SSNA116 - Triaxial compression test with the model of Hoek-Brown modified into axisymmetric

Summary

This test makes it possible to validate the elastoplastic law of behavior of Hoek-Brown modified in rock mechanics. It is about a triaxial compression test for which calculations are carried out only on the solid part of the ground in pure mechanics.

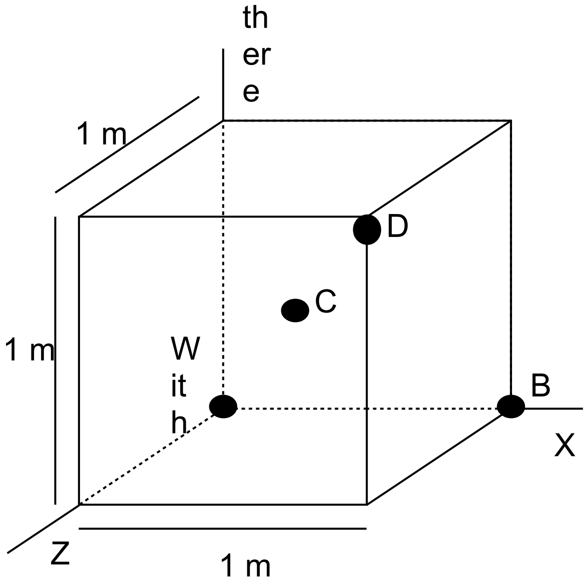
Two levels of containment are applied: $5\,MPa$ and $12\,MPa$. Parameters ϕ^{end}, ϕ^{rup} and ϕ^{res} are taken equal (what returns to a constant voluminal plastic deformation): one can in this case calculate an analytical solution with the problem and thus compare the results got with *Code_Aster* with this reference solution.

For reasons of symmetry, one is interested only in the eighth of a sample subjected to a triaxial compression test. Modeling is axisymmetric.

1 Problem of reference

1.1 Geometry

A cube of dimension here is considered $1\text{m}\times 1\text{m}\times 1\text{m}$.



Coordinates of the points (in m):

	A	B	C	D
x	0	1	0.5	1
y	0	0	0.5	1
z	0	0	0.5	1

1.2 Properties of material

Parameters of the elastic law of behavior:

$E=4500\,MPa$

$\nu=0.3$

Parameters of the law of Hoek-Brown modified:

$\gamma^{rup}=0.005$

$\gamma^{res}=0.017$

$(S\sigma_c^2)^{end}=225\,MPa^2$

$(S\sigma_c^2)^{rup}=482.5675\,MPa^2$

$(m\sigma_c)^{end}=13.5\,MPa$

$(m\sigma_c)^{rup}=83.75\,MPa$

$\beta=3\,MPa$

$\phi^{end}=15°$

$\phi^{rup}=15°$

$\phi^{res}=15°$

$\alpha = 3.3$

1.3 Initial conditions, with the limits and loading

The test breaks up into two phases:

1) Initially, one brings the sample in a homogeneous state $\sigma_{xx}^0=\sigma_{yy}^0=\sigma_{zz}^0$. For that, the corresponding confining pressure is imposed on the front faces ($z=1$), side right-hand side ($x=1$) and higher ($y=1$), while displacements are taken worthless on the faces postpones ($u_z|_{z=0}=0$), side left ($u_x|_{x=0}=0$) and lower ($u_y|_{y=0}=0$).
2) Once the homogeneous state obtained, displacements are maintained blocked on the faces postpones, side left and lower and the confining pressure is always imposed on the front faces and side right-hand side. A displacement is imposed on the higher face ($u_y(t)$) in order to obtain a deformation ε_{yy} equalize with – 25% starting from the beginning of the second phase, by constant increments of deformation $\Delta\varepsilon_{yy}=-2.5E-4$.

