# SSNP15 - Plate in traction-shearing - Von Mises (isotropic work hardening)

**Summary:**

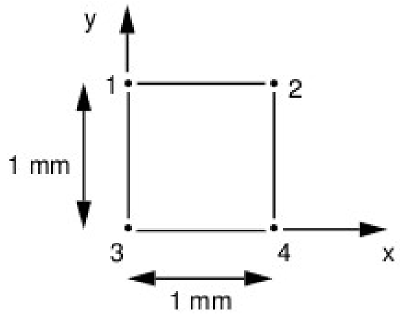
This test $2D$ plane constraints quasi-static, from guide VPCS [1], enters within the framework of the validation of the relations of elastoplastic behavior. An element of volume, consisted of a plastic material with linear isotropic work hardening, is subjected to a tractive effort and a shearing force.

The principal interest of this test lies in the nonradial character of the loading.

# 1 Problem of reference

## 1.1 Geometry

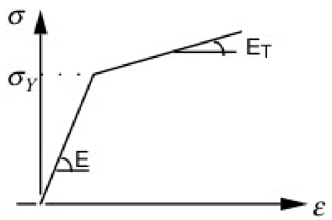
The constraints and deformations are homogeneous in the element of volume. This one can be represented by an element plan or voluminal, for example:




## 1.2 Material properties

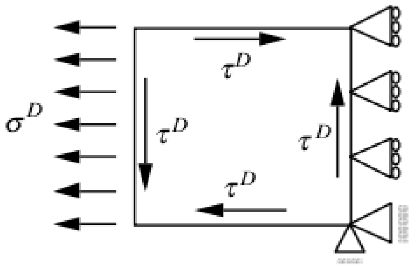
Elastoplastic law of behaviour to linear kinematic work hardening.

$E = 195000\ MPa$
$\nu = 0.3$
$\sigma_\gamma = 181\ MPa$
$E_T = 1930\ MPa$



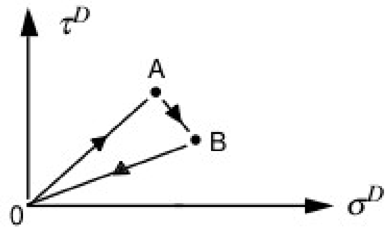

## 1.3 Boundary conditions and loadings

The element of volume is blocked according to $Ox$ along the side $[2,4]$ while being subjected to a traction $\sigma^D$ and a shearing force $\tau^D$.




The way of loading is the following:

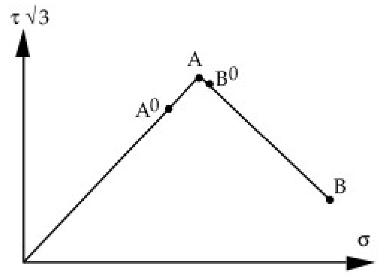
| | $\sigma^D$ [MPa] | $\tau^D$ [MPa] |
|---|---|---|
| $A$ | 151.2 | 93.1 |
| $B$ | 257.2 | 33.1 |



# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

In the plan $(\sigma, \tau\sqrt{3})$, the standard of von Mises results in the classical distance in the octahedral plan between the projection of the state of stresses and hydrostatic line, so that one can immediately predict the phases of load and of discharge at the time of the way of loading, since it is respectively the phases where the standard grows or decrease:

| Way of loading | Digital values $(\sigma, \tau)[\mathrm{MPa}]$ | | | Phases of loading | |
|---|---|---|---|---|---|
|   | $A^0$ | (123.8; | 76.23) | $O-A^0$ | Elastic load |
| | $A$ | (151.2. ; | 93.10) | $A^0-A$ | Plastic load |
| | $B^0$ | (158.23; | 89.12) | $A-B^0$ | Discharge |
| | $B$ | (257.2; | 33.10) | $B^0-B$ | Plastic load |

The loading is done according to a curve parameterized by the moment:
•Phase 1: plasticization of the point $O$ at the point $A$ (moments 0.0 to 1.0).
•Phase 2: discharge of the point $A$ at the point $B$ (moments 1.0 to 2.0).
•Phase 3: total discharge of the point $B$ at the point $C$ (moments 2.0 to 3.0).

| Moment | $\sigma$ [MPa] | $\tau$ [MPa] | Many steps |
|---|---|---|---|
| 0.0 – Not $O$ | 0 | 0 | |
| 0.1 | | | 1 |
| 0.9 | | | 10 |
| 1.0 – Not $A$ | 151.2 | 93.1 | 1 |
| 2.0 – Not $B$ | 257.2 | 33.1 | 40 |
| 3.0 – Not $C$ | 0 | 0 | 1 |

### 2.1.1 Approach of resolution

Mechanically, it is about a test $0\mathrm{D}$ controlled in constraints, the material being elastoplastic with criterion of von Mises and linear isotropic work hardening. For a loading controlled in constraint, one easily determines the cumulated plastic deformation:

$$F(\sigma, p)=\sigma_{éq}-\sigma_y-R' p\leq 0 \quad \Rightarrow \quad p=\frac{\sigma_{éq}-\sigma_y}{R'} \quad \text{en charge} \qquad \textbf{éq 2.1.1-1}$$

The integration of the plastic deformation is of course more delicate. The equation of flow is written:

$$\dot{\varepsilon}^p=\frac{3}{2}\dot{p}\frac{\tilde{\sigma}}{\sigma_{éq}} \quad \Rightarrow \quad \dot{\varepsilon}^p=\frac{3}{2R'}.\frac{\dot{\sigma}_{éq}}{\sigma_{éq}}.\tilde{\sigma} \quad \text{en charge}$$ **éq 2.1.1-2**

Lastly, one will deduce the deformation via the relation from state:

$$\varepsilon=\varepsilon^p+E^{-1}:\sigma \quad \Rightarrow \quad \varepsilon_{xx}=\varepsilon^p_{xx}+\frac{\sigma}{E} \quad \text{et} \quad \varepsilon_{xy}=\varepsilon^p_{xy}+\frac{\tau}{2\mu}$$ **éq 2.1.1-3**

