SSNP118 - Validation of the elements of joint and interface in 2D plan and 3D

Summary:

Validation of the elements of joint and interface. Comparison between the results and the analytical solution.

Joint : $2D$ plan (mesh `QUAD4`) and $3D$ (mesh `HEXA8` or `PENTA6`) with the cohesive laws of behavior: `CZM_LIN_REG` and `CZM_EXP_REG` and a piloting by elastic prediction `PRED_ELAS`.

Interface : $2D$ plan (mesh `QUAD8`) and $3D$ (mesh `HEXA20` or `PENTA15`) with the cohesive laws of behavior: `CZM_OUV_MIX`, `CZM_EXP_MIX`, `CZM_TAC_MIX`, `CZM_FAT_MIX` and `CZM_TRA_MIX` as well as piloting by elastic prediction `PRED_ELAS` for the three first.

Formulation XFEM : $2D$ plan with the cohesive laws of behavior: `CZM_TAC_MIX`, `CZM_OUV_MIX`, `CZM_LIN_MIX`.

Modeling A : `PLAN_JOINT` with `CZM_EXP_REG`
Modeling B : `3D_JOINT` mesh `HEXA8` with `CZM_EXP_REG`
Modeling C : `3D_JOINT` mesh `PENTA6` with `CZM_EXP_REG`

Modeling D : `PLAN_JOINT` with `CZM_LIN_REG`
Modeling E : `3D_JOINT` mesh `HEXA8` with `CZM_LIN_REG`
Modeling F : `3D_JOINT` mesh `PENTA6` with `CZM_LIN_REG`

Modeling G : `PLAN_INTERFACE_S` with `CZM_OUV_MIX` and `CZM_TAC_MIX`
Modeling H : `3D_INTERFACE_S`, mesh `HEXA20` with `CZM_OUV_MIX` and `CZM_TAC_MIX`
Modeling I : `3D_INTERFACE_S`, mesh `PENTA15` with `CZM_OUV_MIX` and `CZM_TAC_MIX`

Modeling J : `PLAN_INTERFACE_S` with `CZM_FAT_MIX`
Modeling K : `3D_INTERFACE_S`, mesh `HEXA20` with `CZM_FAT_MIX`
Modeling L : `3D_INTERFACE_S`, mesh `PENTA15` with `CZM_FAT_MIX`

Modeling M : `PLAN_INTERFACE_S` with `CZM_TRA_MIX`
Modeling N : `3D_INTERFACE_S`, mesh `HEXA20` with `CZM_TRA_MIX`
Modeling O : `3D_INTERFACE_S`, mesh `PENTA15` with `CZM_TRA_MIX`

Modeling P : formulation `XFEM` , mesh `QUAD8` with `CZM_OUV_MIX` then `CZM_TAC_MIX`

Modeling Q : `PLAN_INTERFACE_S` with `CZM_EXP_MIX`
Modeling R : `3D_INTERFACE_S`, mesh `HEXA20` with `CZM_EXP_MIX`
Modeling S : `3D_INTERFACE_S`, mesh PENTA15 with `CZM_EXP_MIX`

Modeling T : formulation `XFEM`, mesh `QUAD4` with `CZM_LIN_MIX`, `C_PLAN`
Modeling U : formulation `XFEM`, mesh `QUAD4` with `CZM_LIN_MIX`, `D_PLAN`

1 Problem of reference

1.1 Geometry

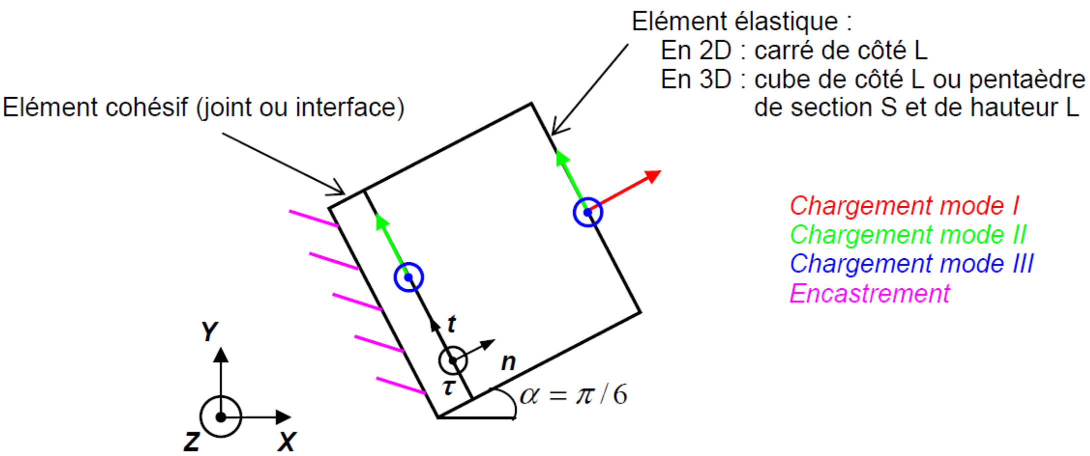

Figure 1 : Representation of the system of two elements in the plan (X, Y). One chooses $L=1\,mm$.

1.1.1 Geometry: case X-FEM

In modelings in formulation `XFEM`, there is no more in the model of cohesive element of joint or interface, but the cohesive law is defined on the interface using the order `DEFI_CONTACT`, as one would do it for a law of contact. Consequently, the square is with a grid with some elements and the line of discontinuity is introduced in the middle of the square, in a way nonin conformity: it cuts elements here elastic.