2 Reference solution

2.1 Calculation of the reference solution

One places here in the case of a triaxial compression test for which the constraints of containment are applied in the directions x and z and for which the direction of imposed deformation is the direction y . One supposes moreover than the parameter η is independent of the parameter of work hardening γ , it is - with - to say $\phi^{end}=\phi^{rup}=\phi^{res}$: it is then possible to calculate an analytical solution with the problem. The criterion of plasticity and flow are written:

$$(\sigma_3-\sigma_1)-\sqrt{S\sigma_c^2(\gamma)-m\sigma_c(\gamma)\sigma_3}-b(\gamma)\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)=0$$

$$\dot{\varepsilon}_1^p=\dot{\lambda}(\eta-1)=\frac{\eta-1}{\eta+1}\dot{\gamma}$$

$$\dot{\varepsilon}_3^p=\dot{\varepsilon}_2^p=\dot{\lambda}(\eta+\frac{1}{2})=\frac{2\eta+1}{2(\eta+1)}\dot{\gamma}$$

$$\dot{\varepsilon}_v^p=3\eta\dot{\lambda}=\frac{3\eta}{\eta+1}\dot{\gamma}$$

An increasing situation of loading is considered for which the preceding equations can be written in a nonincremental way:

$$\varepsilon_1^p=\frac{\eta-1}{\eta+1}\gamma,\quad \varepsilon_3^p=\varepsilon_2^p=\frac{2\eta+1}{2(\eta+1)}\gamma,\quad \varepsilon_v^p=\frac{3\eta}{\eta+1}\gamma$$

The relations of elasticity give:

$$\varepsilon_1-\varepsilon_1^p=\frac{1}{E}(\sigma_1-\sigma_1^0)-\frac{2\nu}{E}(\sigma_3-\sigma_3^0)$$

$$\varepsilon_3-\varepsilon_3^p=\frac{1-\nu}{E}(\sigma_3-\sigma_3^0)-\frac{\nu}{E}(\sigma_1-\sigma_1^0)$$

i.e.:

$$\varepsilon_1 - \frac{\eta - 1}{\eta + 1}\gamma = \frac{1}{E}(\sigma_1 - \sigma_1^0) - \frac{2\nu}{E}(\sigma_3 - \sigma_3^0)$$

$$\varepsilon_3 - \frac{2\eta + 1}{2(\eta + 1)}\gamma = \frac{1 - \nu}{E}(\sigma_3 - \sigma_3^0) - \frac{\nu}{E}(\sigma_1 - \sigma_1^0)$$

with σ_3^0 and σ_1^0 values of σ_1 and σ_3 at the beginning of the loading. It thus remains to calculate σ_1 according to γ by using the criterion of plasticity to obtain γ, σ_1 and ε_3.

1er case: $\gamma \le \gamma^{rup}$

While noting $S\sigma_c^2(\gamma) = A_1 + \gamma A_2$ and $m\sigma_c(\gamma) = B_1 + \gamma B_2$ where A_1, A_2, B_1 and B_2 are given in the reference material of the law of behavior, γ is solution of the polynomial of degree 2:

$$\left(\frac{\eta - 1}{\eta + 1}\right)^2 \gamma^2 - \left(2\varepsilon_1\left(\frac{\eta - 1}{\eta + 1}\right) + \frac{A_2 - \sigma_3 B_2}{E^2}\right)\gamma + \varepsilon_1^2 - \frac{A_1 - \sigma_3 B_1}{E^2} = 0,$$

with γ in the interval $[0, \gamma^{rup}]$.

2ème case: $\gamma^{rup} \le \gamma \le \gamma^{res}$

By taking again the notations of the reference material of the law of Hoek-Brown modified for has, D, C and σ_3^{b-d}, γ is solution of the polynomial of degree 2:

$$\frac{a}{E}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)\gamma^2 + \left(-\left(\frac{\eta - 1}{\eta + 1}\right) + \frac{d}{E}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)\right)\gamma + \varepsilon_1 + \frac{\sqrt{(S\sigma_c^2)^{rup} - \sigma_3 (m\sigma_c)^{rup}}}{E} + \frac{c}{E}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right) = 0$$

avec γ dans l'intervalle $\left[\gamma^{rup}, \gamma^{res}\right]$

3ème case: $\gamma^{res} \le \gamma$

In this case, σ_1 is constant:

$$\sigma_1 = \sigma_3 - \sqrt{(S\sigma_c^2)^{res} - \sigma_3 (m\sigma_c)^{res}} - b^{res}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)$$

and $\gamma = \dfrac{\sigma_1 - \sigma_1^0}{E} - \varepsilon_1$.