### 2.1.2 Treatment of the phase of radial loading

Let us notice that in phase of radial loading, the law of flow [éq 2.1.2-1] is integrated directly:

$$\varepsilon^p=\frac{3}{2}p\frac{\tilde{\sigma}}{\sigma_{éq}}$$ **éq 2.1.2-1**

The cumulated plastic deformation is then given by [éq 2.1.1-1], the plastic deformation by [éq 2.1.2-1] and the total deflection by [éq 2.1.1-3]. With:

| | | | | | |
|---|---|---|---|---|---|
| $E$ | $=195\ 000\,MPa$ | $2\mu$ | $=150\ 000\,MPa$ | $R'$ | $=1\ 949.29\,MPa$ |
| One obtains: | | | | | |
| $p(A)$ | $=2.0547.10^{-2}$ | $\varepsilon^p_{xx}(A)$ | $=1.4054.10^{-2}$ | $\varepsilon_{xx}(A)$ | $=1.4830.10^{-2}$ |
| | | $\varepsilon^p_{xy}(A)$ | $=1.2981.10^{-2}$ | $\varepsilon_{xy}(A)$ | $=1.3601.10^{-2}$ |

### 2.1.3 Treatment of the phase of nonradial loading

In the phase of nonradial loading $B^0-B$, one can parameterize the way of constraint by:

$$\sigma(q)=\sigma^{B^0}+q\underbrace{(\sigma^B-\sigma^{B^0})}_{\text{direction fixe}} \quad \text{avec} \quad 0\leq q\leq 1$$ **éq 2.1.3-1**

As the way of loading remains confined in the traction-shearing plan $(\sigma,\tau)$, one will may find it beneficial to represent the state of stress by a complex number:

$$\Sigma=\sigma+i\sqrt{(3)}\tau \quad \Rightarrow \quad \sigma_{\text{éq}}=|\Sigma| \quad \text{et} \quad \Sigma(q)=\Sigma^{B^0}+q\underbrace{(\Sigma^{B^0}-\Sigma^0)}_{\text{direction fixe}}$$ **éq 2.1.3-2**

The integration of the law of flow [éq 2.1.1-2], followed by an integration by part, makes it possible to express the plastic deformation:

$$\frac{2\text{R}'}{3}[\varepsilon^p]_0^1=\int_0^1\frac{\dot{\sigma}_{\text{éq}}}{\sigma_{\text{éq}}}\tilde{\sigma}\,dq=\left[\ln(\sigma_{\text{éq}})\tilde{\sigma}\right]_0^1-\frac{1}{2}\underbrace{\dot{\tilde{\sigma}}}_{\tilde{\sigma}^B-\tilde{\sigma}^{B^0}}\int_0^1\ln(\sigma^2_{\text{éq}})\,dq$$

The adoption of the complex plan allows an easy calculation of the last integral:

$$\int_0^1 \ln(\sigma_{\text{éq}}^2)\,dq = \int_0^1 \ln(\Sigma\,\bar{\Sigma})\,dq = \int_0^1 \ln(\Sigma)\,dq + \int_0^1 \ln(\bar{\Sigma})\,dq = 2\,\mathrm{Re}\left[\int_0^1 \ln(\Sigma)\,dq\right] = 2\,\mathrm{Re}\left[\frac{\Sigma\ln(\Sigma)-\Sigma}{\Sigma^B-\Sigma^{B^0}}\right]_0^1$$

Finally, the increment of plastic deformation on the way $B^0-B$ is worth:

$$[\varepsilon^p]_{B^0}^B = \frac{3}{2R'}[\ln(\sigma_{\text{éq}})\tilde{\sigma}]_{B^0}^B - \frac{3}{2R'}\mathrm{Re}\left[\frac{\Sigma\ln(\Sigma)-\Sigma}{\Sigma^B-\Sigma^{B^0}}\right]_{B^0}^B (\tilde{\sigma}^B - \tilde{\sigma}^{B^0}) \qquad \textbf{éq 2.1.3-4}$$

## 2.2 Results of reference

By calculating the plastic deformation cumulated by [éq 2.1.1-1], the plastic deformation by [éq 2.1.3-4] and the total deflection by [éq 2.1.1-3], one obtains:

| | | | | | |
|---|---|---|---|---|---|
| $p(B)$ | $=4.2329.10^{-2}$ | $\varepsilon_{xx}^p(B)$ | $=3.3946.10^{-2}$ | $\varepsilon_{xx}(B)$ | $=3.5265.10^{-2}$ |
| One obtains: | | $\varepsilon_{xy}^p(B)$ | $=2.0250.10^{-2}$ | $\varepsilon_{xy}(B)$ | $=2.0471.10^{-2}$ |

One will be interested in the values of the constraints, the deformations and the plastic deformation cumulated at the points $A$ and $B$ way of loading.

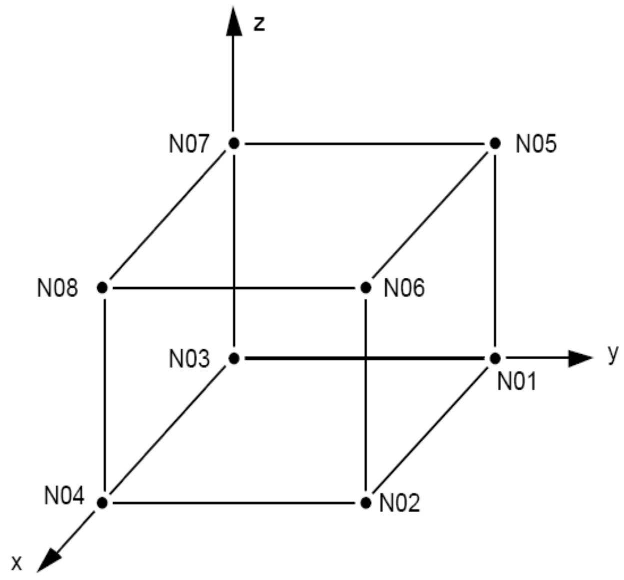
## 2.3 Bibliographical references

1) French company of the Mechanics. Guide of validation of the software packages of structural analysis (VPCS). Technical AFNOR, 1990.

