SSNL106 - Elastoplastic beam in traction and pure inflection

Summary:

This test validates elastoplasticity in a right beam in traction and inflection, for a perfect elastoplastic behavior or with linear work hardening as well as options for the postprocessing of the beams.

- Static analysis,
- Elastic behavior,
- Elastoplastic behavior,
- 3 sections: rectangular, circular full, hollow circular,
- 1 types of work hardening: linear (VMIS_ISOT_LINE)

Modelings make it possible to test the elements POU_D_E (only in elasticity), POU_D_TGM, POU_D_EM, TUYAU_3M (3 and 4 nodes) COQUE_3D.

The test makes it possible to validate the operation of the integration of these laws of behavior and of the algorithm of resolution until complete plasticization of the beam.

one of modelings makes it possible to test the operation of DYNA_NON_LINE on a static calculation quasi - of traction of a beam modelled in POU_D_TGM.

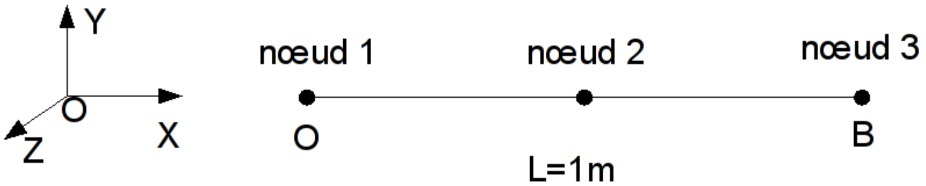
Moreover, modeling G is used to test the operation of the keyword factor ETAT_INIT of STAT_NON_LINE with the field of structure STRX_ELGA.

1 Problem of reference

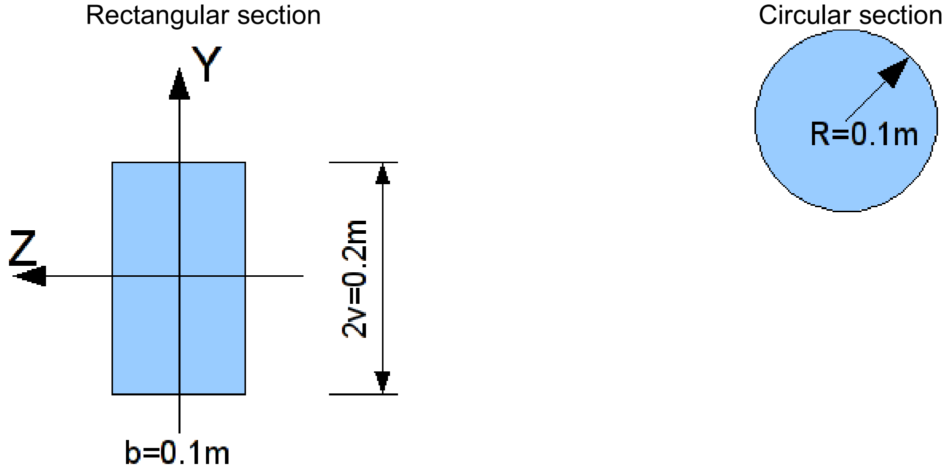
1.1 Geometry

The geometrical values are expressed in meters.

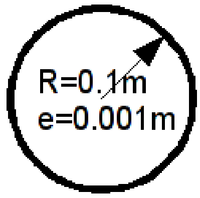
Right beam length $L=1$, of direction x .



One calculates simultaneously 2 types of section:



For modeling D, one calculates 1 section of thin tube:



1.2 Material properties

$E=2.10^{11}\,Pa$
$\nu=0.3$
ECRO_LINE :

$SY=\sigma_y=150\,10^6\,Pa$
H = D_SIGM_EPSI = $2\,10^9\,Pa$ or 0

1.3 Boundary conditions and loadings

Embedding in O

Displacement imposed in B

$$DX^e=\frac{L.\sigma_y}{E}=0.75 10^{-3} m$$

DX vary DX^e with $3DX^e$

Rotation imposed in B

$DRZ^e=0.75 10^{-2} m$

DRZ vary DRZ^e with $20\times DRZ^e$ then decrease until $-2\times DRZ^e$

Note:

