SSNL501 - Beam fixed at the two ends subjected to a uniform pressure

Summary:

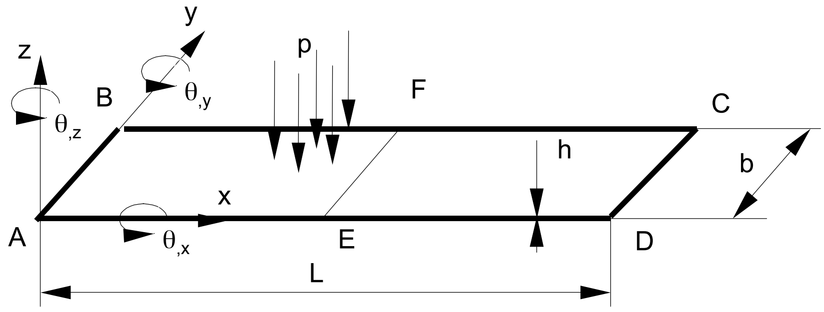
This test represents a quasi-static calculation of a fixed beam subjected to a uniform pressure, made up of a perfectly plastic elastic material. This test makes it possible to validate following modelings finite elements:

- DKT (TRIA3, QUAD4),
- COQUE_3D (TRIA7, QUAD9),
- POU_D_TGM (SEG2).

The limiting pressure is compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



1.2 Properties of material

The properties of material constituting the beam are:

$E=2.10^{11}\,Pa$ Young modulus
$\nu=0.3$ Poisson's ratio

The material follows a perfectly plastic law of elastic behavior:

$\sigma_e=2.35\,10^8\,Pa$ Yield stress
$\varepsilon_e=1.175\,10^{-3}$ Limiting elastic strain

1.3 Boundary conditions and loadings

- Boundary conditions: Dimensioned AB and CD embedded
- Following imposed displacement Z in E ($x=L/2$) :

$$DZ_e=6.609\,10^{-3}\,m \ \left(DZ_e=\frac{q_e L^4}{384EI}\right)$$

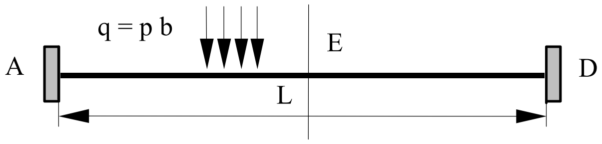
$DZ(E)$ vary 0 with $30\,DZ_e$

1.4 Initial conditions

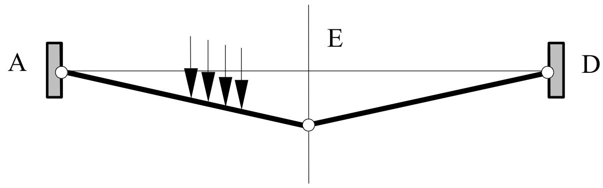
Without object

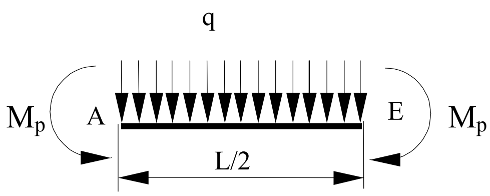
2 Reference solution

2.1 Method of calculating used for the reference solution



(a)





(b) (c)

The ruin of the beam appears when there are plastic kneecaps at the points A , D and E (figure b). The static balance of the left half of the beam, makes it possible to determine the pressure limits (figure c)

$$\sum M_A = 2 M_p - q_L (\frac{L}{2})(\frac{L}{4}) = 0 \qquad \Rightarrow q_L = \frac{16 M_p}{L^2}$$

where:

q_L represent the limiting pressure

M_p represent the plastic moment ($M_p = \sigma_e \frac{bh^2}{4}$)

The appearance of the first plastic point on fibre external of the beam takes place at the points A and D , other fibres being in elastic mode. The pressure limits elastic is of $q_e = 2\sigma_e \frac{bh^2}{L^2}$.