# 3 Modeling A

## 3.1 Characteristics of modeling

Modeling 3D




The loading and the boundary conditions are modelled by:

- Condition of Dirichlet (keyword DDL_IMPO) :
  - Node $N04$ , $x=y=0$ ,
  - Node $N08$ , $x=y=z=0$ ,
  - Node $N02$ , $x=0$ ,
  - Node $N06$ , $x=0$ .
- Condition of Neumann, surface forces (keyword FORCE_FACE) :
  - on the faces (meshs of skin): $(1,5,6,2)$ , $(1,5,7,3)$ , $(3,4,8,7)$ and $(4,8,6,2)$ .

## 3.2 Characteristics of the grid

Many nodes: 8
Many meshs and types: 1 HEXA8, 4 QUAD4

## 3.3 Sizes tested and results

### 3.3.1 Case of VMIS_ISOT_LINE

| Identification | Type of reference | Value of reference | Tolerance |
|---|---|---|---|
| $\sigma_{xx}$ at the moment $A$ | 'ANALYTICAL' | 1,512E+002 | 0.1% |
| $\sigma_{xy}$ at the moment $A$ | 'ANALYTICAL' | 9,310E+001 | 0.1% |
| $p$ at the moment $A$ | 'ANALYTICAL' | 2,0547E-002 | 0.1% |
| Rate of triaxiality $TRIAX$ at the moment $A$ | 'ANALYTICAL' | 2,2800E-001 | 0.1% |
| $\varepsilon_{xx}$ at the moment $A$ | 'ANALYTICAL' | 1,48297E-002 | 0.1% |
| $\varepsilon_{xy}$ at the moment $A$ | 'ANALYTICAL' | 1,36014E-002 | 0.1% |
| $\varepsilon^p_{xx}$ at the moment $A$ | 'ANALYTICAL' | 1,40543E-002 | 0.1% |
| $\varepsilon^p_{xy}$ at the moment $A$ | 'ANALYTICAL' | 1,29807E-002 | 0.1% |
| $p$ at the moment $B$ | 'ANALYTICAL' | 4,23293E-002 | 1.0% |
| Rate of triaxiality $TRIAX$ at the moment $B$ | 'ANALYTICAL' | 3,25349E-001 | 0.1% |
| $\varepsilon_{xx}$ at the moment $B$ | 'ANALYTICAL' | 3,5265E-002 | 1.0% |
| $\varepsilon_{xy}$ at the moment $B$ | 'ANALYTICAL' | 2,0471E-002 | 1.0% |
| $\varepsilon^p_{xx}$ at the moment $B$ | 'ANALYTICAL' | 3,3946E-002 | 1.0% |
| $\varepsilon^p_{xy}$ at the moment $B$ | 'ANALYTICAL' | 2,0250E-002 | 1.0% |

as well as the indicators of load-discharge:

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| INDIC_ENER at the moment $A$ | 'ANALYTICAL' | 0 | 0.1% |
| INDIC_ENER at the moment $B$ | 'ANALYTICAL' | 3,26E-002 | 3.0% |
| INDIC_SEUIL at the moment $A$ | 'ANALYTICAL' | 0 | 0.1% |
| INDIC_SEUIL at the moment $B$ | 'ANALYTICAL' | 3,26E-002 | 3.0% |
| INDIC_ENER at the moment $C$ (discharge supplements) | 'ANALYTICAL' | 4,69E-002 | 3.0% |
| INDIC_SEUIL at the moment $C$ (discharge supplements) | 'ANALYTICAL' | 1.0 | 1.0% |

| | | | |
|---|---|---|---|
| DERA_ELNO/RADI_V at moment 0.1 | 'ANALYTICAL' | 0.0 | 1.0% |

### 3.3.2 Case of **VMIS_ECMI_LINE**

Only energies are calculated, and one compares compared to the case VMIS_ISOT_LINE :

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| ETOT_ELGA/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ETOT_ELGA/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ETOT_ELGA/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ETOT_ELGA/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |
| ETOT_ELNO/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ETOT_ELNO/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ETOT_ELNO/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ETOT_ELNO/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |
| ETOT_ELEM/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ETOT_ELEM/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ETOT_ELEM/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ETOT_ELEM/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |
| ETOT_NOEU/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40650 | 0.1% |

### 3.3.3 Case of **DERA_ELxx**

The indicators of discharge are tested DCHA_V and of loss of radiality DCHA_R in the mesh $CUBE$ :

- at the first point of Gauss (DERA_ELGA),
- with the node $N_2$ (DERA_ELNO).

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| DERA_ELGA/DCHA_V with increment 2 | 'NON_REGRESSION' | 3.07692E-1 | 0,10% |
| DERA_ELNO/DCHA_V with increment 2 | 'NON_REGRESSION' | 3.07692E-1 | 0,10% |
| DERA_ELGA/RADI_V with increment 2 | 'NON_REGRESSION' | 0.0 | 0,10% |

One tests in the mesh $CUBE$ at the point of gauss n°1:

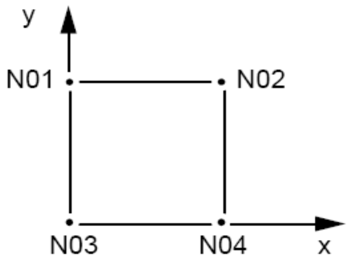
- The indicator of discharge IND_DCHa who allows to know if the discharge remains elastic or if there would be a risk of plasticization if a pure kinematic work hardening were used,
- The indicator VAL_DCHA who indicate the proportion of exit of the criterion.

| Identification | | Type of reference | Value | Tolerance |
|---|---|---|---|---|
| DERA_ELGA | IND_DCHA with increment 10 | 'NON_REGRESSION' | 2 | 0.10% |
| | VAL_DCHA with increment 10 | 'NON_REGRESSION' | 0.0 | 0,001 |
| | IND_DCHA with increment 12 | 'NON_REGRESSION' | -1 | 0.10% |
| | VAL_DCHA with increment 12 | 'NON_REGRESSION' | 0.0 | 0,001 |
| | IND_DCHA with the increment 14 | 'NON_REGRESSION' | -2 | 0.10% |
| | VAL_DCHA with the increment 14 | 'NON_REGRESSION' | 1.057898 | 0.10% |
| | IND_DCHA with increment 52 | 'NON_REGRESSION' | -2 | 0.10% |
| | VAL_DCHA with increment 52 | 'NON_REGRESSION' | 1.057898 | 0.10% |

# 4 Modeling B

## 4.1 Characteristics of modeling

Modeling in plane constraints: C_PLAN




The loading and the boundary conditions are modelled by:

- Condition of Dirichlet (keyword DDL_IMPO) :
  - Node $N04$ , $x=y=0$ ,
  - Node $N02$ , $x=0$ .

