reached at the time of the phase right before the collision k):

$$\Delta t_{k\to k+1} \approx \frac{8R^2}{3bg}\dot{\theta}_k^+ = 4R\sqrt{\frac{2\theta_k}{3bg}} = \frac{4R^2 - 6b^2}{R}\sqrt{\frac{2\theta_{k-1}}{3bg}}$$

Like the ratio θ_k/θ_{k-1} with two successive collisions is identical (to the 1er order) with that of the kinetic energies $E_{kin}^{+k}/E_{kin}^{-k}$, then one also has:

$$\Delta t_{k\to k+1}/\Delta t_{k-1\to k} \approx \sqrt{E_{kin}^{+k}/E_{kin}^{-k}} = \omega_+/\omega_-$$

2.1.2 Case to release starting from a position inclined at rest: elastic body

One does not have a complete theoretical reference under this assumption. On the other hand, by admitting the weakness of the energy stored in vibratory form, there are the following results.

The phase of "coasting flight" (rotation around the corners) is still characterized by the equilibrium equation of the rigid body in rotation (valid for small angles):

$$\frac{4}{3}MR^2\ddot{\theta} = -Mbg + M\,l\,g\,\theta$$

From where reactions: $F_O = Mg + Mb\,\ddot{\theta} \approx Mg\left(1 - \frac{3b^2}{4R^2}\right)$ and $H_O = M\,l\,\ddot{\theta} \approx -\frac{3Mgb\,l}{4R^2}$, that is to say: $F_O \approx 3579{,}25\ N$ and $|H_O| \approx 1149{,}79\ N$. These values are valid for all the phases of coasting flight between two collisions.

One obtains also the angular velocity right before the first collision: $\dot{\theta}\left(t_{coll}\right)=\frac{1}{R}\sqrt{\frac{3}{2}bg\,\theta_{in}}$ and kinetic energy: $E_{kin}^{-}\left(t_{coll}\right)=Mbg\,\theta_{in}=\frac{2}{3}MR^{2}\,\dot{\theta}_{-}^{2}\left(t_{coll}\right)$. The moment of the first collision must be the same one as analyzes some in rigid body: $0{,}05440978\ s$; kinetic energy with the first collision being also: $7{,}373981\ J$.

The contact being maintained over a certain duration in the digital simulation with the assumption of deformable bodies, contrary to the case of the rigid bodies, the restitution of shock does not have same phenomenology.
Also decides one to take the value found by simulation by *Code_Aster* to identify a drainage efficiency, which is employed to reconstitute the succession of the later collisions.

One thus decides to take by the median value of $\sqrt{E_{kin}^{+k}/E_{kin}^{-k}}$ on the 5 studied collisions drainage efficiency: 0.79, that one chooses for the successive predictions. This value higher than that is obtained in assumption of rigid body without rebound, cf [§ 2.1.1], which is legitimate.

While following the assumption of absence of vibration lasting the phase of coasting flight of swinging, the time interval between two successive collisions $k\rightarrow k+1$ is given by the maximum swing angle θ_k reached at the time of the phase right after the collision k :

$$\Delta t_{k\rightarrow k+1}\approx\frac{8R^{2}}{3bg}\dot{\theta}_{k}^{+}=4R\sqrt{\frac{2\theta_{k}}{3bg}}$$

In addition, it is observed numerically that the phase of rebound “delays” the restarting of the rocking movement to each collision. One then adopts a median value of the relationship between the time intervals between two successive collisions of: 0.77 .

The results which one then draws starting from this value are listed in table 2.3.

The vertical percussion is the integral over the duration of contact at the time of the collision of the vertical reactions of shock:

$I_y=\int_{t_0}^{t_1}f_y(t)\,dt\ \ (N.m)$ that one compared to $I_y=-\frac{4R^2-3b^2}{2R^2}Mb\,\omega_{-}$

As their exact obtaining by the digital simulation is delicate, one will test also the relationship with the horizontal percussion $|I_x|/|I_y|\approx 0.3212$, which is weaker than the coefficient of selected friction (one should not thus have slip).

