# SSNA302 - Simply supported circular plate subjected to pressure

**Summary:**

This test consists in applying a transverse pressure to a circular plate, simply supported and made up of an elastic material. It is intended particularly to study the taking into account of nonthe geometrical linearities in the absence of initial curve when the transverse stiffness is only due to the effect of plate.

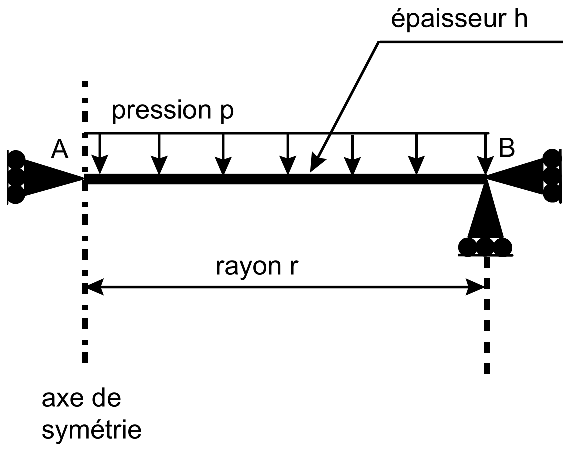
Modeling is made with voluminal elements of type HEXA20 and PENTA15 and surface elements of type QUAD8 and TRIA6 for the application of the pressure.

The reference is software the SAMCEF software. One has for information the results resulting from the theory of the thin hulls.

Under the keyword `BEHAVIOR`, the option `GREEN` is compared with the option `PETIT_REAC`.

# 1 Problem of reference

## 1.1 Geometry




## 1.2 Material properties

Isotropic elastic material:

$E=200000\,Mpa$
$\nu=0.3$

## 1.3 Boundary conditions and loadings

Not $B$ : $u_x=0$
$u_y=0.$

A transverse pressure is applied $p$ on the plate: $p=222.72\,N/mm^2$ . This pressure corresponds to an arrow $w_0$ of $1.5\,mm$ .

Ray $r=10\,mm$
Thickness $h=1\,mm$

The problem is axisymmetric.

# 2 Reference solution

## 2.1 Reference solution

The reference is software the SAMCEF software. One has, for information, in the paragraph [§2.2], the theoretical results related to an assumption of type thin hull. Then, the results got with the SAMCEF software are presented according to whether one chooses an assumption of type thick hull or standard volume. It is the latter which is taken into account for the evaluation of *Code_Aster*.

## 2.2 Analytical solution and results of reference

The following formula gives the arrow $w_0$ in the center of the plate:

$$\frac{w_0}{h}+A\left(\frac{w_0}{h}\right)^3=\frac{Bp}{E}\left(\frac{r}{h}\right)^4 \quad \text{with } A=1.852 \quad \text{and} \quad B=0.696$$

The constraints with mid thickness are worth:

$$\sigma_{rr} = \alpha_r E \frac{w_0}{r^2}$$

$$\sigma_{\theta\theta} = \alpha_t E \frac{w_0}{r^2}$$

The constraints in lower skin are worth

$$\sigma_{rr}' = \beta_r E \frac{w_0 h}{r^2}$$

$$\sigma_{\theta\theta}' = \beta_t E \frac{w_0 h}{r^2}$$

The coefficients are worth:

In the center of the plate:

$$\alpha_r=\alpha_t=0.905$$
$$\beta_r=\beta_t=1.778$$

At the edge of the plate:

$$\alpha_r=0.610$$
$$\alpha_t=0.183$$
$$\beta_r=0$$
$$\beta_t=0.755$$

For a pressure of $222.72\,MPa$, the arrow $w_0$ is worth $1.5\,mm$ and the following constraints are obtained:

| Position | $\sigma_{rr}$ $(MPa)$ | $\sigma_{\theta\theta}$ $(MPa)$ | $\sigma_{rr}'$ $(MPa)$ | $\sigma_{\theta\theta}'$ $(MPa)$ |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| Center | 4072.5 | 4072.5 | 5334.0 | 5334.0 |

These results correspond to an assumption of thin hull.

The following table shows the results got by the SAMCEF software for a modeling of type thick hull and standard volume.

| Identification | Thick hull | Volume |
|---|---|---|
| Arrow $w0 \ (mm)$ | – 1.43041 E-3 | – 1.441838 E-3 |
| SIXX $(MPa)$ center, mid thickness | 3899.88 | 3850.88 |
| SIYY $(MPa)$ center, mid thickness | 3899.53 | 3850.91 |
| SIXX $(MPa)$ center, skin inf | 8085.81 | 8133.60 |
| SIYY $(MPa)$ center, skin inf | 8083.32 | 8133.65 |
| SIXX $(MPa)$ $r=R/2$ , mid thickness | 3596.91 | 3512.79 |
| SIYY $(MPa)$ $r=R/2$ , mid thickness | 3056.37 | 2947.55 |
| SIXX $(MPa)$ $r=R/2$ , skin inf | 7798.18 | 7815.73 |
| SIYY $(MPa)$ $r=R/2$ , skin inf | 7264.69 | 7307.04 |

The values of the constraints are values by element extrapolated with the nodes.

The results got with the SAMCEF software are close one to the other.

The variations of results between the mean theory hull and calculation finite element using the assumption thick hull are important.

One chooses to take as reference the voluminal calculation obtained by the SAMCEF software.