- Condition of Neumann, surface forces (keyword FORCE_CONTOUR) :
  - on the faces (meshs of skin): (1, 2), (2, 4), (4, 3) and (3, 1).

## 4.2 Characteristics of the grid

Many nodes: 4
Many meshs and types: 1 QUAD4, 4 SEG2

## 4.3 Sizes tested and results

| Identification | Moments | Reference | % Tolerance |
|---|---|---|---|
| $\sigma_{xx}$ | $A$ | 151.2 | 0.1 |
| $\sigma_{xy}$ | $A$ | 93.1 | 0.1 |
| $\varepsilon_{xx}$ | $A$ | $1.4830\ 10^{-2}$ | 0.1 |
| $\varepsilon_{xy}$ | $A$ | $1.3601\ 10^{-2}$ | 0.1 |
| $p$ | $A$ | $2{,}055\ 10^{-2}$ | 0.1 |
| Rate of triaxiality $TRIAX$ | $A$ | $2.28\ 10^{-1}$ | 0.1 |
| $\varepsilon_{xx}$ | $B$ | $3.5265\ 10^{-2}$ | 1.0 |
| $\varepsilon_{xy}$ | $B$ | $2.0471\ 10^{-2}$ | 1.0 |
| $p$ | $B$ | $4.2329\ 10^{-2}$ | 1.0 |
| Rate of triaxiality $TRIAX$ | $B$ | $3.25349\ 10^{-1}$ | 0.1 |
| $\varepsilon_{xx}^{p}$ | $B$ | $3.3946\ 10^{-2}$ | 1.0 |
| $\varepsilon_{xy}^{p}$ | $B$ | $2.0250\ 10^{-2}$ | 1.0 |

as well as the indicators of load-discharge:

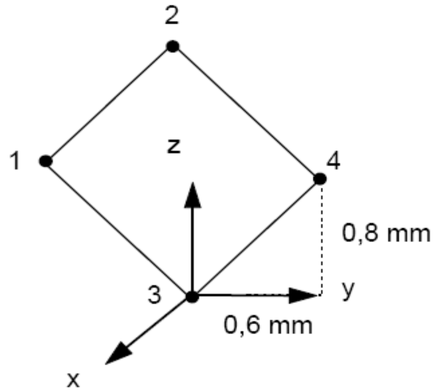
| Identification | Moments | Reference | % tolerance |
|---|---|---|---|
| `INDIC_ENER` | $A$ | 0. | 0.1 |
| `INDIC_SEUIL` | $A$ | 0. | 0.1 |
| `INDIC_ENER` | $B$ | $3.26\ 10^{-2}$ | 3.00 |
| `INDIC_SEUIL` | $B$ | $9.71\ 10^{-2}$ | 1.00 |

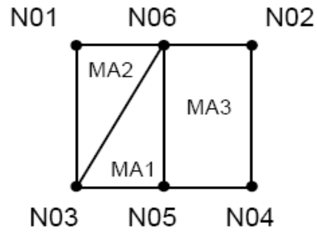
# 5 Modeling C

## 5.1 Characteristics of modeling

Modeling hull DKT-DKQ

## 5.2 Geometry






Dimensions of the structure do not change compared to the problem of reference, only differs its orientation.

Coordinates of the nodes:

| Nodes | $x$ | $y$ | $z$ |
|---|---|---|---|
| *N01* | 0 | – 0.8 | 0.6 |
| *N02* | 0 | – 0.2 | 1.4 |
| *N03* | 0 | 0 | 0 |
| *N04* | 0 | 0.6 | 0.8 |
| *N05* | 0 | 0.3 | 0.4 |
| *N06* | 0 | – 0.5 | 1 |

**Boundary conditions:**

```
DDL_IMPO= (_F (NOEUD=' NO4', DY=0.0, DZ=0.0,),
          _F (TOUT=' OUI', DX=0.0,),),
LIAISON_DDL=_F (NOEUD= ('NO2', 'NO2',),
                DDL= ('DY', 'DZ',),
                COEF_MULT= (0.75, 1.0,),
                COEF_IMPO=0.0,),);
```

**Loading:**

One imposes surface forces (keyword FORCE_ARETE) on the faces (meshs of skin SEG2) $(1,2)$, $(2,4)$, $(4,3)$ and $(3,1)$.

**Specificity DKT and DKQ:**

Two layers in the thickness for plasticity.

## 5.3 Characteristics of the grid

Many nodes: 6
Many meshs and types: 2 TRIA3 and 1 QUAD4

## 5.4 Sizes tested and results

Displacements tested are those of the problem of reference by taking account of the rotation of the structure.

The generalized strains, stresses and efforts are tested in the reference mark user defined by the order ANGL_REP. The values are thus those given by the problem of reference.

