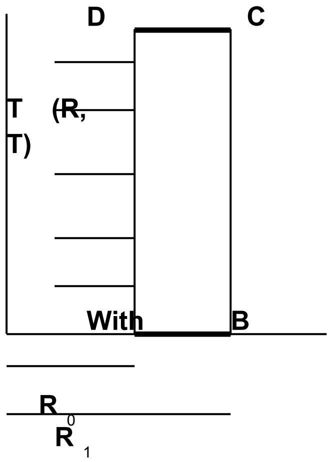
SSNA106 - Subjected hollow roll with a behavior thermoviscoelastic

Summary:

This CAS-test makes it possible to validate the law of LEMAITRE established in *Code_Aster* in the case of linear behavior thermoviscoelastic. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



R_0	$1\,m$
R_1	$2\,m$

1.2 Properties of materials

Young modulus: $E=1\,MPa$
Poisson's ratio: $\nu=0.3$
Dilation coefficient: $\alpha=0.7$

Law of LEMAITRE:

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\lambda^{\frac{1}{m}}}\right)^n \text{ with } \frac{1}{K}=1\,,\ \frac{1}{m}=0\,,\ n=1$$

1.3 Boundary conditions and loading

Boundary conditions:
The cylinder is blocked in DY on the sides $[AB]$ and $[CD]$.

Loading:
The cylinder is subjected to a field of temperature $T(r,t)=t\,r^2$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions $I(t)$ and $K(t)$ so that strains and stresses can be written:

$$\varepsilon(t) = (I+K) * \frac{d\sigma(t)}{d\tau} - K * \frac{d\left(Tr(\sigma(t))\right)}{d\tau}\mathbf{I}_3 + \alpha T(r,t)\mathbf{I}_3$$

where I_3 indicate the matrix identity of row 3

and $*$ the product of convolution: $(f*g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau$

The thermoelastic problem are equivalent, via the transform of Laplace is:

$$\begin{cases} \varepsilon^+ = (I^+ + K^+)\sigma^+ - K^+ Tr(\sigma^+)\mathbf{I_3} + \dfrac{\alpha r^2}{p}\mathbf{I_3} \\ \sigma_r^{+}{}' = \dfrac{d\sigma_r^+}{dr} = \dfrac{1}{r}\left(\sigma_\theta^+ - \sigma_r^+\right) \\ \varepsilon_z^+ = 0 \\ \left(r\varepsilon_\theta^+\right) = \varepsilon_r^+ \end{cases}$$

By eliminating the sign "+":

$$\begin{cases} \sigma_r' + \dfrac{1}{r}(\sigma_r - \sigma_\theta) = 0 \\ (I+K)\sigma_z - K(\sigma_r + \sigma_\theta + \sigma_z) + \dfrac{\alpha r^2}{p} = 0 \\ \left(r\left[(I+K)\sigma_\theta - K\left(\sigma_r + \sigma_\theta + \sigma_z\right) + \dfrac{\alpha r^2}{p}\right]\right)' = (I+K)\sigma_r - K\left(\sigma_r + \sigma_\theta + \sigma_z\right) + \dfrac{\alpha r^2}{p} \end{cases}$$

maybe,

$$\begin{cases} \sigma_r' + \dfrac{1}{r}(\sigma_r - \sigma_\theta) = 0 \\ \sigma_z = \dfrac{K}{I}(\sigma_r + \sigma_\theta) - \dfrac{\alpha r^2}{pI} \\ \left(r\left[(I+K)\sigma_\theta - \dfrac{(I+K)K}{I}\left(\sigma_r + \sigma_\theta\right) + \dfrac{\alpha r^2}{p}\right]\right)' = (I+K)\sigma_r - \dfrac{(I+K)K}{I}\left(\sigma_r + \sigma_\theta\right) + \dfrac{(I+K)}{I}\dfrac{\alpha r^2}{p} \end{cases}$$

$$(I+K)\sigma_\theta + r\left[(I+K)\sigma_\theta - \frac{(I+K)K}{I}(\sigma_r+\sigma_\theta) + \frac{(I+K)}{I}\frac{\alpha r^2}{p}\right] = (I+K)\sigma_r$$

According to the equilibrium equation, one has $\sigma_\theta = r\sigma_r' + \sigma_r$, one obtains:

$$(I+K)\sigma_r' + r\left[(I+K)(r\sigma_r'+\sigma_r) - \frac{(I+K)K}{I}(2\sigma_r + r\sigma_r') + \frac{(I+K)}{I}\frac{\alpha r^2}{p}\right] = 0,$$

$$\left[(2\sigma_r + r\sigma_r') + \frac{\alpha r^2}{p(I-K)}\right] = 0,$$

$2\sigma_r + r\sigma_r' = A + \dfrac{\alpha r^2}{p(K-I)}$ what while integrating compared to R gives:

$$\sigma_r = \frac{A}{2} + \frac{B}{r^2} + \frac{\alpha r^2}{4p(K-I)},$$

boundary conditions $\sigma_r(r_0) = \sigma_r(r_1) = 0$ give:

$$A = -\frac{\alpha}{2p(K-I)}(r_0^2 + r_1^2)$$

$$B = \frac{\alpha r_0^2 r_1^2}{4p(K-I)}$$

One thus has by taking again the initial notations:

$$\begin{cases} \sigma_r^+ = \dfrac{\alpha}{4p(I^+ - K^+)}\left(r_0^2 + r_1^2 - r^2 - \dfrac{r_0^2 r_1^2}{r^2}\right) \\ \sigma_\theta^+ = \dfrac{\alpha}{4p(I^+ - K^+)}\left(r_0^2 + r_1^2 - 3r^2 + \dfrac{r_0^2 r_1^2}{r^2}\right) \\ \sigma_z^+ = \dfrac{\alpha}{p(I^+ - K^+)}\left(\dfrac{K^+}{I^+}\dfrac{(r_0^2 + r_1^2)}{2} - r^2\right) \end{cases}$$

Maybe, by taking the opposite transform,

$$\sigma = \begin{bmatrix} \frac{\alpha}{2k}(1-e^{-bt})\left[r_0^2 + r_1^2 - r^2 - \frac{r_0^2 r_1^2}{r^2}\right] & 0 & 0 \\ 0 & \frac{\alpha}{2k}(1-e^{-bt})\left[r_0^2 + r_1^2 - 3r^2 + \frac{r_0^2 r_1^2}{r^2}\right] & 0 \\ 0 & 0 & \frac{\alpha}{k}\left[(1-e^{-bt})(r_0^2 + r_1^2 - 2r^2) + \frac{r_0^2 + r_1^2}{r^2}(1-e^{-Ekt})\right] \end{bmatrix}$$

One from of deduced ε_V and w :

$$w(r,t) = \frac{1-2\nu}{Ek}\alpha r\left[(1-e^{-bt})\left[r_0^2 + r_1^2 - \frac{r_0^2 r_1^2}{r^2}\right] + (1-e^{-Ekt})\left[\frac{-(r_0^2 + r_1^2)}{4}\right] + \frac{3Ekt}{4(1-2\nu)}\left[\frac{r_0^2 r_1^2}{r^2} + r^2\right]\right]$$

2.2 Results of reference

Displacement DX on the node B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 Bibliographical references

PH. BONNIERES, two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modeling A

3.1 Characteristics of modeling

The problem is modelled in axisymetry

3.2 Characteristics of the grid

120 meshs QUAD4

3.3 Sizes tested and results

Identification	Moments	Reference	Tolerance %
$DX(B)$	0.24	1,110	0.1%

4 Summary of the results

Results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the grid.