
SSNA105 - Hollow roll subjected to a pressure, linear viscoelasticity, contact

Summary:

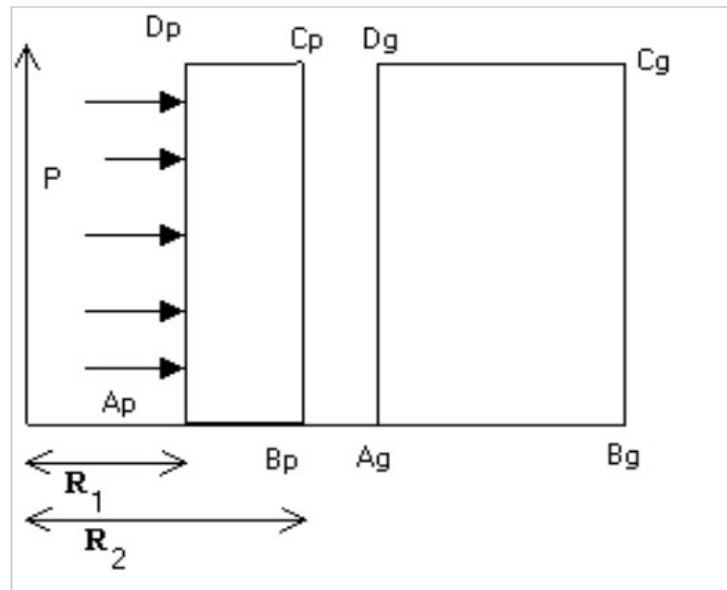
This CAS-test makes it possible to validate the law of LEMAITRE established in *Code_Aster* in the case of linear viscoelastic behavior. The found results are compared with an analytical solution.

This test takes again same modeling as the CAS-test SSNA104A to which one adds a cylinder (pastille) and one treats the contact.

1 Problem of reference

1.1 Geometry

The diagram is not on the scale, the difference between the two cylinders was amplified for a better visibility.



R_1	0.82
R_2	0.92
R_3	1.
R_4	2.

1.2 Properties of materials

The pastille is made up of an elastic material, the sheath consists of a viscoelastic material.
The elastic data coincide for two materials.

Young modulus: $E = 1 \text{ MPa}$

Poisson's ratio: $\nu = 0.3$

Law of LEMAITRE:

$$g(\sigma, \lambda, T) = \left(\frac{1}{K} \frac{\sigma}{\lambda^m} \right)^n \quad \text{with} \quad \frac{1}{K} = 1, \quad \frac{1}{m} = 0, \quad n = 1$$

1.3 Boundary conditions and loading

Boundary conditions:

The cylinder is blocked in DY on the sides $[AP, BP]$, $[AG, BG]$ and $[CP, PD][CG, PG]$.

Loading:

The cylinder is subjected to a pressure interns on $[DP, AP]$, this pressure is calculated so that at the moment $t=0$, the sheath has the same behavior as the cylinder modelled in the test ssna104a.

$$p_1(t) = \begin{cases} \frac{r_3}{r_2} - 1 \frac{E(r_2^2 - r_1^2)}{2r_1^2(1-\nu)} & \text{si } -1 \leq t \leq 0 \\ A[B(r_3 - r_2 + C(D + Ge^{-Ekt} + Ht)) + K] & \text{si } 0 < t \leq 5 \end{cases}$$

with

$$A = \frac{r_2^2 - r_1^2}{2r_1^2(1-\nu)}, \quad B = \frac{E}{r_2(1+\nu)}, \quad C = \frac{P_0 r_3^3}{r_4^2 - r_3^2} \text{ with } P_0 = 1.E-3 \text{ MPa, pressure of the test ssna104a.}$$

$$D = \frac{1}{E} \left[(1+\nu) \frac{r_4^2}{r_3^2} + \frac{3}{2}(1-2\nu) \right], \quad G = -\frac{(1-2\nu)^2}{2E}, \quad H = \frac{3}{2} k \frac{r_4^2}{r_3^2}, \quad K = \frac{P_0 r_2^2}{r_2^2 - r_1^2} \left[1 - 2\nu + \frac{r_1^2}{r_2^2} \right]$$

One treats the contact between the two cylinders.

2 Reference solutions

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [bib1].

Phase without contact

One wants to find the value of $p_1(t)$ to apply to the internal wall of the pastille for which the contact takes place.