The values are tested at the point $A$ way of loading $OA$ . One tests as follows:

Displacements (DEPL). He result easily from the reference solution since the deformation is homogeneous.

| Identification | Reference | % Tolerance |
|---|---|---|
| $DY$ $N01$ | $1.86722\ 10^{-2}$ | 1,00E-004 |
| $DZ$ $N01$ | $-3.25413\ 10^{-2}$ | 1,00E-004 |
| $DY$ $N06$ | $1,224\ 10^{-2}$ | 1,00E-004 |
| $DZ$ $N06$ | $-1.88485\ 10^{-2}$ | 1,00E-004 |
| $DY$ $N02$ | $5.80782\ 10^{-3}$ | 1,00E-004 |
| $DZ$ $N02$ | $-4.35586\ 10^{-3}$ | 1,00E-004 |

Constraints (SIGM_ELNO).

| Identification | Reference | % Tolerance |
|---|---|---|
| $SIXX$ $MA2$ $N01$ | $1,512\ 10^{2}$ | 1.0 |
| $SIXY$ $MA2$ $N01$ | 93.1 | 1.0 |
| $SIXX$ $MA1$ $N03$ | $1,512\ 10^{2}$ | 1.0 |
| $SIXY$ $MA1$ $N03$ | 93.1 | 1.0 |
| $SIXX$ $MA2$ $N03$ | $1,512\ 10^{2}$ | 1.0 |
| $SIXY$ $MA2$ $N03$ | 93.1 | 1.0 |
| $SIXX$ $MA3$ $N02$ | $1,512\ 10^{2}$ | 1.0 |
| $SIXY$ $MA3$ $N02$ | 93.1 | 1.0 |

Efforts generalized by elements with the nodes (EFGE_ELNO).

| Identification | Reference | % tolerance |
|---|---|---|
| $NXX$ $MA2$ $N01$ | $3,024\ 10^{2}$ | 1.0 |
| $NXY$ $MA2$ $N01$ | $1,862\ 10^{2}$ | 1.0 |
| $NXX$ $MA1$ $N03$ | $3,024\ 10^{2}$ | 1.0 |
| $NXY$ $MA1$ $N03$ | $1,862\ 10^{2}$ | 1.0 |
| $NXX$ $MA2$ $N03$ | $3,024\ 10^{2}$ | 1.0 |
| $NXY$ $MA2$ $N03$ | $1,862\ 10^{2}$ | 1.0 |
| $NXX$ $MA3$ $N02$ | $3,024\ 10^{2}$ | 1.0 |
| $NXY$ $MA3$ $N02$ | $1,862\ 10^{2}$ | 1.0 |

Deformations by element with the nodes starting from displacements (EPSI_ELNO).

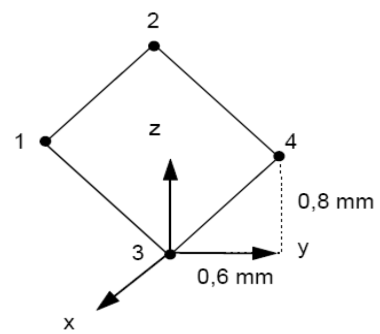
| Identification | Reference | % tolerance |
|---|---|---|
| *EPXX MA2 N01* | $1.48297\ 10^{-2}$ | 0.01 |
| *EPYY MA2 N01* | $-7.25977\ 10^{-3}$ | 0.01 |
| *EPXY MA2 N01* | $1.36014\ 10^{-2}$ | 0.01 |
| *EPXX MA1 N03* | $1.48297\ 10^{-2}$ | 0.01 |
| *EPYY MA1 N03* | $-7.25977\ 10^{-3}$ | 0.01 |
| *EPXY MA1 N03* | $1.36014\ 10^{-2}$ | 0.01 |
| *EPXX MA2 N03* | $1.48297\ 10^{-2}$ | 0.01 |
| *EPYY MA2 N03* | $-7.25977\ 10^{-3}$ | 0.01 |
| *EPXY MA2 N03* | $1.36014\ 10^{-2}$ | 0.01 |
| *EPXX MA3 N02* | $1.48297\ 10^{-2}$ | 0.01 |
| *EPYY MA3 N02* | $-7.25977\ 10^{-3}$ | 0.01 |
| *EPXY MA3 N02* | $1.36014\ 10^{-2}$ | 0.01 |
| *EPXX MA3 N04* | $1.48297\ 10^{-2}$ | 0.01 |
| *EPYY MA3 N04* | $-7.25977\ 10^{-3}$ | 0.01 |
| *EPXY MA3 N04* | $1.36014\ 10^{-2}$ | 0.01 |

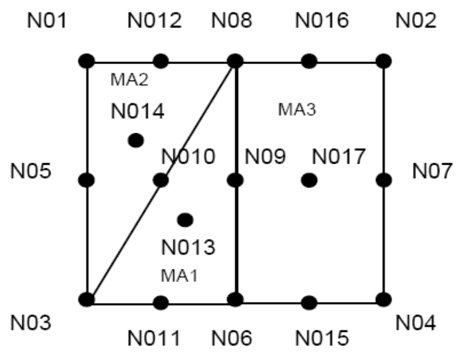
# 6 Modeling D

## 6.1 Characteristics of modeling

Modeling hull COQUE_3D

## 6.2 Geometry




Dimensions of the structure do not change compared to the problem of reference, only differs its orientation.

Coordinates of the nodes:

| Nodes | $x$ | $y$ | $z$ |
|---|---|---|---|
| *N01* | 0 | – 0.8 | 0.6 |
| *N02* | 0 | – 0.2 | 1.4 |
| *N03* | 0 | 0 | 0 |
| *N04* | 0 | 0.6 | 0.8 |
| *N05* | 0 | – 0.4 | 0.3 |
| *N06* | 0 | 0.3 | 0.4 |
| *N07* | 0 | 0.2 | 1.1 |
| *N08* | 0 | – 0.5 | 1 |
| *N09* | 0 | – 0.1 | 0.7 |
| *N010* | 0 | – 0.25 | 0.5 |
| *N011* | 0 | 0.15 | 0.2 |
| *N012* | 0 | – 0.65 | 0.8 |
| *N013* | 0 | – 0.06666 | 0.466666 |
| *N014* | 0 | – 0.433333 | 0.533333 |
| *N015* | 0 | 0.45 | 0.6 |
| *N016* | 0 | – 0.35 | 1.2 |
| *N017* | 0 | 0.05 | 0.9 |