2.1.3 Harmonic case of training

Under investigation, not restored. For more details, to refer to [bib2].

2.2 Sizes and results of reference

Here the list of the studied moments of impact and sizes calculated, under the rigid assumption body. One also compares with the results got by software LMGC90 (v2), of the University of Montpellier, employing one θ- diagram of temporal integration of speed for a step of time $\Delta t=10^{-4}\,s$, and a method of treatment of the contact of speed.

Phase Impact n°	Moments (S)	Vertical displacement (m) maximum of the corner	Vertical speed (m/S) maximum of the corner right before collision	Kinetic energy (J) maximum of the block right before collision
initialization	0.0000	0.003600 in A	0.0000	0.0000
$0 \Rightarrow 1$		0.002011 in O		
1	0.05440978		– 0.133572 in A	7.373981
LMGC90	0.0546	0.1898 in O (– 5.6%)	– 0.13325 (– 0.24%)	7.23255 (–1,918 %)
$1 \Rightarrow 2$		0.001123 in A		
2	0. 13574		– 0.099832 in O	4.119169
LMGC90	0.1324		– 0.10053 (0.71%)	4.04688 (1,756 %)
$2 \Rightarrow 3$		0.000628 in O		
3	0. 196529		– 0.074615 in A	2.301003
LMGC90	0.1929			2.25311 (– 2.081%)
$3 \Rightarrow 4$		0.000351 in A		
4	0. 241961		– 0.055767 in O	1.28536
LMGC90	0.2359			1.26174 (– 1,841 %)
$4 \Rightarrow 5$		0.000196 in O		
5	0.27592		– 0.04168 in A	0.718013
LMGC90	0.2693			

Table 2.1. Analytical solutions under the rigid assumption body, without rebound. Comparison with the solution obtained with software LMGC90 (v2), of the University of Montpellier.

Phase Impact n°	Moments (S)	Ratio angular velocities	Vertical reaction on the corner	Horizontal reaction on the corner	Percussion I_y vertical on the corner
$0 \Rightarrow 1$			3579.25 NR in O	– 1149.8 NR in O	
1	0.054409	0.7474			48.73 N.s in A
$1 \Rightarrow 2$			3579.25 NR in A	1149.8 NR in A	
2	0.13574	0.7474			36.42 N.s in O
$2 \Rightarrow 3$			3579.25 NR in O	– 1149.8 NR in O	
3	0.196529	0.7474			27.22 N.s in A
$3 \Rightarrow 4$			3579.25 NR in A	1149.8 NR in A	
4	0. 241961	0.7474			20.35 N.s in O
$4 \Rightarrow 5$			3579.25 NR in O	– 1149.8 NR in O	
5	0.27592	0.7474			15.21 N.s in A

Table 2.2. Analytical solutions under the rigid assumption body, without rebound.

Phase Impact n°	Moments (S)	Vertical displacement (m) maximum of the corner	Vertical speed (m/S) extreme of the corner right before collision	Kinetic energy (J) maximum of the block right before collision	Percussion I_y vertical on the corner
initialization	0.0000	0.003600 in A	0.0000	0.0000	
$0 \Rightarrow 1$					
1	0.05440978	0.002247 in O	– 0.13357229 in A	7.373981	48.73 N.s in A
$1 \Rightarrow 2$					
2	0.138201	0.001402 in A	– 0.1055221 in O	4.6021	37.52 N.s in O
$2 \Rightarrow 3$					
3	0. 202720	0.000875 in O	– 0.083362 in A	2.8722	28.89 N.s in A
$3 \Rightarrow 4$					
4	0. 252400	0.000546 in A	– 0.065856 in O	1.7925	22.24 N.s in O
$4 \Rightarrow 5$					
5	0.29065	0.000341 in O	– 0.052026 in A	1.1187	17.12 N.s in A

Table 2.3. Solutions interpreted analytically starting from average drainage efficiencies of shock estimated numerically with *Code_Aster*.