For the pastille, one finds:

$$\sigma = \begin{pmatrix} \gamma \left(1 - \frac{r_2^2}{r^2}\right) & 0 & 0 \\ 0 & \gamma \left(1 + \frac{r_2^2}{r^2}\right) & 0 \\ 0 & 0 & 2\nu\gamma \end{pmatrix} \quad \text{where } \gamma = \frac{p_1(t)r_1^2}{r_2^2 - r_1^2}$$

$$\varepsilon_\theta = \frac{1+\nu}{E} \gamma \left[1 - 2\nu + \frac{r_2^2}{r^2}\right] = \frac{w}{r}.$$

The condition of being written contact: $w(r_3) - w(r_2) = 0$, one has $r_3 - r_2 = r_2 \frac{2(1+\nu)\gamma}{E} (1-\nu)$

$$\text{From where } \gamma = \left(\frac{r_3}{r_2} - 1\right) \frac{E}{2(1-\nu)^2}$$

$$p_1 \text{ lim} = \left(\frac{r_3}{r_2} - 1\right) \frac{E(r_2^2 - r_1^2)}{2r_1^2(1-\nu)^2}.$$

Phase with contact

One wants that as from the moment $t=0$, the sheath has same behaviour as in the test ssna104a.

When there is contact, one a:

$$w_P(r_2) = W_G(r_3) + r_3 - r_2,$$

thus by recovering the value of displacements in the test ssna104, one must obtain:

$$w_P(r_2) = r_3 - r_2 + \frac{p_0 r_3^3}{r_4^2 - r_3^2} \left\{ \frac{1}{E} \left((1+\nu) \frac{r_4^2}{r_3^2} + \frac{1-2\nu}{2} (3 - (1-2\nu)e^{-Ekt}) \right) + \frac{3}{2} k \frac{r_4^2}{r_3^2} t \right\}.$$

The stress field of the pastille is given by

$$\sigma = \begin{pmatrix} \gamma_1 \left(1 - \frac{r_2^2}{r^2}\right) - \gamma_0 \left(1 - \frac{r_2^2}{r^2}\right) & 0 & 0 \\ 0 & \gamma_1 \left(1 + \frac{r_2^2}{r^2}\right) - \gamma_0 \left(1 + \frac{r_2^2}{r^2}\right) & 0 \\ 0 & 0 & \sigma_z \end{pmatrix}$$

with $\gamma_1 = \frac{p_1 r_1^2}{r_2^2 - r_1^2}$ and $\gamma_0 = \frac{p_0 r_1^2}{r_2^2 - r_1^2}$.

Like $\varepsilon_z = \frac{1+\nu}{E} \sigma_z - \frac{\nu}{E} (2(\gamma_1 - \gamma_0) + \sigma_z) = 0$, one finds: $\sigma_z = 2\nu(\gamma_1 - \gamma_0)$.

One thus has $\varepsilon_\theta = \frac{1+\nu}{E} \sigma_\theta - \frac{\nu}{E} (\sigma_r + \sigma_\theta + \sigma_z) = \frac{1+\nu}{E} \left[(1-2\nu)(\gamma_1 - \gamma_0) + \gamma_1 \frac{r_2^2}{r^2} - \gamma_0 \frac{r_1^2}{r^2} \right] = \frac{w}{r}$

$w_p(r_2) = \frac{1+\nu}{E} r_2 \left[2(1-\nu)\gamma_1 - \gamma_0 \left(1 - 2\nu + \frac{r_1^2}{r_2^2}\right) \right]$, one finds $p_1(t)$ given by the formula a little higher.

2.2 Results of reference

Displacement DX on the node B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 Bibliographical references

- PH. BONNIERES, two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modeling A

3.1 Characteristics of modeling

The problem is modelled in axisymetry. The contact is treated by the discrete formulation.

3.2 Characteristics of the grid

600 meshes QUAD4
160 meshes SEG2

3.3 Sizes tested and results

Identification	Moments	Reference	Tolerance
$DX(B)$	0.9	2.1400 E-3	RELATIVE – 0,95%
$SIXX(B)$	0.9	0	ABSOLUTE – 9,6E-6
$SIYY(B)$	0.9	2.7912 E-4	RELATIVE – 3.40%
$SIZZ(B)$	0.9	6.6000 E-4	RELATIVE – 2.00%

4 Modeling B

4.1 Characteristics of modeling

The problem is modelled in axisymetry. The contact is treated by the formulation continues.

4.2 Characteristics of the grid

600 meshes QUAD4
16 0 meshes SEG2

4.3 Sizes tested and results

Identification	Moments	Reference	Tolerance (%)
$DX(B)$	0.9	2.14 E-3	1.2%
$SIXX(B)$	0.9	0.0	
$SIYY(B)$	0.9	2.7912 E-4	3.5%
$SIZZ(B)$	0.9	6.66 E-4	2.5%

5 Summary of the results

Results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the grid. The two methods of taking into account of the contact (discrete formulation and continues) give the same results.

Code_Aster

Version
default

Titre : SSNA105 - Cylindre creux soumis à une pression, vi[...]
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