Boundary conditions:

```
DDL_IMPO= (_F (NOEUD=' NO4', DX=0., DY=0., DZ=0., DRX=0., DRY=0., DRZ=0.),
           _F (NOEUD=' NO2', DRX=0., DRY=0., DRZ=0.),
           _F (NOEUD=' NO7', DRX=0., DRY=0., DRZ=0.)),

LIAISON_DDL= (_F (NOEUD= ('NO2', 'NO2',),
                 DDL= ('DY', 'DZ',),
                 COEF_MULT= (0.75, 1. ,),
                 COEF_IMPO=0.),
              _F (NOEUD= ('NO7', 'NO7',),
                 DDL= ('DY', 'DZ',),
                 COEF_MULT= (0.75, 1. ,),
                 COEF_IMPO=0.),),)
```

**Loading:**

One imposes surface forces (keyword FORCE_ARETE) on the faces (meshs of skin SEG3) $(1,2)$, $(2,4)$, $(4,3)$ and $(3,1)$.

## 6.3 Characteristics of the grid

Many nodes: 17
Many meshs and types: 2 TRIA7 and 1 QUAD9

## 6.4 Sizes tested and results

Displacements tested are those of the problem of reference by taking account of the rotation of the structure.

The generalized strains, stresses and efforts are tested in the reference mark user defined by the order ANGL_REP. The values are thus those given by the problem of reference.

The values are tested at the point $A$ way of loading $OA$. One tests as follows:

Displacements (DEPL). He result easily from the reference solution since the deformation is homogeneous.

| Identification | Reference | % tolerance |
|---|---|---|
| $DY$ $N01$ | $1.86722\ 10^{-2}$ | 1,00E-004 |
| $DZ$ $N01$ | $-3.25413\ 10^{-2}$ | 1,00E-004 |
| $DY$ $N08$ | $1{,}224\ 10^{-2}$ | 1,00E-004 |
| $DZ$ $N08$ | $-1.84485\ 10^{-2}$ | 1,00E-004 |
| $DY$ $N02$ | $5.80782\ 10^{-3}$ | 1,00E-004 |
| $DZ$ $N02$ | $-4.35586\ 10^{-3}$ | 1,00E-004 |

Efforts generalized by elements with the nodes (EFGE_ELNO).

| Identification | Reference | % difference |
|---|---|---|
| $NXX$ $MA1$ $N01$ | $3{,}024\ 10^{2}$ | 1.0 |
| $NXY$ $MA1$ $N01$ | $1{,}862\ 10^{2}$ | 1.0 |
| $NXX$ $MA1$ $N03$ | $3{,}024\ 10^{2}$ | 1.0 |
| $NXY$ $MA1$ $N03$ | $1{,}862\ 10^{2}$ | 1.0 |
| $NXX$ $MA2$ $N03$ | $3{,}024\ 10^{2}$ | 1.0 |
| $NXY$ $MA2$ $N03$ | $1{,}862\ 10^{2}$ | 1.0 |
| $NXX$ $MA3$ $N02$ | $3{,}024\ 10^{2}$ | 1.0 |

| | | |
|---|---|---|
| $NXY$ $MA3$ $N02$ | 1,862 $10^{2}$ | 1.0 |

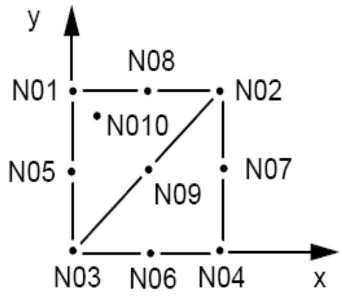
Deformations by element with the nodes starting from displacements (EPSI_ELNO).

| Identification | Reference | % tolerance |
|---|---|---|
| $EPXX$ $MA1$ $N01$ | 1.48297 $10^{-2}$ | 0.01 |
| $EPYY$ $MA1$ $N01$ | – 7.25977 $10^{-3}$ | 0.01 |
| $EPXY$ $MA1$ $N01$ | 1.36014 $10^{-2}$ | 0.01 |
| $EPXX$ $MA1$ $N03$ | 1.48297 $10^{-2}$ | 0.01 |
| $EPYY$ $MA1$ $N03$ | – 7.25977 $10^{-3}$ | 0.01 |
| $EPXY$ $MA1$ $N03$ | 1.36014 $10^{-2}$ | 0.01 |
| $EPXX$ $MA2$ $N03$ | 1.48297 $10^{-2}$ | 0.01 |
| $EPYY$ $MA2$ $N03$ | – 7.25977 $10^{-3}$ | 0.01 |
| $EPXY$ $MA2$ $N03$ | 1.36014 $10^{-2}$ | 0.01 |
| $EPXX$ $MA3$ $N02$ | 1.48297 $10^{-2}$ | 0.01 |
| $EPYY$ $MA3$ $N02$ | – 7.25977 $10^{-3}$ | 0.01 |
| $EPXY$ $MA3$ $N02$ | 1.36014 $10^{-2}$ | 0.01 |
| $EPXX$ $MA3$ $N04$ | 1.48297 $10^{-2}$ | 0.01 |
| $EPYY$ $MA3$ $N04$ | – 7.25977 $10^{-3}$ | 0.01 |
| $EPXY$ $MA3$ $N04$ | 1.36014 $10^{-2}$ | 0.01 |

# 7 Modeling E

## 7.1 Characteristics of modeling

Modeling COQUE_3D (MEC3TR7H)




Boundary conditions:

```
DDL_IMPO: (NODE: N04, DX: 0. , DY: 0.)
                   (NODE: N02, DX: 0.)
                   (NODE: NO7, DX: 0.)
                   (NODE: (N01, N02, N03), DZ: 0.)
```

Loading

```
FORCE_NODALE:      (NODE: N01,  FX: -9.683333,  FY: - 15.516666)
                   (NODE: N02,  FX: 15.516666,  FY: 15.516666)
                   (NODE: N03,  FX: - 40.716666,  FY: - 15.516666)
                   (NODE: N04,  FX: - 15.516666,  FY: 15.516666)
                   (NODE: N05,  FX: - 100.8,      FY: - 62.066666)
                   (NODE: N06,  FX: - 62.066666)
                   (NODE: N07,  FX: 62.066666)
                   (NODE: N08,  FX: 62.066666)
```

