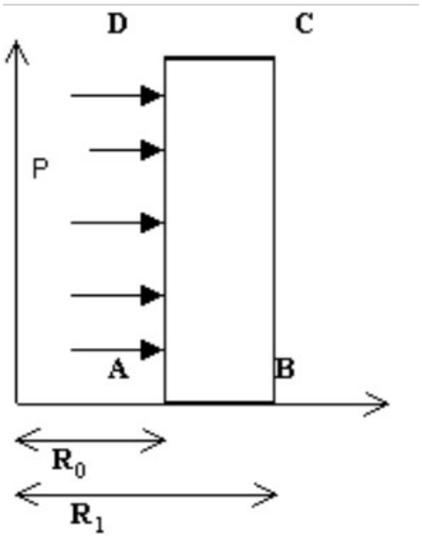
# SSNA104 - Hollow roll subjected to a pressure, linear viscoelasticity

**Summary:**

This CAS-test makes it possible to validate the laws of LEMAITRE and LEMA_SEUIL established in *Code_Aster* in the case of linear viscoelastic behavior. The found results are compared with an analytical solution.

# 1 Problem of reference

## 1.1 Geometry




Dimensions of the cylinder:

| | |
|---|---|
| $R_0$ | $1\,m$ |
| $R_1$ | $2\,m$ |

**Figure 1.1-a: Cut of the hollow roll and loading**

## 1.2 Properties of materials

Young modulus: $E=1\,MPa$
Poisson's ratio: $\nu=0.3$

Law of LEMAITRE :

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\lambda^{\frac{1}{m}}}\right)^n \text{ with } \frac{1}{K}=1\ ,\ \frac{1}{m}=0\ ,\ n=1$$

Law LEMA_SEUIL :

$$g(\sigma,\lambda,T)=A\left(\frac{2}{\sqrt{3}}\sigma\right)\Phi \text{ with } A=\frac{\sqrt{3}}{2}\ ,\ \Phi=1 \text{ on all the grid}$$

$$S=10^{-10}$$

being given the value of the various parameters materials, the two laws are absolutely identical and can thus be compared with the same analytical solution.

## 1.3 Boundary conditions and loading

**Boundary conditions:**
The cylinder is blocked in $DY$ on the sides $[AB]$ and $[CD]$.

**Loading:**
The cylinder is subjected to a pressure interns on $[DA]P0=1.\mathrm{E}-3\ MPa$

# 2 Reference solutions

## 2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions $I(t)$ and $K(t)$ so that strains and stresses can be written:

$$\varepsilon(t)=(I+K)*\frac{d\sigma(t)}{d\tau}-K*\frac{d(Tr(\sigma(t)))}{d\tau}\boldsymbol{I}_3$$

where $I_3$ indicate the matrix identity of row 3

and $*$ the product of convolution: $(f*g)(t)=\int_0^t f(t-\tau)\,g(\tau)\,d\tau$

One finds $I(t)=\frac{1}{E}+kt$ , $K(t)=\frac{\nu}{E}+\frac{1}{2}kt$

The pressure is imposed $P_0$ at the moment $t=0$ , the internal pressure is worth $p(t)=H(t)P_0$ where $H(t)=\begin{cases}0 & si \quad t-\tau<0\\ 1 & si \quad t-\tau\geq 0\end{cases}$ with in this case $\tau=0$

One uses the transform of Laplace Carson $f^+(n)=L(f(t))=n\int_0^\infty f(t)e^{-nt}dt$

From where $p^+=P_0$

The solution of the elastic problem are equivalent is:

$$\sigma^+=\begin{pmatrix}\gamma\left(1-\frac{r_1^2}{r^2}\right) & 0 & 0\\ 0 & \gamma\left(1+\frac{r_1^2}{r^2}\right) & 0\\ 0 & 0 & \sigma_Z^+\end{pmatrix} \text{ where } \gamma=\frac{P_0 r_0^2}{r_1^2-r_0^2}$$

One determines $\sigma_Z^+$ by the condition on $\varepsilon_Z^+$ data by the boundary conditions:

$$\varepsilon_Z^+=0=(I^++K^+)\sigma_Z^+-K^+(2\gamma+\sigma_Z^+)=I^+\sigma_Z^+-2K^+\gamma$$

From where $\sigma_Z^+=\gamma\left(1+\frac{(2\nu-1)p}{p+Ek}\right)$ .

One finds by the transform of opposite Laplace $\sigma_z(t)=\gamma(1-(1-2\nu)e^{-Eht})$, in the same way by applying the transform of Laplace reverses on $\sigma_r$ and $\sigma_\theta$, one finds

$$\sigma^+=\begin{pmatrix} \gamma\left(1-\frac{r_1^2}{r^2}\right) & 0 & 0 \\ 0 & \gamma\left(1+\frac{r_1^2}{r^2}\right) & 0 \\ 0 & 0 & \gamma(1-(1-2\nu)e^{-Eht}) \end{pmatrix}$$

One from of deduced:

$$\dot{\varepsilon_V}=\begin{pmatrix} \frac{3}{2}k\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-\frac{r_1^2}{r^2}\right) & 0 & 0 \\ 0 & \frac{3}{2}k\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-\frac{r_1^2}{r^2}\right) & 0 \\ 0 & 0 & -k\gamma((1-2\nu)e^{-Eht}) \end{pmatrix}$$

and while integrating with $\varepsilon_V(0)=0$ ;

$$\dot{\varepsilon_V}=\begin{pmatrix} \frac{3}{2}\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-k\frac{r_1^2}{r^2}t\right) & 0 & 0 \\ 0 & \frac{3}{2}\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-k\frac{r_1^2}{r^2}t\right) & 0 \\ 0 & 0 & -\gamma\frac{(1-2\nu)}{E}(1-e^{-Eht}) \end{pmatrix}.$$

One from of deduced radial displacement

$$w(r,t)=r\gamma\left[\frac{1}{E}\left[(1+\nu)\frac{r_1^2}{r^2}+\frac{1-2\nu}{2}(3-(1-2\nu)e^{-Ekt})\right]+\frac{3}{2}k\frac{r_1^2}{r^2}t\right]$$

## 2.2 Results of reference

Displacement $DX$ on the node $B$ and constraints $SIXX$, $SIYY$ and $SIZZ$ in $B$

## 2.3 Uncertainty on the solution

$0\%$ : analytical solution

## 2.4 Bibliographical references

PH. BONNIERES: Two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

# 3 Modeling A

## 3.1 Characteristics of modeling

The problem is modelled in axisymetry.

## 3.2 Characteristics of the grid

1000 meshs QUAD4

## 3.3 Sizes tested and results

| Identification | Moments | Reference |
|---|---|---|
| $DX(B)$ | 0.9 | 2.14498 E-3 |
| $SIXX(B)$ | 0.9 | 0.0 |
| $SIYY(B)$ | 0.9 | 2.7912 E-4 |
| $SIZZ(B)$ | 0.9 | 6.66 E–4 |

# 4 Modeling B

## 4.1 Characteristics of modeling

The problem is modelled in axisymetry

## 4.2 Characteristics of the grid

1000 meshs QUAD4

## 4.3 Sizes tested and results

| Identification | Moments | Reference |
|---|---|---|
| $DX(B)$ | 0.9 | 2.14498 E-3 |
| $SIXX(B)$ | 0.9 | 0.0 |
| $SIYY(B)$ | 0.9 | 2.7912 E-4 |
| $SIZZ(B)$ | 0.9 | 6.66 E–4 |

# 5 Summary of the results

Results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the grid.