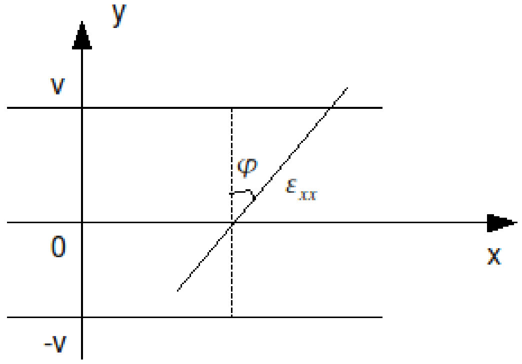
In pure inflection, MZ and DRZ do not depend on x . Curve $\varphi=\frac{d(DRZ)}{dx}=DRZ(B)$

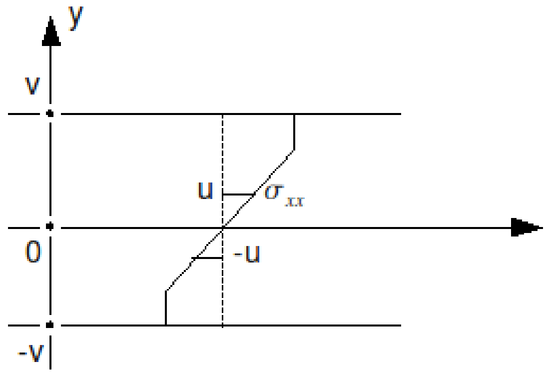
2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Pure inflection - linear Work hardening

Analytical solution:





$\varepsilon_{xx} = \varphi \quad y \qquad \varphi$: curve

Calculation of the moment by:

$$M(u) = \int_s \sigma_{xx}(y) . y \; ds$$

$\sigma_{xx} = E \, \varepsilon_{xx}$ for $0 \leq |y| \leq u$

$$\sigma_{xx} = \sigma_y + H \left(\varepsilon_{xx} - \frac{\sigma_y}{E} \right)$$

for $u < |y| \leq v$

One obtains:

for the rectangular section:

$$\frac{M}{M_e} = \left(1 - \frac{H}{E}\right)\left(\frac{3}{2} - \frac{1}{2}\left(\frac{\varphi_e}{\varphi}\right)^2\right) + \frac{H\varphi}{E} / \varphi \quad \text{with} \quad \varphi_e = \frac{M_e}{EI} \; M_e = \frac{I_z \cdot \sigma^y}{v}$$

for the circular section:

$$M(\mu) = \frac{R^3 \sigma^y}{E}\left[\frac{\pi}{4}\frac{H}{\mu} + \frac{4}{3}(E-H)\left(1-\mu^2\right)^{3/2} + \frac{E-H}{2\mu}\left(Arc\sin\mu - \mu\left(1-2\mu^2\right)\sqrt{1-\mu^2}\right)\right]$$

with $\mu = \dfrac{u}{R} = \dfrac{\sigma_y}{ER\varphi} = \dfrac{\varphi_e}{\varphi}$

In discharge, after having reached the limiting load charges some, one obtains a limiting load of contrary sign.

for the tubular section:

(assumption of beam of Navier-Bernoulli)

The load limits ($H = 0$) is worth:

$$\frac{M}{M_e} = \frac{4}{\pi}$$

The total solution for a thin tube is [bib1]:

$$\frac{M(\mu)}{M_e}=\frac{\lambda}{\mu}+\frac{2(1-\lambda)}{\pi\mu}\left(arc\sin\mu+\mu\sqrt{1-\mu^2}\right) \text{ with } \lambda=\frac{E_T}{E}=\frac{H}{E+H}$$

2.1.2 Traction - Work hardening linear

Analytical solution: one has immediately $N=S\sigma_y\left(1-\frac{H}{E}\right)+\frac{HS}{L}.DX$.

2.1.3 Pure traction - Elasticity

Analytical solution:

$$N=\frac{E.S.\delta u}{L} \quad \sigma=\frac{E.\delta u}{L}$$, with $\delta u=7.5\text{E}-03$

2.1.4 Pure inflection - Elasticity

Analytical solution:

$$Mfz=\frac{3.0\,\delta u\,E\,I}{L^2} \quad \sigma=\frac{M_{fz}.h}{I}$$, with $\delta u=7.5\text{E}-03$

with $h=R$ for the circular section, $h=v$ for the rectangular section.