2.2 Results of reference

Limiting pressure $q_L = 39\,166.67\,N/m$

2.3 Uncertainties on the solution

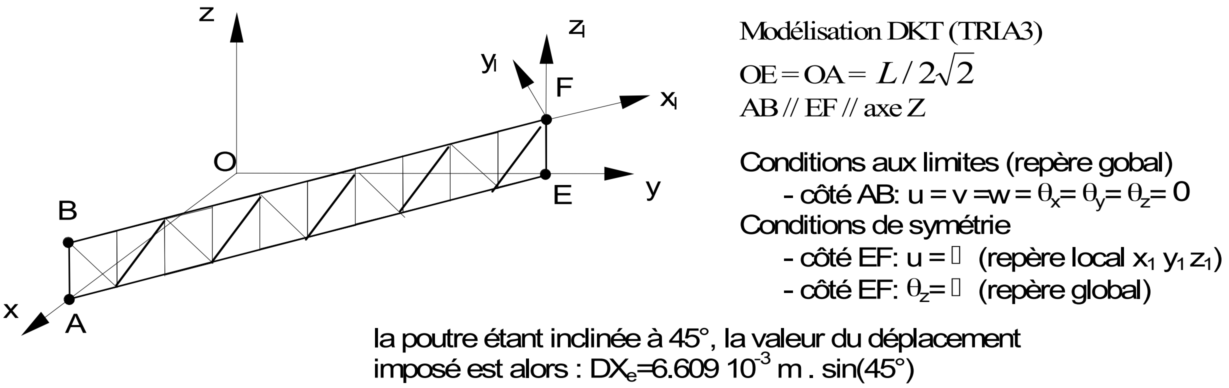
Analytical solution

2.4 Bibliographical references

1) WILLIAM A. NASH: Theory and problems of Strength of material, Schaum' S outline series, 2/ed, McGRAW-HILL

3 Modeling A

3.1 Characteristics of modeling



3.2 Characteristics of the grid

Many nodes: 43
Number of meshs and type: 20 TRIA3

3.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	Aster	% difference
5	ETA_PILOTAGE	5	1.0	1.11133	11.11
25	ETA_PILOTAGE	15	1.0	1,142	11.42

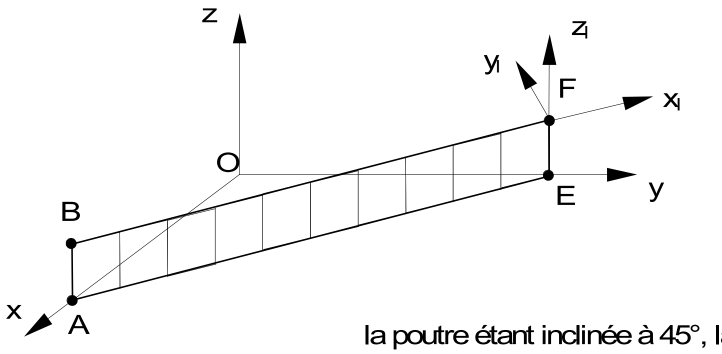
3.4 Remarks

In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.
Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the results DKT TRIA3 when the grid is refined.

4 Modeling B

4.1 Characteristics of modeling



Modélisation DKT (QUAD4)
$OE = OA = L/2\sqrt{2}$
AB // EF // axe Z

Conditions aux limites (repère gobal)
- côté AB: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie
- côté EF: u = (repère local x_1 y_1 z_1)
- côté EF: θ_z = (repère global)

la poutre étant inclinée à 45°, la valeur du déplacement imposé est alors : $DX_e = 6.609\ 10^{-3}$ m . sin(45°)

4.2 Characteristics of the grid

Many nodes: 43
Number of meshs and type: 10 QUAD4

4.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	*Aster*	% difference
5	`ETA_PILOTAGE`	5	1.0	1.0837	8.37
25	`ETA_PILOTAGE`	25	1.0	1.0998	9.98

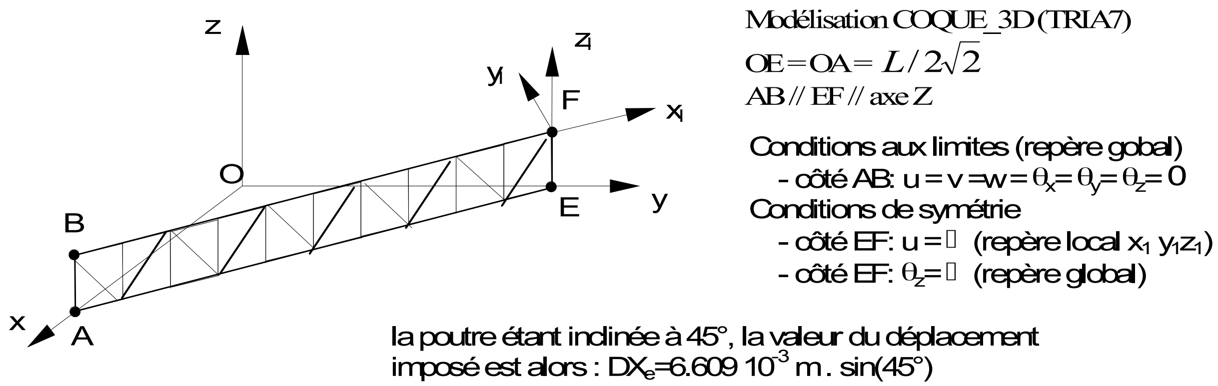
4.4 Remarks

In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement (‘`DDL_IMPO'`). This method provides for each value of imposed displacement, a multiplying coefficient of the loading (‘`ETA_PILOTAGE'`). The value of the loading imposed in ‘`AFFE_CHAR_MECA'` is equal to the limiting pressure, consequently the value of reference of the parameter ‘`ETA_PILOTAGE'` is equal to 1.
Calculations were stopped when the value of the parameter `ETA_PILOTAGE` stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the results `DKT QUAD4` when the grid is refined.

