

SSNL118 - Bar subjected to a field speed of wind

Summary:

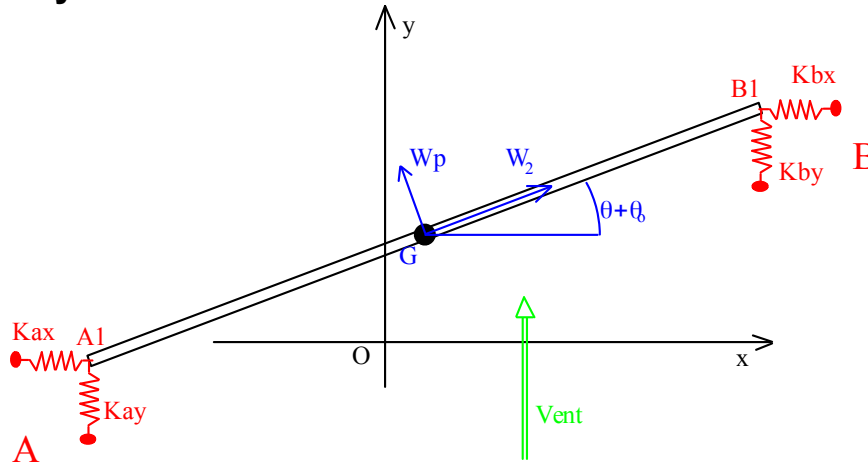
This test relates to the validation of the application of the loadings of wind on the linear elements. The loading is described by fields speeds of wind.

This problem makes it possible to test:

- linear finite elements [bars, cables, beams (except the curved beams)] with following loadings of natural "wind",
- loadings representing speeds of wind:
 - reading of the data of the fields of wind,
 - projection of the fields of wind attached to the group of dots on the deformed grid of the structure,
 - calculation relative speed,
- the taking into account of the function giving the force distributed according to the relative speed of the structure,
- the reactualization of the geometry to take account of great displacements and great rotations.

1 Problem of reference

1.1 Geometry



Length of the bar: 1.5m

Stiffnesses of the discrete ones: kax , kay , kbx , kby

1.2 Properties of materials

Material for the linear element: $E = 2.0E+08 \text{ Pa}$, $\rho = 1000.0 \text{ kg/m}^3$

Mechanical characteristics of the bar: *section*='CERCLE', *rayon*=0.5m, *ep*=0.5m

Stiffness of the springs:

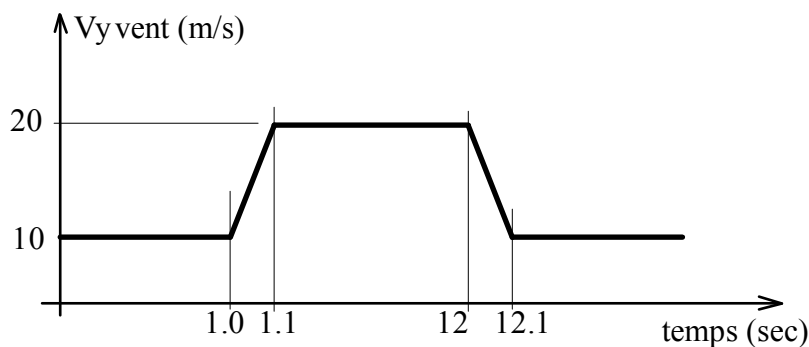
Kxa	Kya	Kxb	Kyb
10 N/m	20 N/m	25 N/m	30 N/m

1.3 Boundary conditions and loadings

At the points A and B : blocking of the degrees of freedom: dx , dy , dz

At the points $A1$ and $B1$: blocking of the degrees of freedom: dz

Characteristics of the field speed of wind, along the axis y :



1.4 Initial conditions

The bar forms an angle of 30° ($\theta_0 = 30^\circ$) compared to the axis x .

2 Reference solution

2.1 Equilibrium equations

Effort at the point $A1$

$$F_a = \begin{pmatrix} -k_{xa} \cdot \delta x_a \\ -k_{ya} \cdot \delta y_a \\ L \cdot (\delta y_a \cdot k_{ya} \cdot \cos(\theta_0 + \theta) - \delta x_a \cdot k_{xa} \cdot \sin(\theta_0 + \theta)) / 2 \end{pmatrix} \quad \text{avec les déplacements du point } A1$$

$$\begin{pmatrix} \delta x_a = L \cdot \cos(\theta_0) / 2 - L \cdot \cos(\theta_0 + \theta) / 2 + x \\ \delta y_a = L \cdot \sin(\theta_0) / 2 - L \cdot \sin(\theta_0 + \theta) / 2 + y \end{pmatrix}$$