2.2 Bibliographical references

1. J.H. LAU and T.T. LAU: Newspaper of Pressure Vessel Technology vol. 106 p188-195 - May 1984.

3 Modeling A

3.1 Characteristics of modeling

2 elements POU_D_TGM by type of section. There are thus 2 groups of elements comprising each one 2 elements.

Group	$GR1$:	rectangular section
	$GC1$:	circular section
Simple traction with work hardening		on $GR1$ and $GC1$ (ECRO_LINE)
Pure inflection without work hardening		on $GR1$ and $GC1$ (ECRO_LINE)
Elasticity		on $GR1$ and $GC1$

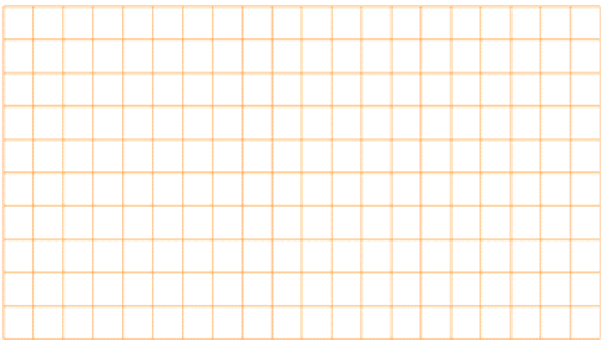
3.2 Characteristics of the grid

- Grid of the beam

  2×2 elements POU_D_TGM

- Grid of the sections

111 nodes, 188 TRIA3



231 nodes, 200 QUAD4

3.3 Sizes tested and results

- Simple traction (with $DX^e=0.75\text{E}-03$)

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification		Type of Reference	Reference	Tolerance
$2\,DX^e$	11	GR1	R3	EFGE_ELNO	NR	ANALYTICAL	$3.\text{E}+06$	$0.10\,\%$
$3\,DX^e$	21	GR1	R3	EFGE_ELNO	NR	ANALYTICAL	$3.\text{E}+06$	$0.10\,\%$
$2\,DX^e$	11	GC1	C3	EFGE_ELNO	NR	ANALYTICAL	$4.82\text{E}+06$	$2.5\,\%$
$3\,DX^e$	21	GC1	C3	EFGE_ELNO	NR	ANALYTICAL	$4.87\text{E}+06$	$2.5\,\%$
$3\,DX^e$	21	GC1	C3	DEGE_ELNO	EPXX	ANALYTICAL	$2.25\text{E}-03$	$0.10\,\%$

$DX_{(B)}$	N°ordre	MESH	Not	Under - point	Identification		Type of Reference	Reference	Tolerance
$3\,DX^e$	21	SR1	1	1	VARI_ELGA	V1	ANALYTICAL	$1.5\text{E}-03$	$0.1\,\%$
$3\,DX^e$	21	SC1	1	1	VARI_ELGA	V1	ANALYTICAL	$1.5\text{E}-03$	$1.5\,\%$

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification	Type of Reference	Reference	Tolerance
$2\,DX^e$	11	GR1	R3	REAC_NODA DX	ANALYTICAL	$3.E+06$	0.10%
$3\,DX^e$	21	GR1	R3	REAC_NODA DX	ANALYTICAL	$3.E+06$	0.10%
$2\,DX^e$	11	GC1	C3	REAC_NODA DX	ANALYTICAL	$4.82E+06$	2.5%
$3\,DX^e$	21	GC1	C3	REAC_NODA DX	ANALYTICAL	$4.87E+06$	2.5%

- Pure inflection (with $DRZ^e=0.75E-02$)

$DRZ_{(B)}$	N°ordre	GROUP_MA	NODE	Identification	Type of Reference	Reference	Tolerance
DRZ^e	1	GR1	R3	MFZ (Nm)	ANALYTICAL	$1.0E+05$	0.5%
$5\,DRZ^e$	21	GR1	R3	MFZ	ANALYTICAL	$1.48E+05$	1.0%
$10\,DRZ^e$	31	GR1	R3	MFZ	ANALYTICAL	$1.495E+05$	1.0%
$20\,DRZ^e$	41	GR1	R3	MFZ	ANALYTICAL	$1.499E+05$	1.0%
$-2.\,DRZ^e$	71	GR1	R3	MFZ	ANALYTICAL	$-1.5E+05$	1.0%
$10\,DRZ^e$	31	GR1	R3	KZ	ANALYTICAL	0.075	0.1%
DRZ^e	1	GC1	C3	MFZ	ANALYTICAL	$1.178E+05$	2.5%
$5\,DRZ^e$	21	GC1	C3	MFZ	ANALYTICAL	$1.96E+05$	2.0%
$10\,DRZ^e$	31	GC1	C3	MFZ	ANALYTICAL	$1.99E+05$	1.5%
$20\,DRZ^e$	41	GC1	C3	MFZ	ANALYTICAL	$1.998E+05$	1.5%
$-2.\,DRZ^e$	71	GC1	C3	MFZ	ANALYTICAL	$-2.0E+05$	2.0%