## 2.3 Bibliographical references

1) Theory of Plates and Shells, Timoshenko S.P., 2Nd edition, p 412

# 3 Modeling A

## 3.1 Characteristics of modeling A

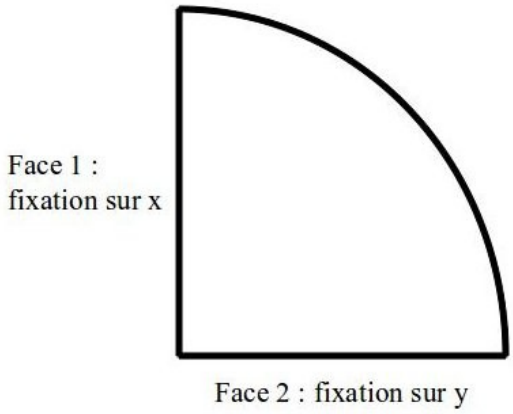



**Figure 3.1-3.1-a**

Only one quarter of plate is modelled. One introduces conditions of symmetry on the two faces shown above.

Moreover one fixes according to $x, y, z$ all nodes of the edge located at semi thickness.

In order to most accurately represent the assumptions of thin hull, one introduces linear constraints on the degrees of freedom of the nodes of the edge. Those Ci are written:

for two nodes $i$ and $j$ located on both sides of the average layer:

$$u_i+u_j=0$$
$$v_i+v_j=0$$

where U and v indicate displacements along the axes $x$ and $y$.

The syntax used in the data file of *Code_Aster* is the following one:

LIAISON_DDL (NODE: $N_i$ $N_j$ ) , DDL: ('DX', 'DX') , COEF_MULT: (1, 1), COEF_IMPO: 0

One applies the pressure linearly by using 6 increments.

## 3.2 Characteristics of the grid

| | |
|---|---|
| Many nodes: | 2091 |
| Many meshs and types: | 368 HEXA20, 28 PENTA15<br>92 QUAD8, 7 TRIA6 |

## 3.3 Values tested

| Identification | Reference |
|---|---|
| | |

| | |
|---|---|
| Arrow $w0$ $(mm)$ | – 1.441838E-3 |
| SIXX $(MPa)$ center, mid thickness | 3850.9 |
| SIYY $(MPa)$ center, mid thickness | 3850.9 |
| SIXX $(MPa)$ center, skin inf | 8133.6 |
| SIYY $(MPa)$ center, skin inf | 8133.6 |
| SIXX $(MPa)$ $r=R/2$ , mid thickness | 3512.8 |
| SIYY $(MPa)$ $r=R/2$ , mid thickness | 2947.5 |
| SIXX $(MPa)$ $r=R/2$ , skin inf | 7815.7 |
| SIYY $(MPa)$ $r=R/2$ , skin inf | 7307.0 |

## 3.4 Remarks

One obtains the same precision on the results by using only one increment of loading.

# 4 Modeling B

## 4.1 Characteristics of modeling B

Idem modeling A, but by treating nonthe geometrical linearities by the keyword BEHAVIOR option PETIT_REAC.

## 4.2 Characteristics of the grid

Many nodes: 2091
Many meshs and types: 368 HEXA20, 28 PENTA15
92 QUAD8, 7 TRIA6

## 4.3 Values tested

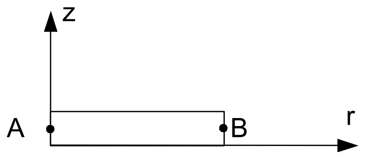
| Identification | Reference | *Aster* | % difference |
|---|---|---|---|
| Arrow $w0$ $(mm)$ | – 1.441838E-3 | – 1.3692E-3 | – 5,036 |
| SIXX $(MPa)$ center, mid thickness | 3850.9 | 4219.3 | 9,568 |
| SIYY $(MPa)$ center, mid thickness | 3850.9 | 4219.4 | 9,569 |
| SIXX $(MPa)$ center, skin inf | 8133.6 | 7950.5 | – 2,251 |
| SIYY $(MPa)$ center, skin inf | 8133.6 | 7950.5 | – 2,252 |
| SIXX $(MPa)$ $r=R/2$, mid thickness | 3512.8 | 3838.4 | 9,268 |
| SIYY $(MPa)$ $r=R/2$, mid thickness | 2947.5 | 3314.2 | 12,440 |
| SIXX $(MPa)$ $r=R/2$, skin inf | 7815.7 | 7593.9 | – 2,839 |
| SIYY $(MPa)$ $r=R/2$, skin inf | 7307.0 | 7123.7 | – 2,509 |

## 4.4 Remarks

One notes an important difference between the reference solution and the solution provided by *Code_Aster*.
It is checked that this variation tends towards 0 when one increases the number of increments.

# 5 Modeling C

## 5.1 Characteristics of modeling C




- axisymmetric modeling
- boundary conditions:

$$B : \quad DX=0 \quad DY=0$$

As for modeling A (3D), of the linear constraints are introduced for better representing a kinematics of hull. They relate to the nodes of the edge external of the disc. If $i$ and $j$ 2 nodes on both sides of the average layer indicate, they are written:

$u_i+u_j=0$ where $u$ indicate radial displacement

## 5.2 Characteristics of the grid

Many nodes: 149
Many meshs and types: 40 QUAD8, 10 SEG3

## 5.3 Values tested

| Identification | Reference |
|---|---|
| Arrow $w0$ $(mm)$ | – 1,430 |
| SIXX $(MPa)$ center, mid thickness | 3900 |
| SIZZ $(MPa)$ center, mid thickness | 3900 |

## 5.4 Remarks

In the absence of an axisymmetric calculation of reference the SAMCEF software, one is based on calculation in thick hull (always the SAMCEF software) and one does not compare that the arrow and the constraints with the center with mid thickness.

# 6 Summary of the results

The performances in time calculation and precision of the results are satisfactory while using the keyword BEHAVIOR option GREEN. On the other hand, treatment of nonthe geometrical linearities by the keyword BEHAVIOR option PETIT_REAC provides results rather far away from the reference solution by adopting a discretization in 6 pas de time which leads to costs in already important times calculation.