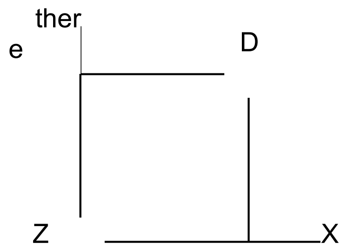
2.2 Results of reference

Constraints $\sigma_{xx}(\sigma_3)$, $\sigma_{yy}(\sigma_1)$ and $\sigma_{zz}(\sigma_3)$ at the point D.
Displacements $\varepsilon_{xx}(\varepsilon_3)$ and $\varepsilon_{yy}(\varepsilon_1)$ at the point D.

3 Modeling A

3.1 Characteristics of modeling

Modeling $2D$ axisymmetric



Cutting: $1m$ in height, $1m$ in width
Loading of phase 1: $\sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 = -5\ MPa$ (confining pressure)
Boundary conditions: $u_x|_{x=0} = u_y|_{y=0} = u_z|_{z=0} = 0$

3.2 Characteristics of the grid

Many nodes: 4
Many meshs and types: 1 QUAD4 and 4 SEG2

3.3 Sizes tested and results

Localization	Sequence number	Constraint (MPa)	Reference solution
Not D	12	σ_{xx}	-5
	70	σ_{xx}	-5
	12	σ_{zz}	-5
	70	σ_{zz}	-5
	12	σ_{yy}	-18.50
	16	σ_{yy}	-22.5675778
	32	σ_{yy}	-30.8797526
	41	σ_{yy}	-34.9342281
	42	σ_{yy}	-32.9136722
	46	σ_{yy}	-26.8215156
	52	σ_{yy}	-22.7560224
	70	σ_{yy}	-20.721512

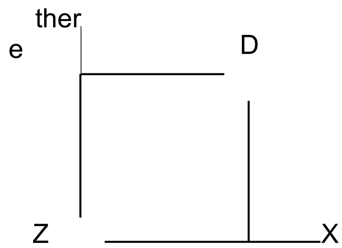
Localization	Sequence number	Deformation	Reference solution
Not D	12	ε_{xx}	0.9 E-3
	16	ε_{xx}	1.24644 E-3

32	ε_{xx}	3.48682 E-3
41	ε_{xx}	4.81373 E-3
42	ε_{xx}	5.22653 E-3
46	ε_{xx}	6.66403 E-3
52	ε_{xx}	8.27551 E-3
70	ε_{xx}	12.01865 E-3
12	ε_{yy}	-0,003
70	ε_{yy}	-0.0175

4 Modeling B

4.1 Characteristics of modeling

Modeling $2D$ axisymmetric



Cutting: $1\mathrm{m}$ in height, $1\mathrm{m}$ in width
Loading of phase 1: $\sigma_{xx}^{0}=\sigma_{yy}^{0}=\sigma_{zz}^{0}=-12\,\mathrm{MPa}$ (confining pressure)
Boundary conditions: $u_x\big|_{x=0}=u_y\big|_{y=0}=u_z\big|_{z=0}=0$

4.2 Characteristics of the grid

Many nodes: 4
Many meshs and types: 1 `QUAD4` and 4 `SEG2`

4.3 Sizes tested and results

Localization	Sequence number	Constraint (MPa)	Reference solution
Not D	16	σ_{xx}	-12
	80	σ_{xx}	-12
	16	σ_{zz}	-12
	80	σ_{zz}	-12
	16	σ_{yy}	-30
	20	σ_{yy}	-33.4287301
	36	σ_{yy}	-43.5095082
	49	σ_{yy}	-50.4230084
	52	σ_{yy}	-48.4775526
	56	σ_{yy}	-46.4935733
	60	σ_{yy}	-45.0479008
	70	σ_{yy}	-43.1174944
	80	σ_{yy}	-42.8023313

Localization	Sequence	Deformation	Reference solution

	number		
Not D	16	ε_{xx}	1.2 E-3
	20	ε_{xx}	1.61504 E-3
	36	ε_{xx}	3.66549 E-3
	49	ε_{xx}	5.46863 E-3
	52	ε_{xx}	6,265 E-3
	56	ε_{xx}	7.26131 E-3
	60	ε_{xx}	8.19982 E-3
	70	ε_{xx}	10.36527 E-3
	80	ε_{xx}	12.35726E-3
	16	ε_{yy}	-0,004
	80	ε_{yy}	-0.02

5 Summary of the results

The got results make it possible to validate the model of Hoek-Brown modified integrated in *Code_Aster* in the typical case of a constant voluminal plastic deformation.