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification	Type of Reference	Reference	Tolerance
DRZ^e	1	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.0E+05$	0.2%
$5\,DRZ^e$	21	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.48E+05$	1.0%
$10\,DRZ^e$	31	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.495E+05$	1.0%
$20\,DRZ^e$	41	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.499E+06$	1.0%
DRZ^e	1	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.178E+05$	2.5%
$5\,DRZ^e$	21	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.96E+05$	2.0%
$10\,DRZ^e$	31	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.99E+05$	1.5%
$20\,DRZ^e$	41	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.998E+06$	1.5%

4 Modeling B

4.1 Characteristics of modeling

Modeling is carried out with 2 elements POU_D_E by type of section. There are thus 2 groups of elements comprising each one 2 elements.

Group	$GR1$:	rectangular section
	$GC1$:	circular section
Elasticity		on $GR1$ and $GC1$

4.2 Characteristics of the grid

- Grid of the beam

  2×2 elements POU_D_EM

4.3 Sizes tested and results

- Simple traction with $DX^e=0.75\text{E}-03$

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification		Type of Reference	Reference
$3\,DX^e$	21	SC2	C3	DEGE_ELNO	EPXX	ANALYTICAL	$2.25\text{E}-03$
$3\,DX^e$	21	SC1	C2	DEGE_ELNO	EPXX	ANALYTICAL	$2.25\text{E}-03$
$3\,DX^e$	21	SR2	R3	DEGE_ELNO	EPXX	ANALYTICAL	$2.25\text{E}-03$
$3\,DX^e$	21	SR1	R2	DEGE_ELNO	EPXX	ANALYTICAL	$2.25\text{E}-03$

5 Modeling C

5.1 Characteristics of modeling

2 elements POU_D_EM by type of section. There are thus 2 groups of elements comprising each one 2 elements.

Group $GR1$:	rectangular section
$GC1$:	circular section
Simple traction with work hardening	on $GR1$ and $GC1$ (ECRO_LINE)
Pure inflection without work hardening	on $GR1$ and $GC1$ (ECRO_LINE)
Elasticity	on $GR1$ and $GC1$

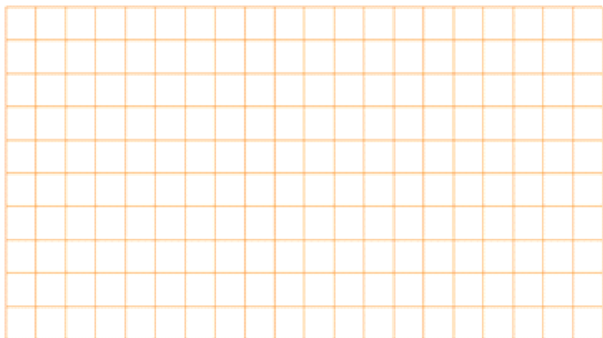
5.2 Characteristics of the grid

- Grid of the beam

  2×2 elements POU_D_EM

- Grid of the sections

111 nodes, 188 TRIA3



231 nodes, 200 QUAD4

5.3 Sizes tested and results

- Simple traction (with $DX^e=0.75\text{E-}03$)

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification		Type of Reference	Reference	Tolerance
$2DX^e$	11	GR1	R3	EFGE_ELNO	NR	ANALYTICAL	$3.\text{E}+06$	$0.10\,\%$
$3DX^e$	21	GR1	R3	EFGE_ELNO	NR	ANALYTICAL	$3.\text{E}+06$	$0.5\,\%$
$2DX^e$	11	GC1	C3	EFGE_ELNO	NR	ANALYTICAL	$4.82\text{E}+06$	$2.5\,\%$
$3DX^e$	21	GC1	C3	EFGE_ELNO	NR	ANALYTICAL	$4.87\text{E}+06$	$2.5\,\%$
$3DX^e$	21	GC1	C3	DEGE_ELNO	EPXX	ANALYTICAL	$2.25\text{E-}03$	$0.10\,\%$