5 Modeling C

5.1 Characteristics of modeling



5.2 Characteristics of the grid

Many nodes: 83
Number of meshs and type: 20 TRIA7

5.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	*Aster*	% difference
5	ETA_PILOTAGE	5	1.0	1.1143	11.43
15	ETA_PILOTAGE	15	1.0	1.1682	16.82

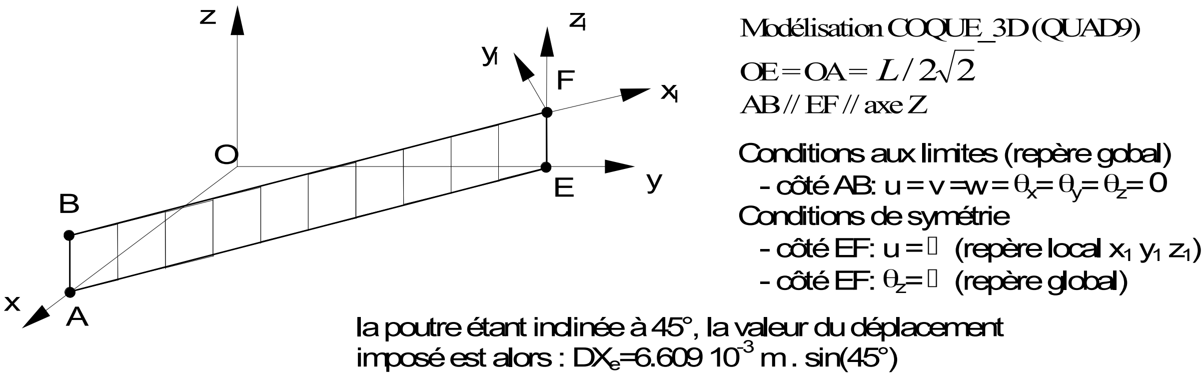
5.4 Remarks

In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.
Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the hulls results 3D TRIA7 when the grid is refined.

6 Modeling D

6.1 Characteristics of modeling



6.2 Characteristics of the grid

Many nodes: 54
Number of meshs and type: 10 QUAD9

6.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	*Aster*	% difference
5	ETA_PILOTAGE	5	1.0	1.0978	9.78
25	ETA_PILOTAGE	25	1.0	1.1085	10.85

6.4 Remarks

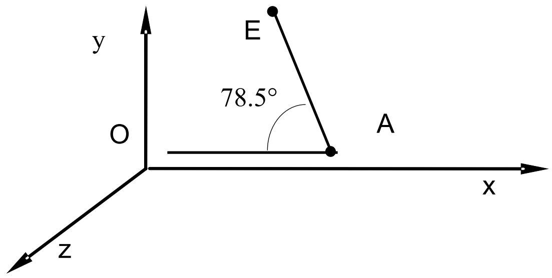
In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement (‘DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading (‘ETA_PILOTAGE'). The value of the loading imposed in ‘AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter ‘ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the hulls results 3D QUAD9 when the grid is refined.

7 Modeling E

7.1 characteristics of modeling



Modélisation POU_D_TGM
A= 2.1 0.7 0.
AE =0.75 m

Conditions aux limites (repère gobal)
- noeud A: u = v =w = θ_x= θ_y= θ_z= 0

Conditions de symétrie
- noeud E: u = 0 , θ_z= 0

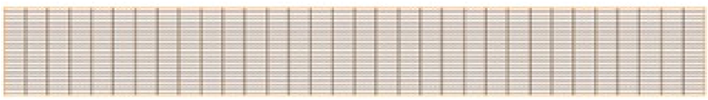
la poutre étant inclinée à 78.5°, la valeur du déplacement imposé est alors : DX_e=6.609 10^{-3} m . sin(78.5°)

7.2 characteristics of the grid

- grid of the beam

  Many nodes: 21
  Number of meshs and type: 10 SEG2

- grid of the section



  Many nodes: 355
  Number of meshs and type: 280 QUAD4

7.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	% Tolerance
5	ETA_PILOTAGE	10	1.0	14.
5.24	ETA_PILOTAGE	12	1.0	14.

In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The solution beam improves appreciably when the grid is refined.

8 Summary of the results

Concerning the evolution of normal displacement in the center of the plate according to the parameter of piloting, one observes that:

- modelings comprising of the quadrangles give better results, compared to the meshs triangles.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself, or when calculation was not possible any more. Taking into account the grids used, the got results are satisfactory. The errors observed are for modelings:

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- DKT : 11.4% for the mesh TRIA3 (With) and 9.9% for the mesh QUAD4 (B),
- COQUE_3D : 16% for the mesh TRIA7 (C) and 10.8% for the mesh QUAD9 (D),
- POUT_D_TGM : 9% (E).

But it is noted that with a finer grid at the ends and the center of the plate, place or plasticization appears, it is possible to minimize the error compared to the reference solution.