2.3 Uncertainties on the solution

Analytical solution obtained in rigid assumption of body without rebound; quasi-analytical elastic solution exploiting an energy value obtained by simulation, characterizing the restitution of shock.

2.4 Bibliographical references

1 F. VOLDOIRE: VHTR 2015: Graphite core seismic design: Non-linear dynamics of blocks; oscillations and contact-impacts. Bibliographical survey, analytical results. Technical note EDF/AMA HT-62/04/010/A, March 2005.

2 F. VOLDOIRE: VHTR 2015: Graphite core seismic design: Non-linear dynamics of blocks; oscillations and contact-impacts. First finite element modelling methodologies. Technical note EDF/AMA HT-62/05/018/A, February 2006.

3 [P. Chabrand, O. Chertier, and F. Dubois. Complementarity methods for multibody friction and contact problems in finite deformations. Int. J. Num. Meth. Eng., 51:553-578, 2001.

4 F. VOLDOIRE, Mr. Kham: Project OMERSI. Deliverable T342: CAS-test of structures tilting and slipping into Code_Aster, coupling Code_Aster LMGC90. CR-AMA-07.289, 2/2008.

3 Modeling B

3.1 Characteristics of modeling

One chooses a plane modeling, with finite elements 2D in plane constraints (modeling C_PLAN) and of the specific discrete elements, spring seat of shock.

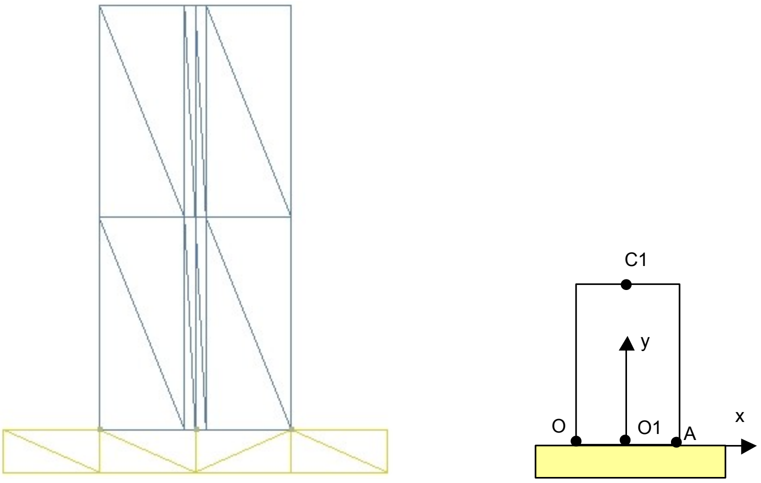

Figure 4.1-a: Modeling and grid

3.2 Characteristics of the grid

Made grid: 25 nodes, 34 meshs SEG2 and 20 meshs TRIA6 . Here the list of the groups of nodes and useful meshs in modeling:

Name groups	Contents
O	Low left lower node of the block and mesh-point
With	Node lower low right of the block and mesh-point
O1	Node lower medium of the block and mesh-point
C	Node higher medium of the block
CTBLO	Meshs SEG2 contour of the block
CONHAUT	Meshs SEG2 bases block
CONBAS	Meshs SEG2 of the table under the base of the block
BLOCK	Meshs of the block

A new grid is created ma_poi to add the 3 mesh-points POI1 on the nodes O, With and O1, which will be support of the discrete elements of spring DIS_T. The table is not part of the built model. One calls Mo the model associated with the problem.

