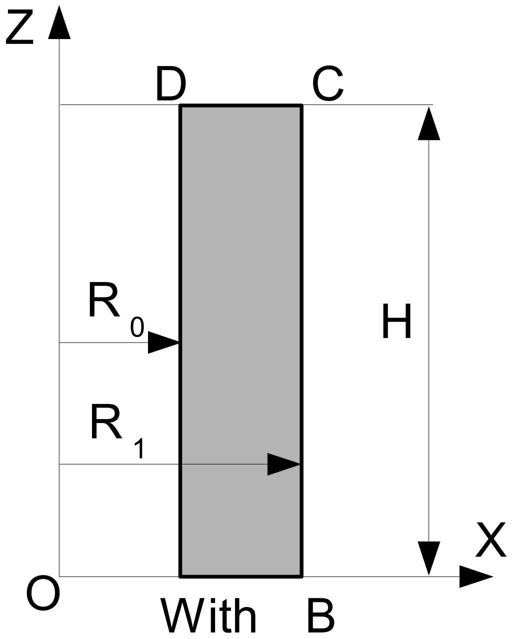
SSNA107 – Hollow roll in nonlinear viscoelasticity

Summary:

This CAS-test makes it possible to validate the law of LEMAITRE established in nonlinear Code_Aster in the case of viscoelastic behavior. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



Geometry of the cylinder (m) :

$R_0=1$
$R_1=1.02$
$H=1$

1.2 Properties of material

Rubber band

- Young modulus: $E=1.0\times10^6 Pa$
- Poisson's ratio: $\nu=0.3$

LEMAITRE

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\left(\frac{1}{\lambda^m}\right)}\right)^n \text{ with } n=2 \text{ ; } \frac{1}{K}=1 \text{ ; } \frac{1}{m}=0$$

CIN1_CHAB

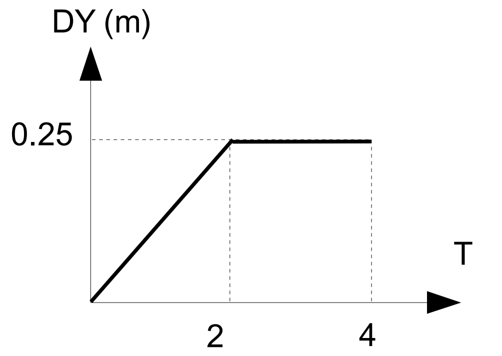
$R\_0=0.$
$R\_I=0.$
$B=0.$
$C\_I=0.$
$K=0.$
$W=0.$
$G\_0=0.$

$A\_I=0.$

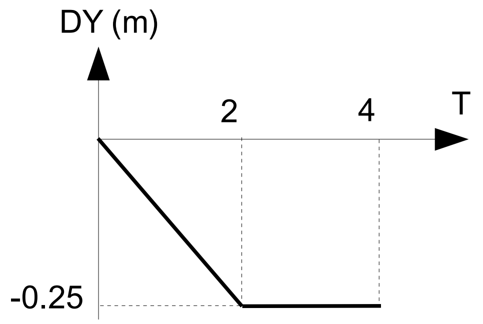
1.3 Boundary conditions and loadings

Imposed displacement (m) :

Dimensioned CD



Dimensioned AB



2 Reference solution

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document $[1]$

The tensor of constraints is written:

$$\sigma=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \sigma_z\end{pmatrix}$$

Because of the loading, one a:

$$\begin{cases}\varepsilon_z-\varepsilon_{vz}=\dfrac{\sigma_z}{E}\\ \varepsilon_\theta-\varepsilon_{v\theta}=-\dfrac{\nu}{E}\sigma_z\\ \varepsilon_r-\varepsilon_{vr}=-\dfrac{\nu}{E}\sigma_z\end{cases} \quad \text{and} \quad \dot{\varepsilon}_v=\frac{3}{2}g(\sigma*)\frac{\sigma^D}{\sigma*}$$

thus

$$\begin{cases}\dot{\varepsilon}_{vz}=g(\sigma_z)\\ \dot{\varepsilon}_{v\theta}=-\dfrac{1}{2}g(\sigma_z)\\ \dot{\varepsilon}_{vr}=-\dfrac{1}{2}g(\sigma_z)\end{cases}$$