## 7.2 Characteristics of the grid

Many nodes: 11, Many meshs and types: 2 TRIA7

## 7.3 Sizes tested and results

The displacement of the node $N01$ results easily from the reference solution since the deformation is homogeneous. It is tested not in $A$ but also in a point $A^0$ $(\sigma^D = 123.8\ \ \tau^D = 76.2)$ way $OA$ . The cumulated plastic deformation is also tested.

| Identification | Moments | Reference | % tolerance |
|---|---|---|---|
| $DX \quad N01$ | $A^0$ | – 6,349 $10^{-4}$ | 0.5 |
| $DY \quad N01$ | $A^0$ | – 1,207 $10^{-3}$ | 0.5 |
| $DY \quad N01$ | $A$ | – 3,431 $10^{-2}$ | 0.5 |
| $p$ | $A$ | 2,055 10-2 | 1.0 |

## 7.4 Notice

Only the portion $OA$ way of the loading is actually tested.

# 8 Modeling F

## 8.1 Characteristics of modeling

This modeling is identical to modeling A. the only difference is at the level of the management of the step of time. The selected temporal discretization is minimal: $0{,}1\ ;0{,}9\ ;1\ ;2\ ;3$ .
One causes a recutting of the step of time so to convergence, the maximum increment of cumulated plastic deformation exceeds $0{,}1\%$ .

## 8.2 Sizes tested and results

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| $\sigma_{xx}$ at the moment $A$ | 'AUTRE_ASTER' | 151.2 | 0.1% |
| $\sigma_{xy}$ at the moment $A$ | 'AUTRE_ASTER' | 93.1 | 0.1% |
| $\varepsilon_{xx}$ at the moment $A$ | 'AUTRE_ASTER' | 1.48297E-2 | 0.1% |
| $\varepsilon_{xy}$ at the moment $A$ | 'AUTRE_ASTER' | 1.36014E-2 | 0.1% |
| $p$ at the moment $A$ | 'AUTRE_ASTER' | 2.05473E-2 | 0.1% |
| $\varepsilon_{xx}^{p}$ at the moment $A$ | 'AUTRE_ASTER' | 1.4054E-2 | 0.1% |
| $\varepsilon_{xy}^{p}$ at the moment $A$ | 'AUTRE_ASTER' | 1.2981E-2 | 0,10% |

Moreover, in one second series of calculations, one tests the indicator of error due to nonthe radiality of the loading: starting from a coarse temporal discretization, like previously, one activates the subdivision of the step of time if the error due to nonthe radiality exceeds 2% (RESI_RADI_RELA=0.02). This test is carried out for 2 equivalent behaviors: VMIS_CINE_LINE, VMIS_ECMI_LINE..The results are identical for the two behaviors:

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| $\sigma_{xx}$ at the moment $A$ | 'AUTRE_ASTER' | 151.2 | 0.1% |
| $\sigma_{xy}$ at the moment $A$ | 'AUTRE_ASTER' | 93.1 | 0.1% |
| $\varepsilon_{xx}$ at the moment $A$ | 'AUTRE_ASTER' | 1.48297E-2 | 0.1% |
| $\varepsilon_{xy}$ at the moment $A$ | 'AUTRE_ASTER' | 1.36014E-2 | 0.1% |
| $p$ at the moment $A$ | 'AUTRE_ASTER' | 2.05473E-2 | 0.1% |
| $\varepsilon_{xx}^{p}$ at the moment $A$ | 'AUTRE_ASTER' | 1.4054E-2 | 0.1% |
| $\varepsilon_{xy}^{p}$ at the moment $A$ | 'AUTRE_ASTER' | 1.2981E-2 | 0,10% |

One tests moreover indicators of loss of radiality DERA_ELGA:

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| DERA_ELGA/ERR_RADI at moment 1, | 'NON_REGRESSION' | 0 | 0,00% |

| | | | |
|---|---|---|---|
| DERA_ELGA/ERR_RADI at the moment $1,5$ D | 'NON_REGRESSION' | 9,50E-003 | 0,00% |

# 9 Modeling G

## 9.1 Characteristics of modeling

This modeling is similar to modeling A. the principal difference is at the level of the management of the step of time. One the 1st calculation is carried out with a temporal discretization approximately 2 times coarser than that of modeling A. Ensuite, one extracts from the result the list from the moments really calculated (by taking account of possible under-cuttings of the step of time during the 1st calculation) and one creates one 2nd list of moments, 2 times finer than this extracted. To finish, one carries out one the 2nd calculation, identical to 1st, but with the finer list of moments.

## 9.2 Sizes tested and results

The tests relate to the deformations at the end of the load $\varepsilon_{xx}(B)$ and $\varepsilon_{xy}(B)$ for 2 calculations. The tolerances of the 1st calculation are 2 times looser than those of the 2nd calculation.

### 9.2.1 Calculation with the coarse list

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| $\varepsilon_{xx}$ at the moment $A$ | 'ANALYTICAL' | $3,5265.10^{-2}$ | 0,4% |
| $\varepsilon_{xy}$ at the moment $A$ | 'ANALYTICAL' | $2,0471.10^{-2}$ | 1,2% |

These doubled tests of tests of not-regression.

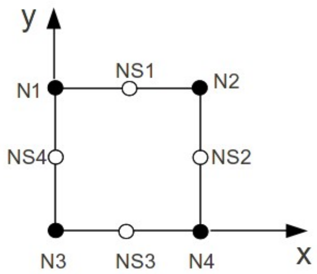
### 9.2.2 Calculation with the refined list

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| $\varepsilon_{xx}$ at the moment $A$ | 'ANALYTICAL' | $3,5265.10^{-2}$ | 0,2% |
| $\varepsilon_{xy}$ at the moment $A$ | 'ANALYTICAL' | $2,0471.10^{-2}$ | 0,6% |

# 10 Modeling H

## 10.1 Characteristics of modeling

Modeling in plane constraints with under-integration: C_PLAN_SI
The loading and the boundary conditions are modelled by:




- Condition of Dirichlet (keyword DDL_IMPO) :
  - Node $N04$, $x=y=0$,
  - Nodes $N02$, $NS2$ $x=0$.