3.3 Characteristics of the loadings

Contact-friction between block and table is treated by specific shock absorbers placed in O, With and O1. A material is affected DIS_CONTACT and of the characteristics DISCRETE and ORIENTATION with the elements of springs (to place them vertically). The selected characteristics are:

- in AFFE_CARA_ELEM :
  DISCRETE with CARA=‘K_T_D_N’, VALE= (30000000000.0, 10000000.0,0.0,),)
  CARA=‘A_T_D_N’, VALE= (0.1, 1.0, 0.0,),),),

ORIENTATION with CARA='VECT_X_Y', VALE= (0.0, - 1.0, 0.0, 1.0, 0.0, 0.0,),),);

- in DEFI_MATERIAU :
  DIS_CONTACT with RIGI_NOR=30000000000.0, RIGI_TAN=10000000.0, AMOR_NOR=5000000.0, COULOMB=0.9, GAME=0.0.

The normal stiffness is such as the 3 springs in parallel gives again the value of the spring determined with [§2.1].

Then, in DYNA_NON_LINE, the relation of behavior will be employed DIS_CHOC on these elements. The discrete elements, being three-dimensional, are blocked in z.

Gravity is simulated by a slope, precondition to the slope of the block, is operated by an imposed displacement. These phases are stabilized with their face value by a strong digital damping, precondition under investigation dynamic.

3.4 Characteristics of integration in time

One chooses a diagram, implicit in time, of Newmark, modified average acceleration.

For the phases of installation under the action of vertical gravity and of initialization of releasing (by shift in initial rotation of the block), one chooses a step of time of $0.0125\,s$, and a diagram of integration in time in α- method keyword " HHT ": ALPHA=-0.30, MODI_EQUI=' NON' for gravity of 0 to G (of $t=-2.0\ s$ with $t=-1.0\,s$), then one chose ALPHA=-0.60 for the phase of slope of the block (of $t=-1.25\,s$ with $t=0\ \ s$).

The initialization of releasing with $t=0\ \ s$ is done starting from a worthless speed in the block and of the displacement induced by the slope. For the phases of coasting flight (swinging), the step of time is regulated by $\Delta t=\sqrt{\dfrac{h}{50\,g}}$ where h is a dimension characteristic of the fall. One chooses: $\Delta t=0.00125\ \ s$. One takes the parameters of Newmark of average acceleration: $\alpha=0.$, via the keyword " HHT ".

For the phases including the collisions and the light immediately consecutive rebounds, induced by the penetration caused by the penalization of the contact (due to the elements of springs), one changes integration, while choosing a diagram of modified average acceleration: α- method " HHT ", with MODI_EQUI=' NON' or true diagram " HHT ", with MODI_EQUI=' OUI' and $\alpha=-0.1$ (i.e. by introducing a light digital damping), with a step of refined time $\Delta t_r\ =0.000025\,\mathrm{s}$. Thus, there is the succession:

N° impact	Interval (S)	method	value of	Pas de time
	0.00000 0.00250	α- method	$\alpha\ =- 0.1$	2.5 10^{-5} S
1	0.05375 0.05875	α- method	$\alpha\ =- 0.1$	2.5 10^{-5} S
2	0.13375 0.15000	α- method	$\alpha\ =- 0.1$	2.5 10^{-5} S
3	0.19250 0.21000	HHT	$\alpha\ =- 0.1$	2.5 10^{-5} S
4	0.23250 0.24250	HHT	$\alpha\ =- 0.1$	2.5 10^{-5} S
5	0.27000 0.27750	HHT	$\alpha\ =- 0.1$	2.5 10^{-5} S

One notes a light rebound shortly after each collision (and a light penetration). It is necessary so that simulation is correct that this rebound is integrated with the step of refined time. Moreover, the choice of the step of time is not independent of the value of the normal stiffness of shock and the normal damping of shock, cf [§ 4.3]. It is noted that the moments of collision arrive more precociously than with the method of contact of Lagrange not penalized, cf [§ 4.4].

The final moment is: $0.285\,s$, in order to have the first 5 collisions.

Non-linearities are integrated with the method of Newton, with tangent matrix reactualized with each iteration. The necessary precise details on balance are: `resi_glob_rela`=10^{-6}, `resi_glob_maxi`=10^{-3} (what is rather strict!). The iteration count of maximum Newton is: 15.