If $t\le t_0$, one has $\varepsilon_z=\frac{\varepsilon_0}{t_0}t$,

That is to say $a=\sqrt{\frac{\varepsilon_0}{t_0}}$

One obtains $\varepsilon_z=a^2t$

While replacing, one finds:

$$\dot{\varepsilon}_{vz}=g((a^2t-\varepsilon_{vz})E)$$

One poses $E=1$ and $z=a^2t-\varepsilon_{vz}$, one obtains: $\dot{z}=a^2-z^2$

While integrating with $z(0)=0$ one obtains:

$$z=a\tanh(at)$$

For $t\le t_0$

$$\begin{cases}\sigma_r=\sigma_\theta=0\\ \sigma_z=a\tanh(at)\\ \varepsilon_r=\varepsilon_\theta=a[(\frac{1}{2}-\nu)\tanh(at)-\frac{1}{2}at]\\ \varepsilon_z=a^2 t\\ w=ar[(\frac{1}{2}-\nu)\tanh(at)-\frac{1}{2}at]\end{cases}$$

If $t\ge t_0$

$$\varepsilon_z=a^2 t_0$$
$$\dot{\varepsilon}_{vz}=g(a^2 t_0-\varepsilon_{vz})=(a^2 t_0-\varepsilon_{vz})^2$$

What gives while integrating:

$$\varepsilon_{vz}=a^2 t_0-\frac{1}{\frac{1}{a\tanh(at_0)}+t-t_0}$$

One thus has with the final one

$$\begin{cases}\sigma_r=\sigma_\theta=0\\ \sigma_z=\frac{1}{\frac{1}{a\tanh(at_0)}+t-t_0}\\ \varepsilon_r=\varepsilon_\theta=(\frac{1}{2}-\nu)\frac{1}{\frac{1}{a\tanh(at_0)}+t-t_0}-\frac{1}{2}a^2 t_0\\ \varepsilon_z=a^2 t_0\\ w=r\left[(\frac{1}{2}-\nu)\frac{1}{\frac{1}{a\tanh(at_0)}+t-t_0}-\frac{1}{2}a^2 t_0\right]\end{cases}$$

2.2 Reference variables

- Displacement DX with the node B

- Constraints $SIXX$, $SIYY$ and $SIZZ$ with the node B

2.3 Results of reference

Size	Not	Moments	Reference
DX	B	4	-0.2109
$SIXX$	B	4	$0.$
$SIYY$	B	4	0.2616
$SIZZ$	B	4	$0.$

2.4 Uncertainty on the solution

Analytical solution

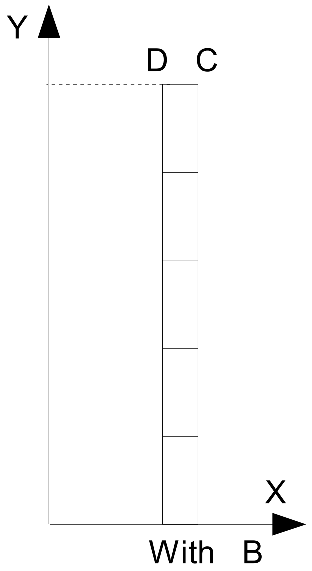
2.5 Bibliographical references

[1] PH. BONNIERES, Mr. ZIDI: Introduction of viscoplasticity into the module of thermomechanics of Cyrano3 : Principle, description and validation, Note HI-71/8334.

