

## SSNL130 – Indeformable plate on a carpet of springs

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### Summary:

The objective is to test and validate the possibilities of the order `AFFE_CARA_ELEM`, option `RIGI_PARASOL` in `2D` and in `3D` and affected of behavior `DIS_CHOC`.

This case test models a plate, considered as indeformable, posed on a carpet of springs.

- The springs are modelled by `DIS_T (K_T_D_L)`, that makes it possible to impose boundary conditions at the ends of the springs which are not related to the solid.
- The behavior `DIS_CHOC` a unilateral behavior of the springs allows, which leaves a possibility of separation of the plate with respect to the carpet of spring.

## 1 Problem of reference

### 1.1 Geometry

A rectangular plate of width  $a$  and length  $b$ , pressed on a carpet of springs.

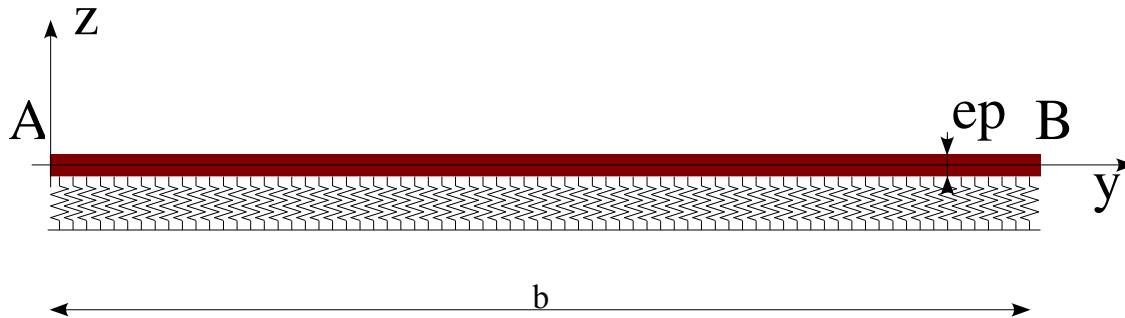


Figure 1.1-a : Diagram of the plate and the springs in the plan  $(y, z)$ .

Dimensions:

$$a = 1.00 \text{ m}$$

$$b = 2.00 \text{ m}$$

$$ep = 0.30 \text{ m}$$

### 1.2 Properties of material

Young modulus:  $2.0E+11 \text{ Pa}$

Poisson's ratio: 0.3

Total stiffness of the carpet of springs:  $K = 10000.0 \text{ N/m}$

### 1.3 Boundary conditions and loadings

The loading is a loading of pressure of the form  $P = p.(y-b)^2$ , with  $p = 5\text{N/m}^4$

Displacements imposed at the ends of the springs off-line to the plate:

- in the time interval  $[0, 1]$  displacement is imposed on 0.0 following  $DX$ ,  $DY$  and  $DZ$ ,
- in the time interval  $[1, 2]$  displacement is imposed on 0.0 following  $DX$  and  $DY$ .

According to  $DZ$  it is imposed by the function  $Dz = (t-1.0)*0.5E-02$ .

### 1.4 Initial conditions

Without object for a static analysis.

## 2 Reference solution

### 2.1 Method of calculating of the continuous problem

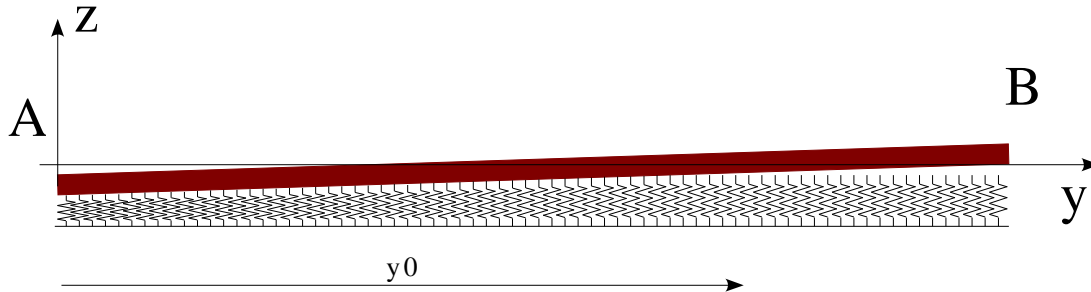


Figure 2.1-a : Diagram of the plate and the springs after loading.