One envisages subdivision only for the initial phase of slope of the block.
The full number of step of time for the study of swinging is of 2140 pas.

3.5 Sizes tested and results

3.5.1 Case of release-swinging starting from a position inclined at rest

Several values are tested not per moments, but by extreme values on a time interval, because that is more relevant. Tests of not-regression, with weak tolerance, were added in order to track the evolutions of the algorithms.

Values of **displacements** (`depl`), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
D_y group_no O (m)	Value min on [0, 0.33] S	0.000000	– 3.0079 10^{-6}	0.00 %
D_y group_no O (m)	Value max on [0, 0.33] S	Rigid body: 0.002011 With retiming: 0.00225	0.00198832	– 1.13% – 11.51%
D_y group_no A (m)	Value min on [0, 0.33] S	0.00000	– 4.94 10^{-6}	0.00 %
D_y group_no A (m)	Value max on [0, 0.33] S	0.00360	0.0035958	– 0.117%

Values of **speeds** (`quickly`), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
V_y group_no O (m/s)	Value min on [0, 0.33] S	Rigid body: – 0.09983 With retiming: – 0.10552	– 0.098934	– 0.90% – 6.24%
V_y group_no O (m/s)	Value min on [0.17, 0.33] S	Rigid body: – 0.074614 With retiming: – 0.08336	– 0.051964	– 30.36% – 37.66 %
V_y group_no A (m/s)	Value min on [0, 0.33] S	– 0.13357	– 0.132746	– 0.617%

Values of **reactions** with the corners: on the discrete elements of springs (`sief_elga`), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
NR grou_no O (NR)	Value max on [0, 0.05] S	3580.0	3604.36	0.680%
NR grou_no A (NR)	Value max on [0.06, 0.12] S	3580.0	3611.13	0.870%
VY grou_no O (NR)	Value min on [0, 0.05] S	– 1150.0	-1152.71	0.236%
VY grou_no A (NR)	Value max on [0.07, 0.12] S	1150.0	1258.71	9.45%

Values of**kinetic energy** on the block alone (`total`), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference (J)	*Aster*	Relative error %
Kinetic energy (J)	Value max on [0.0, 0.28] S	Rigid body: 7.37398	7.28314	– 1.23%
Kinetic energy (J)	Value max on [0.10, 0.28] S	Rigid body: 4.1192 With retiming: 4.6021	4.04539	– 1.79% – 12.1%
Kinetic energy (J)	Value max on	Rigid body: 2.3010	2.17093	– 5.653%

	[0.15, 0.28] S	With retiming: 2.8722		– 24.42%
Kinetic energy (J)	Value max on [0.22, 0.28] S	Rigid body: 1.2854 With retiming: 1.7925	1.11604	– 13.18% – 37.7%
Kinetic energy (J)	Value max on [0. 26 , 0.28] S	Rigid body: 0.7180 With retiming: 1. 1187	0.57368	– 20.1% – 48.7%

Values of **moments of collision**, Type of reference `'ANALYTICAL'` :

Identification	energy (J)	Reference (S)	*Aster*	Relative error %
Moment 1 era collision (S)	Value max on [0.0, 0.33] S	Rigid body: 0.0544098	0.054375	– 0,064 %
Moment 2 ème collision (S)	Value max on [0.10, 0.33] S	Rigid body: 0.13574 With retiming: 0.13820	0.134900	– 0.619% – 2.388%
Moment 3 ème collision (S)	Value max on [0.15, 0.33] S	Rigid body: 0.196529 With retiming: 0.20272	0.19365	– 1.465% – 4.474%
Moment 4 ème collision (S)	Value max on [0.22, 0.33] S	Rigid body: 0.241961 With retiming: 0.25240	0.23560	– 2.629% – 6.66%
Moment 5 ème collision (S)	Value max on [0. 28 , 0.33] S	Rigid body: 0.27592 With retiming: 0.29065	0.26570	– 3.70% – 8.58%