3 Modeling A

3.1 Characteristics of modeling A

Modeling `AXIS`
Viscoelastic relation of behavior of `LEMAITRE`



3.2 Characteristics of the grid

Many nodes	12		
Many meshs	17	That is to say:	
		`SEG2`	12
		`QUAD4`	5

Groups of nodes:
A, B, C, D

Groups of meshs:

$MAIL$	: surface	$ABCD$
DAB	: segment	AB
DBC	: segment	BC
DCD	: segment	CD
DDA	: segment	DA

3.3 Sizes tested and results

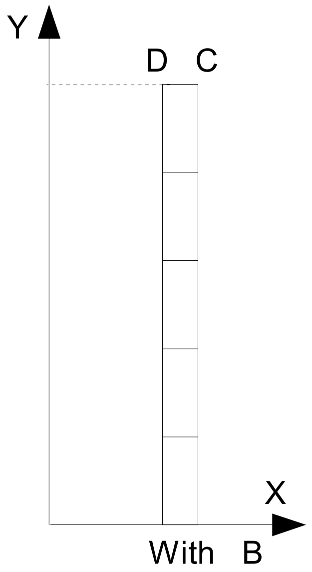
Size	Not	Moments	Reference	Aster	Variation %
DX	B	4	-0.2109	-0.2109	0.001%
$SIXX$	B	4	$0.$	4,82E-9	-
$SIYY$	B	4	0.2616	0.2616	0.002%

SIZZ	*B*	4	0.	4,82E-9	-

4 Modeling B

4.1 Characteristics of modeling B

Modeling `AXIS`
Viscoelastic relation of behavior of `VISC_CIN1_CHAB`



4.2 Characteristics of the grid

Many nodes	12			
Many meshs	17	That is to say:		
			`SEG2`	12
			`QUAD4`	5

Groups of nodes:
A, B, C, D

Groups of meshs:

$MAIL$	: surface	$ABCD$
DAB	: segment	AB
DBC	: segment	BC
DCD	: segment	CD
DDA	: segment	DA

4.3 Sizes tested and results

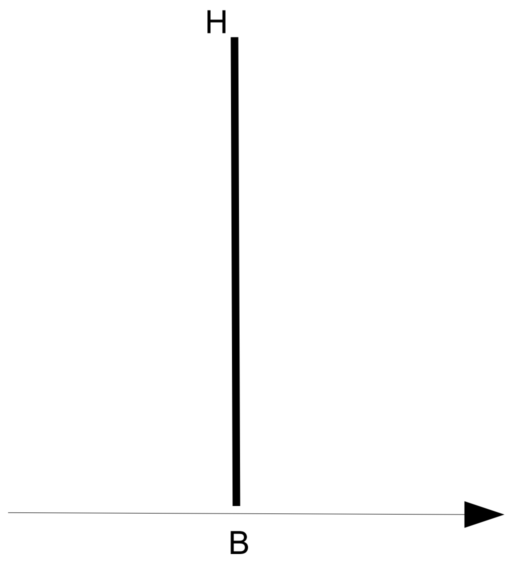
Size	Not	Moments	Reference	Aster	Variation %
DX	B	4	-0.2109	-0.2109	0.026%
$SIXX$	B	4	$0.$	1,55E-10	-
$SIYY$	B	4	0.2616	0.2616	0.130%

SIZZ	*B*	4	0.	1,55E-10	-

5 Modeling C

5.1 Characteristics of modeling A

Modeling COQUE_AXIS
Viscoelastic relation of behavior of LEMAITRE



5.2 Characteristics of the grid

Many nodes 5
Many meshs 2 SEG3

Groups of nodes:
B, H

5.3 Sizes tested and results

Size	Not	Moments	Reference	Aster	Variation %
DX	B	4	-0.2109	-0.20877887227	0.0001%
$SIYY$	B	4	$0.$	2.1120599907551 E-10	4.61E-09
$SIXX$	B	4	0.2616	$0.21644117387 2$	0.13%
$SIZZ$	B	4	$0.$	4,82E-9	-

6 Summary of the results

The results calculated by Code_Aster are in excellent agreement with the analytical solutions.