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification	Type of Reference	Reference	Tolerance
$2\,DX^e$	11	GR1	R3	REAC_NODA DX	ANALYTICAL	$3.E+06$	0.10%
$3\,DX^e$	21	GR1	R3	REAC_NODA DX	ANALYTICAL	$3.E+06$	0.50%
$2\,DX^e$	11	GC1	C3	REAC_NODA DX	ANALYTICAL	$4.82E+06$	2.5%
$3\,DX^e$	21	GC1	C3	REAC_NODA DX	ANALYTICAL	$4.87E+06$	2.5%

- Pure inflection (with $DRZ^e=0.75E-02$)

$DRZ_{(B)}$	N°ordre	GROUP_MA	NODE	Identification	Type of Reference	Reference	Tolerance
DRZ^e	1	GR1	R3	MFZ (Nm)	ANALYTICAL	$1.0E+05$	0.5%
$5\,DRZ^e$	21	GR1	R3	MFZ	ANALYTICAL	$1.48E+05$	1.0%
$10\,DRZ^e$	31	GR1	R3	MFZ	ANALYTICAL	$1.495E+05$	1.0%
$20\,DRZ^e$	41	GR1	R3	MFZ	ANALYTICAL	$1.499E+05$	1.5%
$-2.\,DRZ^e$	71	GR1	R3	MFZ	ANALYTICAL	$-1.5E+05$	2.0%
$10\,DRZ^e$	31	GR1	R3	KZ	ANALYTICAL	0.075	0.1%
DRZ^e	1	GC1	C3	MFZ	ANALYTICAL	$1.178E+05$	2.5%
$5\,DRZ^e$	21	GC1	C3	MFZ	ANALYTICAL	$1.96E+05$	1.5%
$10\,DRZ^e$	31	GC1	C3	MFZ	ANALYTICAL	$1.99E+05$	1.0%
$20\,DRZ^e$	41	GC1	C3	MFZ	ANALYTICAL	$1.998E+05$	2.0%
$-2.\,DRZ^e$	71	GC1	C3	MFZ	ANALYTICAL	$-2.0E+05$	2.0%

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification	Type of Reference	Reference	Tolerance
DRZ^e	1	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.0E+05$	0.5%
$5\,DRZ^e$	21	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.48E+05$	1.0%
$10\,DRZ^e$	31	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.495E+05$	1.0%
$20\,DRZ^e$	41	GR1	R3	REAC_NODA DRZ	ANALYTICAL	$1.499E+06$	1.5%
DRZ^e	1	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.178E+05$	2.5%
$5\,DRZ^e$	21	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.96E+05$	1.5%
$10\,DRZ^e$	31	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.99E+05$	1.0%
$20\,DRZ^e$	41	GC1	C3	REAC_NODA DRZ	ANALYTICAL	$1.998E+06$	1.0%

6 Modeling D

6.1 Characteristics of modeling

2 elements PIPE for the tubular section.
Simple traction: (ECRO_LINE)
Pure inflection: without work hardening
Moreover, one blocks the degrees of freedom which correspond to the mode 3d' ovalization: $U03$, $V03$, $W03$.

6.2 Characteristics of the grid

2 elements PIPE (METUSEG3)

6.3 Sizes tested and results

The nodal reactions are tested:

Identification	Node	Number order	Type of reference	Value of reference	Tolerance
DX	B	11	ANALYTICAL	$9.47152E+4$	0.10%
DX	B	21	ANALYTICAL	$9.5653E+4$	0.10%
DRZ	B	1	ANALYTICAL	$4.64217E+3$	0.10%
DRZ	B	21	ANALYTICAL	$5.9106E+3$	0.50%

One tests respectively equivalent deformations, strains and equivalent stresses :

Size	Mesh	Node	Number order	Type of reference	Value of reference	Tolerance
$EPYY$	$SC1$	$C1$	1	NON_REGRESSION	$-2.25E-4$	0.01%
$INVA_2$	$SC1$	$C1$	1	NON_REGRESSION	$5.89491E-4$	0.01%
$VMIS$	$SC1$	$C1$	1	NON_REGRESSION	0.0	0.10%