3.6 Remarks

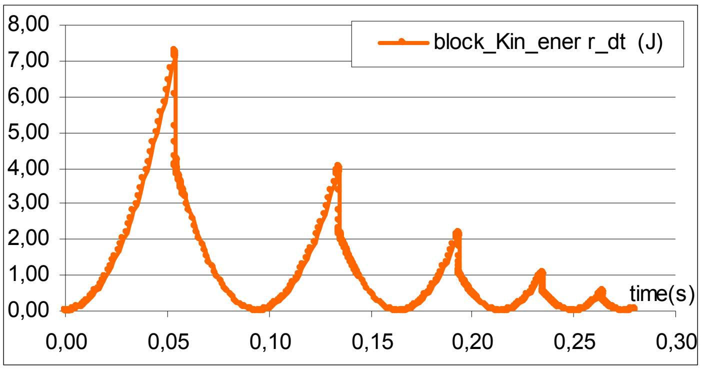

Figure 4.6-a: Evolution of the kinetic energy (J) of the block.

4 Modeling C

4.1 Characteristics of modeling

One chooses a plane modeling, with finite elements 2D in plane constraints (modeling `C_PLAN`). Contact-friction between block and table is treated by the method "continuous hybrid formulation", cf [R5.03.52] and an implicit diagram of integration, formulated in displacement.

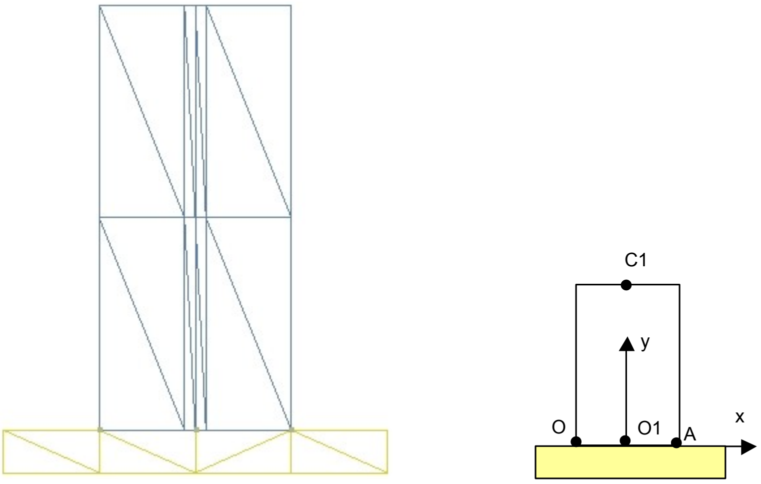

Figure 5.1-a: Modeling and grid

4.2 Characteristics of the grid

One calls `Mo` the model (see fig. 5.1-a) associated with the problem. Made grid: 25 nodes, 34 meshs `SEG2` and 12 meshs `QUA4`. Here the list of the groups of nodes and useful meshs in modeling:

Name groups	Contents
`O`	Low left lower node of the block
`With`	Node lower low right of the block
`O1`	Node lower medium of the block
`C`	Node higher medium of the block
`S0`	Node of the table in contact with O
`S1`	Node of the table in contact with O1
`S2`	Node of the table in contact with A
`CTGROUND`	Nodes of the bottom of the table
`CTBLO`	Meshs SEG2 contour of the block
`CONHAUT`	Meshs SEG2 bases block
`CONBAS`	Meshs SEG2 of the table under the base of the block
`BLOCK`	Meshs of the block
`GROUN`	Meshs of the table

4.3 Characteristics of the loadings

Contact-friction between block and table is treated by the method continues, pairing being by master-slave method, the selected normal being that of the Master:

- Group meshs Masters: `CONBAS` ;
- Group meshs slaves: `CONHAUT`.

A geometrical reactualization is adopted.
The table is blocked in X at the S1 point, and in there on its basis `CTGROUND` .

Gravity is simulated by a slope, precondition to the slope of the block, is operated by an imposed displacement. These phases are stabilized with their face value by a strong digital damping, precondition under investigation dynamic.