The resolution of the problem consists in calculating vertical displacements of the corners of the plate and the position of the point of separation with respect to the carpet of springs.

The equilibrium equations are the following ones:

Effort resulting due to the loading:

$$F_p = \iint_s P \cdot ds = a \cdot p \cdot \frac{b^3}{3} \quad [2.1-1]$$

Moment resulting at the point  $A$  had with the loading:

$$M_{p_A} = \iint_s P \cdot y \cdot ds = a \cdot p \cdot \frac{b^4}{12} \quad [2.1-2]$$

The plate is regarded as rigid, its displacement is form  $z(y) = U_a \left(1 - \frac{y}{y_0}\right)$ . With  $U_a$  the vertical displacement of the point  $A$  and  $y_0$  the position of separation.

Effort of reaction of the springs:

$$F_r = \iint_s \frac{K}{a \cdot b} U_a \left(1 - \frac{y}{y_0}\right) ds = K U_a \frac{y_0}{2b} \quad [2.1-3]$$

Moment of reaction of the springs at the point  $A$  :

$$M_{r_A} = \iint_s \frac{K}{a \cdot b} U_a \left(1 - \frac{y}{y_0}\right) y ds = K U_a \frac{y_0^2}{6b} \quad [2.1-4]$$

The resolution of the equations 2.1-1, 2.1-2, 2.1-3, 2.1-4 (balance of the efforts and the moments) gives the following result:

$$y_0 = \frac{3b}{4} \quad U_a = -\frac{8pa b^3}{9K} \quad \text{one from of deduced} \quad U_b = -\frac{U_a}{3}$$

### 2.2 Method of calculating of the discretized problem

In this analysis the carpet of springs is not regarded any more as continuous. The springs are regularly distributed. As previously vertical displacements of the corners of the plate and the position of the line of separation with respect to the carpet of springs will be calculated.

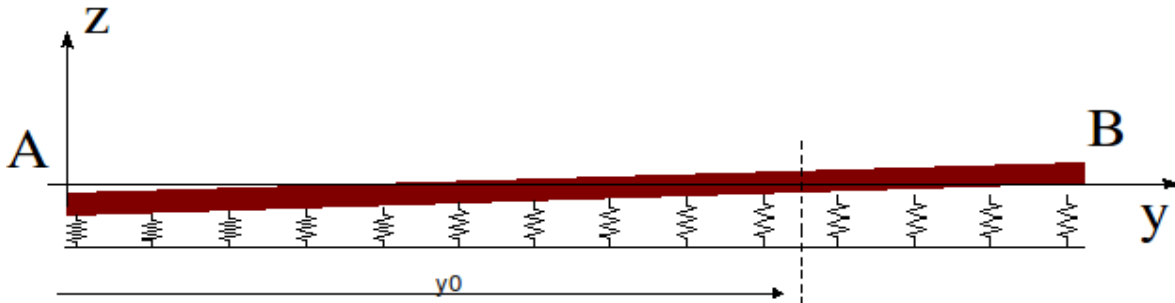


Figure 2.2-a : Diagram of the plate and the springs after loading.

