SSNL126 - Elastoplastic buckling of a right beam

Summary:

A slim right beam of circular section is subjected to a compressive force at an end, and is embedded at the other end. The behavior of material is elastoplastic, with a linear isotropic work hardening. During the rise in load, one calculates the critical loads of elastic buckling, then plastic.

Two modelings make it possible to test the criterion of buckling in elastoplasticity:

Voluminal modeling a: grid, small deformations and small displacements.
Voluminal modeling b: grid, small deformations and great displacements (GREEN).

1 Problem of reference

1.1 Geometry

Right beam, length $L=1\text{m}$
Circular section of ray $R=0.01\text{m}$.

1.2 Material properties

Elastoplastic material with isotropic linear work hardening:

Young modulus: $E=210000\,MPa$
Poisson's ratio: $\nu_{.}=0$. (assumption of beam of Euler-Bernoulli)
Elastic limit: $\sigma_Y=4\,MPa$
Tangent module: $E_T=70000\,MPa$

1.3 Boundary conditions and loadings

- Boundary conditions: embedding on all basic surface
- Surface force on the higher face: with $t=1\text{s}$, $F=6.5\,MPa$

This load is applied in 10 pas de time équi-distribute.

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution:

In small displacements:

- in elastic mode (for $F < \sigma_Y$) the theoretical breaking value corresponds to the load of Euler. Within the framework of a kinematics of beam, the critical load is worth:

  $F_{cr} = \frac{\pi^2 EI}{4 L^2}$, therefore critical pressure: $P_{cr} = \frac{\pi^2 EI}{4 SL^2}$

  with $I = \frac{\pi R^4}{4}$ and $S = \pi R^2$ that is to say $P_{cr} = \frac{\pi^2 ER^2}{16 L^2}$

- in elastoplastic mode, as one considers a uniform compression without elastic discharge and because of law of behavior, the critical load of buckling is worth:

  $F_{cr} = \frac{\pi^2 Et.I}{4 L^2}$ that is to say a pressure criticizes: $P_{cr} = \frac{\pi^2 EtR^2}{16 L^2}$

2.2 Results of reference

Values of the critical load for the two loading cases.

In elastic mode, for $F < 4\,MPa$, that is to say $t < 0.61538462$, one must obtain: $P_{cr} = 12.95\,MPa$.

In plastic mode the critical value of pressure of buckling is: $4{,}32\,MPa$.

The critical coefficients according to the loading are:

Pas de time	**Surface force (in MPa)**	**Critical coefficient**	**Critical load (in MPa)**
1	0.65	19.9290	12.9539
2	1.3	9.9645	12.9539
3	1.95	6.6430	12.9539
4	2.6	4.9823	12.9539
5	3.25	3.9858	12.9539
6	3.9	3.3215	12.9539
7	4.55	0.9490	4.3180
8	5.2	0.8304	4.3180
9	5.85	0.7381	4.3180
10	**6.5**	0.6643	4.3180

3 Modeling A

3.1 Characteristics of modeling

Voluminal grid 3D.

3.2 Characteristics of the grid

Many nodes: 600
Many meshs and types: 90 HEXA20

3.3 Values tested

Moment	Reference
0.2	– 9.9645
1	– 0.6643

4 Modeling B

4.1 Characteristics of modeling

Voluminal grid 3D. Great displacements and deformations (but small rotations)

The surface force applied is worth here $-20\,MPa$ with $t=1\text{s}$, in order to pass, during the evolution of the loading, by the critical point.

This load is applied in 10 pas de time équirépartis.

Two complete calculations are carried out: one with a purely elastic behavior, in order to be able to compare the result with the elastic solution of reference, and the other with an elastoplastic behavior.

4.2 Characteristics of the grid

Many nodes: 600
Many meshs and types: 90 HEXA20

4.3 Values tested

In elastic behavior

One tests the end value of the critical coefficient: (test of nonregression)

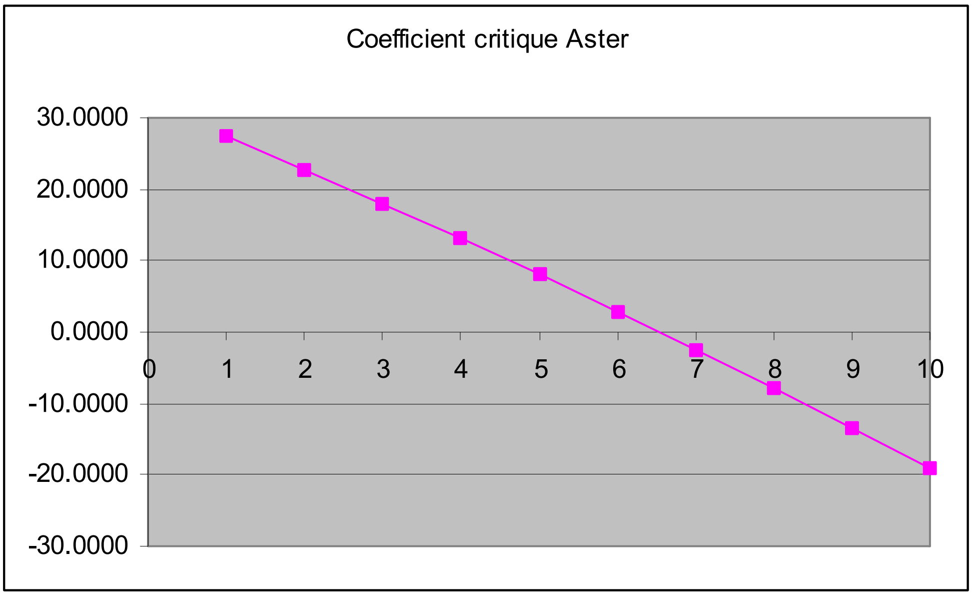
Moment	Reference
1	– 19.0657

In the case of great displacements or great deformations, the value of the critical coefficient must be interpreted differently case small displacements: the structure becomes unstable when the "critical load" is cancelled.

The evolution of this coefficient in the course of time is the following one:

Pas de time	Surface force (in MPa)	Critical coefficient *Aster*	Critical load Euler
1	2	27.5797	12.9539
2	4	22.8250	12.9539
3	6	17.9808	12.9539
4	8	13.0407	12.9539
5	10	7.9975	12.9539
6	12	2.8434	12.9539
7	14	– 2.4301	12.9539
8	16	– 7.8324	12.9539
9	18	– 13.3738	12.9539
10	20	– 19.0657	12.9539

The critical coefficient thus passes well by 0 between moments 6 and 7, and more precisely (cf curves following) in the neighbourhoods of moment 6.5, which corresponds well to critical load in elasticity.



In elastoplasticity, one tests the moments when the critical coefficient changes sign. The tests are of nonregression since one does not have analytical solution in this case.

Moment	Reference
0.4	4.9917
0.5	– 1.3186

5 Summary of the results

The results as of modeling in small displacements is in conformity with the analytical reference (less than 2% of variation in plasticity). The results in great displacements cannot be compared with a reference solution, but the change of sign of the critical coefficient is in conformity with the expected solution.