4.4 Characteristics of integration in time

One chooses a diagram, implicit in time, of Newmark, modified average acceleration.

For the phases of installation under the action of vertical gravity and of initialization of releasing (by shift in initial rotation of the block), one chooses a step of time of $0.0125\,s$, and a diagram of integration in time in α- method keyword " `HHT` ": `ALPHA=-0.30, MODI_EQUI=' NON'` for gravity of 0 to G (of $t=-2.0\ s$ with $t=-1.0\,s$), then one chose `ALPHA=-0.60` for the phase of slope of the block (of $t=-1.25\,s$ with $t=0\ \ s$).
The initialization of releasing with $t=0\,s$ is done starting from a worthless speed in the block and of the displacement induced by the slope. For the phases of coasting flight (swinging), the step of time is regulated by $\Delta t=\sqrt{\dfrac{h}{50\,g}}$ where h is a dimension characteristic of the fall. One chooses: $\Delta t=0.0025\ \ s$. One takes the parameters of Newmark of average acceleration: $\alpha=0.$, via the keyword " `HHT` ".

For the phases including the collisions, one changes integration, while choosing a diagram of modified average acceleration: α- method " `HHT` ", with `MODI_EQUI=' NON'`, with $\alpha=-0.2$ and a step of refined time $\Delta t_r=0.000010\ \ s$. Thus, there is the succession:

N° impact	Interval (S)	α - method	Pas de time
	0.0000 - 0.0025	$\alpha=-0.1$	10^{-5} S
1	0.0525 - 0.0600	$\alpha=-0.2$	10^{-5} S
2	0.1400 - 0.1450	$\alpha=-0.2$	10^{-5} S
3	0.2050 - 0.2150	$\alpha=-0.2$	10^{-5} S
4	0.2575 - 0.2650	$\alpha=-0.2$	10^{-5} S
5	0.2975 - 0.3100	$\alpha=-0.2$	10^{-5} S

One notes a light rebound shortly after each collision. It is necessary so that simulation is correct that this rebound is integrated with the step of refined time.

The final moment is: $0.32\,s$, in order to have the first 5 collisions. However in order to have a less expensive test, one stops after the second collision: one stops in practice at the moment $0.145\,s$.

Like `SOLVEUR`, `METHOD 'LDLT'` do not go with the method continues, one takes `METHOD 'MULT_FRONT'`, with `RENUM 'MDA'`, because `RENUM 'MONGREL'` can be expensive in System time according to the machines.

4.5 Sizes tested and results

Several values are tested not per moments, but by extreme values on a time interval, because that is more relevant. Tests of not-regression, with weak tolerance, were added in order to track the evolutions of the algorithms. One limits oneself here to three phases of coasting flight.

Values of **displacements** (`depl`), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
D_y group_no O (m)	Value min on [0, 0.33] S	0.000000	– 1.8916210 $^{-10}$	0.00 %
D_y group_no O (m)	Value max on [0, 0.33] S	Rigid body: 0.002011 With retiming: 0.00225	0.00233214	15.97% 3.79%
D_y group_no A (m)	Value min on	0.00000	– 1.0265 10 $^{-10}$	0.00 %

	[0, 0.33] S			
D_y group_no A (m)	Value max on [0, 0.33] S	0.00360	0.0035999	– 0.06%

Values of **speeds** (`quickly`), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
V_y group_no O (m/s)	Value min on [0, 0.33] S	Rigid body: – 0.09983 With retiming: – 0.10552	– 0.107117	7.30% 4.16%
V_y group_no A (m/s)	Value min on [0, 0.33] S	– 0.13357	– 0.132807	– 0.571%

Values of **reactions** with the corners (`CONT_NOEU`), component normal (RN) and tangential (X-ray), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
RN grou_no O (NR)	Value max on [0, 0.05] S	3580.0	3599.52	0.545%
RN grou_no A (NR)	Value max on [0.06, 0.12] S	3580.0	3597.66	0.493%
X-ray grou_no O (NR)	Value min on [0, 0.05] S	– 1150.0	– 1137.17	– 1.12%
X-ray grou_no A (NR)	Value max on [0.07, 0.12] S	1150.0	1141.47	– 0.742%