7 Modeling E

7.1 Characteristics of modeling

2 elements PIPE with 4 nodes for the tubular section.
Simple traction: (ECRO_LINE)
Pure inflection: without work hardening
Moreover, one blocks the degrees of freedom which correspond to the mode 3d' ovalization: $U03$, $V03$, $W03$

7.2 Characteristics of the grid

2 elements PIPE (meshs SEG4)

7.3 Sizes tested and results

The nodal reactions are tested:

Size	Number order	Node	Type of reference	Value of reference	Tolerance
DX	11	B	ANALYTICAL	$9.47152\text{E}+4$	0.10%
DX	21	B	ANALYTICAL	$9.5653\text{E}+4$	0.10%
DRZ	1	B	ANALYTICAL	$4.64217\text{E}+3$	0.10%
DRZ	21	B	ANALYTICAL	$5.9106\text{E}+3$	0.50%

One tests respectively equivalent deformations, strains and equivalent stresses :

Size	Number order	Mesh	Node	Type of reference	Value of reference	Tolerance
$EPYY$	1	$M1$	$N1$	NON_REGRESSION	$-2.25\text{E}-4$	0.01%
$INVA\_2$	1	$M1$	$N1$	NON_REGRESSION	$5.89491\text{E}-4$	0.01%
$VMIS$	1	$M1$	$N1$	NON_REGRESSION	0.0	0.10%
KY	1	$M1$	$N1$	ANALYTICAL	$-7.5\text{E}-3$	0.10%

Size	Number order	Mesh	Not	Type of reference	Value of reference	Tolerance
KY	1	$M1$	3	ANALYTICAL	$-7.5\text{E}-3$	0.1%

8 Modeling F

8.1 Characteristics of modeling

Modeling consists of 112 elements COQUE_3D for the tubular section, and 2 elements pipe to apply the boundary conditions. The length of the grid hulls is of $0.98\mathrm{m}$. The length of each element pipe is of $0.01\mathrm{m}$.

A connection COQUE_TUYAU is applied at each end of the grid hulls, with an element pipe. Moreover, one blocks the degrees of freedom of the pipes which correspond to the mode 3d' ovalization: $U03$, $V03$, $W03$

Simple traction:	(ECRO_LINE)
Pure inflection:	without work hardening

8.2 Characteristics of the grid

112 meshs QUAD9 and 2 meshs SEG3.

8.3 Sizes tested and results

Traction:

$DX_{(B)}$	N°ordre	Identification	Type of Reference	Reference	Tolerance
$2\,DX^e$	11	NR	ANALYTICAL	$9.47\mathrm{E}+04$	1.10%
$3\,DX^e$	21	NR	ANALYTICAL	$9.565\mathrm{E}+04$	1.10%

Inflection:

$DRZ_{(B)}$	N°ordre	Identification	Type of Reference	Reference	Tolerance
DRZ^e	1	MFZ	ANALYTICAL	$4.642\mathrm{E}+03$	0.10%
$2.8\,DRZ^e$	19	MFZ	ANALYTICAL	$5.7824\mathrm{E}+03$	0.50%

9 Modeling G

9.1 Characteristics of modeling

2 elements POU_D_EM for the tubular section.
The section is with a grid in QUAD4 : it is discretized by a mesh in the thickness, and 90 meshs on the circumference.

Simple traction:	(ECRO_LINE)
Pure inflection:	without work hardening

9.2 Characteristics of the grid

2 meshs SEG2 for the beam. 90 meshs QUAD4 for the section.

9.3 Sizes tested and results

Traction:

$DX_{(B)}$	N°ordre	Identification	Type of Reference	Reference	Tolerance
$2\,DX^e$	11	NR	ANALYTICAL	$9.47\mathrm{E}+04$	$0.10\,\%$
$3\,DX^e$	21	NR	ANALYTICAL	$9.565\mathrm{E}+04$	$0.10\,\%$

Inflection:

$DRZ_{(B)}$	N°ordre	Identification	Type of Reference	Reference	Tolerance
DRZ^e	1	MFZ	ANALYTICAL	$4.642\mathrm{E}+03$	$0.10\,\%$
$5\,DRZ^e$	21	MFZ	ANALYTICAL	$5.9106\mathrm{E}+03$	$0.50\,\%$

10 Modeling H

10.1 Characteristics of modeling

2 elements `POU_D_TGM` for the tubular section.
The section is with a grid in `QUAD4` : it is discretized by a mesh in the thickness, and 90 meshs on the circumference.