- Condition of Neumann, surface forces (keyword FORCE_CONTOUR) :
  - on the faces (meshs of skin): (1, 2), (2, 4), (4, 3) and (3, 1).

## 10.2 Characteristics of the grid

Many nodes: 8
Many meshs and types: 1 QUAD8, 4 SEG3

## 10.3 Sizes tested and results

### 10.3.1 Case of VMIS_ISOT_LINE

| Identification | Type of reference | Value of reference | Tolerance |
|---|---|---|---|
| $\sigma_{xx}$ at the moment $A$ | 'ANALYTICAL' | 1,512E+002 | 0.1% |
| $\sigma_{xy}$ at the moment $A$ | 'ANALYTICAL' | 9,310E+001 | 0.1% |
| $p$ at the moment $A$ | 'ANALYTICAL' | 2,0547E-002 | 0.1% |
| Rate of triaxiality $TRIAX$ at the moment $A$ | 'ANALYTICAL' | 2,2800E-001 | 0.1% |
| $\varepsilon_{xx}$ at the moment $A$ | 'ANALYTICAL' | 1,48297E-002 | 0.1% |
| $\varepsilon_{xy}$ at the moment $A$ | 'ANALYTICAL' | 1,36014E-002 | 0.1% |
| Plastic indicator $V2$ at the moment $A$ | 'ANALYTICAL' | 1.0 | 0.1% |
| $p$ at the moment $B$ | 'ANALYTICAL' | 4,23293E-002 | 1.0% |
| Rate of triaxiality $TRIAX$ at the moment $B$ | 'ANALYTICAL' | 3,25349E-001 | 0.1% |
| $\varepsilon_{xx}$ at the moment $B$ | 'ANALYTICAL' | 3,5265E-002 | 1.0% |

| | | | |
|---|---|---|---|
| $\varepsilon_{xy}$ at the moment $B$ | 'ANALYTICAL' | 2,0471E-002 | 1.0% |
| $\varepsilon_{xx}^{p}$ at the moment $B$ | 'ANALYTICAL' | 3,3946E-002 | 1.0% |
| $\varepsilon_{xy}^{p}$ at the moment $B$ | 'ANALYTICAL' | 2,0250E-002 | 1.0% |

as well as the indicators of load-discharge:

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| INDIC_ENER at the moment $A$ | 'ANALYTICAL' | 0 | 0.1% |
| INDIC_ENER at the moment $B$ | 'ANALYTICAL' | 3,26E-002 | 3.0% |
| INDIC_SEUIL at the moment $A$ | 'ANALYTICAL' | 0 | 0.1% |
| INDIC_SEUIL at the moment $B$ | 'ANALYTICAL' | 3,26E-002 | 3.0% |
| INDIC_ENER at the moment $C$ (discharge supplements) | 'ANALYTICAL' | 4,69E-002 | 3.0% |
| INDIC_SEUIL at the moment $B$ | 'ANALYTICAL' | 9,71E-002 | 1.0% |
| INDIC_SEUIL at the moment $C$ (discharge supplements) | 'ANALYTICAL' | 1.0 | 1.0% |
| DERA_ELNO/RADI_V at moment 0.1 | 'ANALYTICAL' | 0.0 | 1.0% |

### 10.3.2 Case of VMIS_ECMI_LINE

One calculates energies in the course of resolution and one compares compared to the case VMIS_ISOT_LINE :

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| ETOT_ELGA/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ETOT_ELGA/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ETOT_ELGA/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ETOT_ELGA/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |
| ETOT_ELNO/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ETOT_ELNO/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ETOT_ELNO/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ETOT_ELNO/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |

| | | | |
|---|---|---|---|
| ETOT_ELEM/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ETOT_ELEM/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ETOT_ELEM/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ETOT_ELEM/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |

One recomputes energies by POST_ELEM and one compares compared to the case VMIS_ISOT_LINE :

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| ENER_TO2/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ENER_TO2/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ENER_TO2/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ENER_TO2/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |
| ENER_TO3/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ENER_TO3/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ENER_TO3/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ENER_TO3/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |
| ENER_TO4/TOTALE at moment 0.1 | 'AUTRE_ASTER' | 1,16403E-03 | 0.1% |
| ENER_TO4/TOTALE at moment 0.9 | 'AUTRE_ASTER' | 1.84340 | 0.1% |
| ENER_TO4/TOTALE at moment 2.0 | 'AUTRE_ASTER' | 9.58487 | 0.1% |
| ENER_TO4/TOTALE at moment 3.0 | 'AUTRE_ASTER' | 9.40794 | 0.1% |

The indicators of load-discharge are validated:

| Identification | Type of reference | Value | Tolerance |
|---|---|---|---|
| INDIC_ENER at the moment $A$ | 'ANALYTICAL' | 0 | 0.1% |
| INDIC_ENER at the moment $B$ | 'ANALYTICAL' | 3,26E-002 | 3.0% |
| INDIC_SEUIL at the moment $A$ | 'ANALYTICAL' | 0 | 0.1% |
| INDIC_ENER at the moment $C$ (discharge supplements) | 'ANALYTICAL' | 4,69E-002 | 3.0% |
| INDIC_SEUIL at the moment $B$ | 'ANALYTICAL' | 9,71E-002 | 1.0% |
| INDIC_SEUIL at the moment $C$ (discharge supplements) | 'ANALYTICAL' | 1.0 | 1.0% |

| | | | |
|---|---|---|---|
| DERA_ELNO/RADI_V at moment 0.1 | 'ANALYTICAL' | 0.0 | 1.0% |

# 11 Summary of the results

The results are identical whatever the selected modeling. The results are close to the reference solution since the variations are overall lower than $0.6\%$ .