Values of **percussions** with the corners (`CONT_NOEU` integrated around the moments of collision), Type of reference `'ANALYTICAL'` :

Identification	Moment (S)	Reference	*Aster*	Relative error %
RN grou_no A (NR)	Value integrated on [0.05440, 0.05455]	48.73	41.9301	– 13.9%
X-ray grou_no A (NR)	Value integrated on [0.05440, 0.05455]	15.65	11.2919	– 27.8%

Values of**kinetic energy** on the block alone (`total`):

Identification	Moment (S)	Reference (J)	*Aster*	Relative error %
Kinetic energy (J)	Value max on [0.0, 0.33] S	Rigid body: 7.37398	7.28973	– 1.14%
Kinetic energy (J)	Value max on [0.1, 0.33] S	Rigid body: 4.1192 With retiming: 4.6021	4.74225	15.1% 3.04%

Values of **moments of collision, Type of reference `'ANALYTICAL'` :**

Identification	energy (J)	Reference (S)	*Aster*	Relative error %
Moment 1 era collision (S)	Value max on [0.0, 0.33] S	Rigid body: 0.0544098	0.054400	– 0,018 %
Moment 2 ème collision (S)	Value max on [0.10, 0.33] S	Rigid body: 0.13574 With retiming: 0.13820	0.14154	5.05% 2.42%

4.6 Remarks

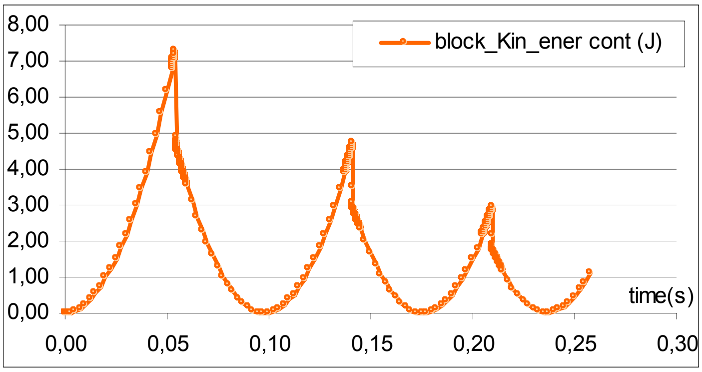

Figure 5.6-a: Evolution of the kinetic energy (J) block (method continues in displacements).

5 Summary of the results

Results got with *Code_Aster*, in dynamics with diagram of temporal integration implicit, expressed on displacements, are about in conformity with those expected by comparison with the analytical solution: error lower than 12% over the moments of collision (on all five modelled collision). One gets results contained in the beach of the two possible analytical models: that of the rigid body without any rebound and that adjusted by taking account of an elastic rebound, from where a stronger drainage efficiency of shock.

One notes differences in prediction between the methods of treatment of the contact selected. The method of penalization (by discrete springs of shock), which functions only by also envisaging damping located, cause a drop in the drainage efficiency of shock, therefore approach plus the values of the moments of collision of the analytical model of rigid body without rebound.

These values are very close to those obtained by a software dedicated, LMGC90, developed at the University of Montpellier, resting on one θ - diagram of temporal integration of speed and proposing two methods to treat the contact (concerned, of speed), which give here the both same predictions.

On the other hand, with the “exact” method of contact of *Code_Aster* – continues – associated with a diagram of implicit temporal integration of the Newmark-HHT family, expressed on displacements, one rather gets results governed by the analytical solution adjusted by taking account of the rebound.

Certain results are slightly different from one machine to another: about % on energies, moments of collision and speeds from the 3ème rebound. The reactions are more stable. The method by discrete springs of shock seems less sensitive to the choice of the object computer.