Effort at the point $B1$

$$F_b = \begin{pmatrix} -k_{xb} \cdot \delta x_b \\ -k_{yb} \cdot \delta y_b \\ L \cdot (-\delta y_b \cdot k_{yb} \cdot \cos(\theta_0 + \theta) + \delta x_b \cdot k_{xb} \cdot \sin(\theta_0 + \theta)) / 2 \end{pmatrix} \quad \text{avec les déplacements du point } B1$$

$$\begin{pmatrix} \delta x_b = -L \cdot \cos(\theta_0) / 2 + L \cdot \cos(\theta_0 + \theta) / 2 + x \\ \delta y_b = -L \cdot \sin(\theta_0) / 2 + L \cdot \sin(\theta_0 + \theta) / 2 + y \end{pmatrix}$$

Effort due to the wind

- Speed of the wind in a point $M \in \text{barre}$

$$V_r = \begin{pmatrix} V_{vx} \\ V_{vy} \\ 0 \end{pmatrix} \quad \text{with } V_{vx}, V_{vy} : \text{speed of the wind following the axis } x \text{ and centers it } y.$$

- Speed relative perpendicular to the bar to the point M :

$$V_p = \begin{pmatrix} \sin(q_0 + q) \cdot (-V_{vy} \cdot \cos(q_0 + q) + V_{vx} \cdot \sin(q_0 + q)) \\ \cos(q_0 + q) \cdot (V_{vy} \cdot \cos(q_0 + q) - V_{vx} \cdot \sin(q_0 + q)) \\ 0 \end{pmatrix}$$

- Force due to the wind in a point M

$$F_{vent(M)} = F_{cx(M)} \frac{V_p}{\|V_p\|} \quad \text{in our case one chooses } F_{cx(M)} = \|V_p\|$$

one thus obtains $F_{vent(M)} = V_p$

- Resultant of the force due to the wind on the bar

$$F_{vent} = \begin{pmatrix} L \cdot \sin(q_0 + q) \cdot (-V_{vy} \cdot \cos(q_0 + q) + V_{vx} \cdot \sin(q_0 + q)) \\ L \cdot \cos(q_0 + q) \cdot (V_{vy} \cdot \cos(q_0 + q) - V_{vx} \cdot \sin(q_0 + q)) \\ 0 \end{pmatrix}$$

Equilibrium equation: $F_a + F_b + F_{vent} = 0$

2.2 Sizes and results of reference

Displacements of the points AI and BI at the moments: $1.s$, $1.05s$ and $2.s$. These moments correspond respectively to speeds of wind of 10 , 15 and $20m/s$

The resolution of the 3 projection, equilibrium equations of $Fa + Fb + Fvent = 0$, is done by iterations. The 3 unknown factors of the problem are the position of the centre of gravity of the bar G : (x, y) and variation of the angle: θ .

In *Code_Aster*, the effect of the wind is taken into account by a force distributed along the linear element. The expression of the module of this force distributed is the following one:

$$Fcx_{(v)} = \frac{1}{2} \cdot \rho \cdot V^2 \cdot Cx(v) \cdot D_h$$

where $Fcx_{(v)}$: is the module of the force distributed along the cable in N/m , depend on speed.

ρ : is the density of the air in kg/m^3 .

V : is the relative speed of the cable in m/s .

$Cx(v)$: is the coefficient of drag of the cable, depend on relative speed.

D_h : is the hydraulic diameter of the cable in m .

To obtain a simple analytical reference solution, the function $Fcx_{(Vp)}$ is taken equalizes with $\|V_p\|$. In the command file of *Code_Aster* the function of $Fcxv$ is thus in the following way defined:

```
FCXV=DEFI_FONCTION (  
  NOM_PARA=' VITE',  
  VALE= ( 0.0, 0.0,  
          10.0, 10.0),  
  PROL_GAUCHE=' LINEAIRE',  
  PROL_DROITE=' LINEAIRE',  
)
```

2.3 Uncertainties on the solution

None. The resolution of the equilibrium equation is done by iterations with an error lower than $1.0E-09$.

2.4 Bibliographical reference

- HM77/01/046/A. "M7-01-70 Project. Evolution of *Code_Aster* for the best taken into account of the loadings of dynamic wind on the linear elements".

3 Modeling A

3.1 Characteristics of modeling and the grid

The linear element: 'BAR'
The discrete ones: 'DIS_T'

4 Results of modeling A

4.1 Sizes tested and results

Balance is calculated at the moments: $1.s$, $1.05.s$ and $2.s$.

Balance with $1.s$	Analytical
$\delta xa(m)$	- 0.2092
$\delta ya(m)$	0.3276
$\delta xb(m)$	- 0.1418
$\delta yb(m)$	0.1965
Balance with $1.05.s$	Analytical
$\delta xa(m)$	- 0.2885
$\delta ya(m)$	0.5050
$\delta xb(m)$	- 0.1942
$\delta yb(m)$	0.3105
Balance with $2.s$	Analytical
$\delta xa(m)$	- 0.3502
$\delta ya(m)$	0.6890
$\delta xb(m)$	- 0.2327
$\delta yb(m)$	0.4324

5 Synthesis

The test shows the good taking into account of the loadings of type speed of wind on the linear elements.