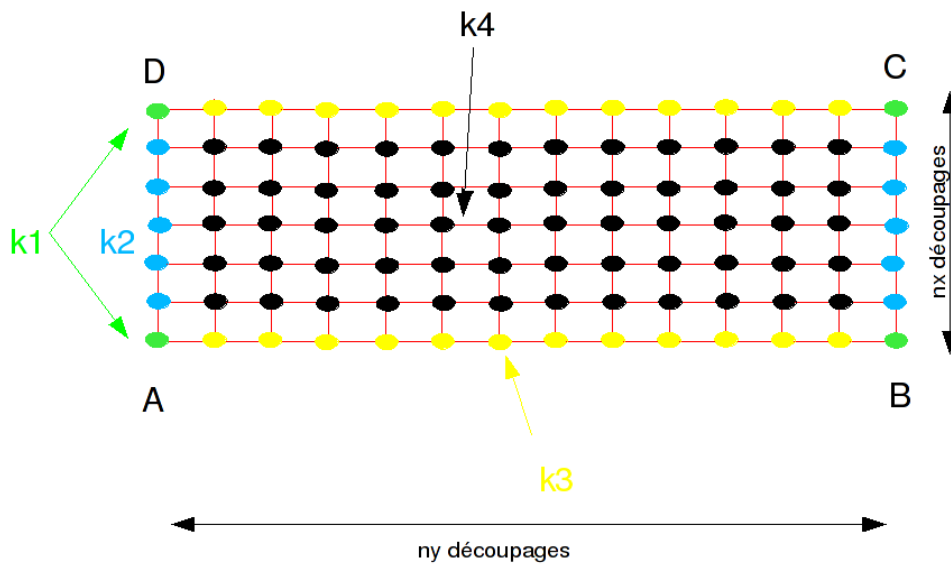


Figure 2.2-b : Discretization of the plate in the plan  $(x, y)$ .

The figure above locates the springs according to their stiffness. This stiffness is calculated by the option `RIGI_PARA_SOL` order `AFFE_CARA_ELEM`. The assignment of the values is made according to the surface of the zone that they affect. If  $\bar{K}$  is the total stiffness of the carpet of spring, one thus has:

$$k4 = \frac{K}{nx ny} \quad k2 = k3 = \frac{k4}{2} = \frac{K}{2 nx ny} \quad k1 = \frac{k4}{4} = \frac{K}{4 nx ny} \quad [2.2-1]$$

The equilibrium equations are the following ones:  
Effort of reaction springs:

$$Fr_{(j)} = U_a \cdot \left[ K'_x + K''_x \cdot \sum_{j=1}^n \left( 1 - j \frac{b}{ny y_0} \right) \right] \quad [2.2-2]$$

Moment of reaction of the springs along the line  $AB$  :

$$Mr_{(j)} = U_a \cdot K''_x \cdot \sum_{j=1}^n \left( 1 - j \frac{b}{ny y_0} \right) \cdot j \frac{b}{ny} \quad [2.2-3]$$

with  $K'_x = (2k1 + k2(nx-1))$   $K''_x = (2k3 + k4(nx-1))$

$$n \frac{b}{ny} \leq y_0 \leq (n+1) \frac{b}{ny}$$

The resolution of the equations (balance of the efforts and the moments) gives the solution of balance:

$$U_a = \frac{p a b^3 ny (3 ny - 8 n - 4)}{6 K (1 + n + n^2)} \quad y_0 = \frac{b n (1 + n) (3 ny - 8 n - 4)}{3 ny (ny + 2 n (ny - 2) - 4 n^2)} \quad [2.2-4]$$

where  $n$  and  $y_0$  must observe the following conditions:

$$n \frac{b}{ny} \leq y_0 \leq (n+1) \frac{b}{ny} \quad 0 \leq y_0 \leq b \quad n \text{ entirety}$$

## 2.3 Sizes and results of reference

The sizes tested will be vertical displacements with the 4 corners of the plate.

## 2.4 Uncertainties on the solution

No, the solution is analytical.

## 3 Modeling A

### 3.1 Characteristics of modeling

The plate is modelled by elements `DKT`. The springs are modelled by `SEG2` affected of a modeling `DIS_T` whose characteristics are `K_T_D_L`. They are the discrete ones in translation having a diagonal matrix, to see the documentation of `AFFE_CARA_ELEM`.