Simple traction:	(`ECRO_LINE`)
Pure inflection:	without work hardening

10.2 Characteristics of the grid

2 meshs `SEG2` for the beam. 90 meshs `QUAD4` for the section.

10.3 Sizes tested and results

Traction:

$DX_{(B)}$	N°ordre	Identification	Type of Reference	Reference	Tolerance
$2\,DX^e$	11	NR	ANALYTICAL	$9.47E+04$	0.10%
$3\,DX^e$	21	NR	ANALYTICAL	$9.565E+04$	0.10%

Inflection:

$DRZ_{(B)}$	N°ordre	Identification	Type of Reference	Reference	Difference
DRZ^e	1	MFZ	ANALYTICAL	$4.642E+03$	0.10%
$5\,DRZ^e$	21	MFZ	ANALYTICAL	$5.9106E+03$	0.50%

11 Modeling I

11.1 Characteristics of modeling

2 elements POU_D_TGM by type of section. There are thus 2 groups of elements comprising each one 2 elements.

Group $GR1$:	rectangular section
$GC1$:	circular section
Simple traction with work hardening	on $GR1$ and $GC1$ (ECRO_LINE)
Pure inflection without work hardening	on $GR1$ and $GC1$ without work hardening

11.2 Characteristics of the grid

- Grid of the beam

  2×2 elements POU_D_TGM

- Grid of the sections

111 nodes, 188 TRIA3

231 nodes, 200 QUAD4

11.3 Notice

The characteristic of modeling L is to test the operation of DYNA_NON_LINE in the quasi-static calculation of traction of a beam modelled in POU_D_TGM. This kind of modeling has as a characteristic to reveal worthless pivots on the lines of the matrix of mass corresponding to the degrees of freedom of warping. In this case, the initialization of the diagram of NEWMARK is not done any more by inversion of the matrix of mass, which is singular, but by zero setting of initial acceleration.

11.4 Sizes tested and results

11.4.1 Values tested

- Simple traction (with $DX^e=0.75\mathrm{E}-03$)

$DX_{(B)}$	N°ordre	GROUP_MA	NODE	Identification		Type of Reference	Reference	Tolerance
$2\,DX^e$	11	GR1	R3	EFGE_ELNO	NR	ANALYTICAL	$3.E+06$	$0.10\,\%$
$3\,DX^e$	21	GR1	R3	EFGE_ELNO	NR	ANALYTICAL	$3.E+06$	$0.10\,\%$
$2\,DX^e$	11	GC1	C3	EFGE_ELNO	NR	ANALYTICAL	$4.82E+06$	$2.5\,\%$
$3\,DX^e$	21	GC1	C3	EFGE_ELNO	NR	ANALYTICAL	$4.87E+06$	$2.5\,\%$
$3\,DX^e$	21	GC1	C3	DEGE_ELNO	EPXX	ANALYTICAL	$2.25E-03$	$0.10\,\%$

$DX_{(B)}$	N°ordre	MESH	Not	Under - point	Identification		Type of Reference	Reference	Tolerance
$3\,DX^e$	21	GR1	1	1	VARI_ELGA	V1	ANALYTICAL	$1.5E-03$	$20.0\,\%$
$3\,DX^e$	21	GC1	1	1	VARI_ELGA	V1	ANALYTICAL	$1.5E-03$	$1.5\,\%$

11.4.2 Observations

It is noticed that the results in traction resulting from DYNA_NON_LINE are identical to those given by STAT_NON_LINE.

12 Summary of the results

Modelings in multifibre beams provide a solution to less 2.5% analytical solution, for a very weak time calculation, in comparison with modelings pipe and hull. The only approximation comes from the grid of the section.

With regard to modelings pipe and hull, the conclusions are the same ones, but this time the difference with the analytical solution comes from this solution which is valid for a very thin beam of tubular section, without effect of ovalization. This ovalization is blocked at the ends, which makes it possible to obtain a solution with less 0.4% analytical solution.