### 3.2 Characteristics of the grid

The plate is cut out with  $ny=16$  and  $nx=4$ . Dimensions of the plate are  $a=1\text{ m}$  and  $b=2\text{ m}$

### 3.3 Sizes tested and results

For the step of time  $n^{\circ}1$ , displacements of the ends of the springs, off-line to the plate, are imposed on zero. Results of `Code_Aster` are compared with the discrete solution, which corresponds to the solution of the modelled problem. This solution is obtained for  $n=12$ , equation 2.2-4.

Nature of the results	$U_A=U_D$	$U_B=U_C$
Continuous solution	$\frac{-4}{1125}$	$\frac{4}{3375}$
Discrete solution ( $n=12$ )	$\frac{-208}{58875}$	$\frac{176}{153075}$
Tolerance	2.0E-04	7.0E-03

For the step of time  $n^{\circ}2$ , displacements of the ends of the springs, off-line to the plate, are moved of  $+5.0\text{E-}03\text{ m}$ . Results of `Code_Aster` are compared with the discrete solution, which corresponds to the solution of the modelled problem.

Nature of the results	$U_A=U_D$	$U_B=U_C$
Continuous solution	$\frac{-4}{1125} + \frac{5}{1000}$	$\frac{4}{3375} + \frac{5}{1000}$
Discrete solution ( $n=12$ )	$\frac{-208}{58875} + \frac{5}{1000}$	$\frac{176}{153075} + \frac{5}{1000}$
Tolerance	4.0E-04	2.0E-04

## 4 Modeling B

### 4.1 Characteristics of modeling

The plate is modelled in 2D in plane deformations, by elements QUAD4. The springs are modelled by SEG2 affected of a modeling 2D\_DIS\_T whose characteristics are K\_T\_D\_L. They are the discrete ones in translation having a diagonal matrix, to see the documentation of AFFE\_CARA\_ELEM.

### 4.2 Characteristics of the grid

The plate is cut out with  $n_y = 16$ . The length of the plate is  $b = 2 m$ .

### 4.3 Sizes tested and results

For the step of time  $n^{\circ}1$ , displacements of the ends of the springs, off-line to the plate, are imposed on zero. Results of Code\_Aster are compared with the discrete solution, which corresponds to the solution of the modelled problem. The solution of balance is obtained for  $n = 12$ , equation 2.2-4.

Nature of the results	$U_A$	$U_B$
Continuous solution	$\frac{-4}{1125}$	$\frac{4}{3375}$
Discrete solution ( $n = 12$ )	$\frac{-208}{58875}$	$\frac{176}{153075}$
Tolerance	8.0E-07	8.0E-07

For the step of time  $n^{\circ}2$ , displacements of the ends of the springs, off-line to the plate, are moved of  $+5.0E-03 m$ . Results of Code\_Aster are compared with the discrete solution, which corresponds to the solution of the modelled problem.

Nature of the results	$U_A$	$U_B$
Continuous solution	$\frac{-4}{1125} + \frac{5}{1000}$	$\frac{4}{3375} + \frac{5}{1000}$
Discrete solution ( $n = 12$ )	$\frac{-208}{58875} + \frac{5}{1000}$	$\frac{176}{153075} + \frac{5}{1000}$
Tolerance	8.0E-07	8.0E-07

## 5 Summary of the results

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The use of the discrete, affected elements on nodes or segments, with a material of the type `DIS_CONTACT` and used with `STAT_NON_LINE` (behavior `COMP_INCR` and relation `DIS_CHOC`) allows to model a unilateral behavior of the springs.

In 2D as in 3D, the use of the keyword `RIGI_PARASOL` order `AFFE_CARA_ELEM` allows to assign to the springs stiffnesses proportional to the length or the surface of the elements to which they are connected.

The behavior being unilateral, it is necessary that *Code\_Aster* makes several iterations to find the position of balance. It is also possible to encounter problems of convergence related to a loss of precision, due to a bad conditioning of the matrix of stiffness during iterations. Stiffness of the springs being able to cancel itself of an iteration with the other.