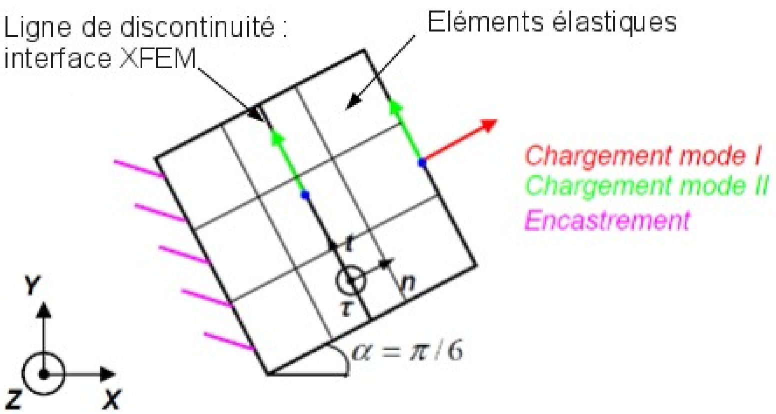

Figure 2 : Modeling X-FEM in the plan (X, Y). One chooses $L=1\,mm$.

1.2 Properties of material

1.2.1 Cohesive laws

1.2.1.1 Material `RUPT_FRAG`

Cubic *:* rubber band
Young modulus: $E=0.5\,MPa$ (except for modelings J, K and L where one takes $E=100\,MPa$, this choice is purely practical)
Poisson's ratio $\nu=0$

Element of joint : laws `CZM_EXP_REG, CZM_LIN_REG`
Element of interface : laws `CZM_OUV_MIX, CZM_EXP_MIX, CZM_TAC_MIX, CZM_FAT_MIX`
Mixed law for linear finite elements : law `CZM_LIN_MIX`

Density of critical energy of surface:	$G_c=0.9\,N/mm$	(keyword: `GC`)
critical stress:	$\sigma_c=1.1\,MPa$	(keyword: `SIGM_C`)

Element of joint:

penalization of adherence	`PENA_ADHERENCE` = $10^{-3}\,mm$	(keyword: `PENA_ADHERENCE`)
(small parameter of regularization of energy in 0, to see [R7.02.11])		
penalization of the contact	`PENA_CONTACT` = 1 (value by default)	(keyword: `PENA_CONTACT`)

Element of interface:

Penalization of Lagrangian	`PENA_LAGR=` 100 (value by default)	(keyword: `PENA_LAGR`)
Rigidity of the slip	`RIGI_GLIS=` 10 (value by default)	(keyword: `RIGI_GLIS`)

Mixed law for linear finite elements:

Penalization of Lagrangian	`PENA_LAGR=` 10	(keyword: `PENA_LAGR`)

NB: The data materials do not have of course authority to represent a material in particular. They are only intended for digital tests of validation.

1.2.1.2 Material `RUPT_DUCT`

Cubic *:* rubber band
Young modulus: $E=10^6\,MPa$ (this choice is purely practical)
Poisson's ratio $\nu=0$

Element of interface : law `CZM_TRA_MIX`

Density of critical energy of surface:	$G_c=0.9\,N/mm$	(keyword: `GC`)
critical stress:	$\sigma_c=9\,MPa$	(keyword: `SIGM_C`)
Extrinsic coefficient of form	0.0625	(keyword: `COEF_EXTR`)
Coefficient of form plate plastic	0.3125	(keyword: `COEF_PLAS`)
Penalization of Lagrangian	`PENA_LAGR=` 100 (value by default)	(keyword: `PENA_LAGR`)
Rigidity of the slip	`RIGI_GLIS=` 10 (value by default)	(keyword: `RIGI_GLIS`)

NB: The data materials do not have of course authority to represent a material in particular. They are only intended for digital tests of validation.

1.3 Boundary conditions and loadings

Embedding : Imposed displacements are worthless on the face of the cohesive element opposed to the elastic element.

In mode I : An imposed displacement $\boldsymbol{U}$ is applied to the face of the elastic element opposed to the joint (see figure 1).

`DX` = 2.16506351 `DY` = 1,250 `DZ` = 0

In mode II : Imposed displacement $\boldsymbol{U}$ is applied to all the nodes of the voluminal element.

`DX` = -1,250 `DY` = 2.16506350946110 `DZ` = 0

In mode III : Imposed displacement $\boldsymbol{U}$ is applied to all the nodes of the voluminal element.

`DX` = 0.0 `DY` = 0.0 `DZ` = 2.5

For the laws `CZM_OUV_MIX`, `CZM_EXP_MIX` and `CZM_TAC_MIX`, one uses the same standardized vectors with 1.

For the law of tiredness `CZM_FAT_MIX` (in mode I only) one uses the same vectors standardized with $0{,}094$. This value corresponds to the amplitude of the loading because this one is multiplied by a cyclic, worthless function into zero, which is worth 1 at the odd moments and 0 at the even moments.

For the ductile law `CZM_TRA_MIX` (in mode I only) one uses the same standardized vectors with 1. One applies a cyclic loading to test all the states of the law but also a monotonous loading. One carries out the tests with the first loading.

1.3.1 Loading: case X-FEM

Embedding : Imposed displacements are worthless on the left face of the square.

In mode I : An imposed displacement $\boldsymbol{U}$ is applied to the straight lines face of the square (see figure 2).

`DX` = 2.16506351 `DY` = 1,250 `DZ` = 0

2 Reference solution

2.1 Case general

In this part, one details the analytical solution in mode I pure in its form $3\mathrm{D}$. For calculations $2\mathrm{D}$ plan, the solution is identical, the forced component of the jump and the following vector τ do not intervene, and it is enough to replace surface S by the length L in the solution.
For the loadings in mode of shearing, the elastic element does not play a part. One carries out only one test on the tangential law of behavior. For the cohesive laws one imposes a jump of displacement and one checks the cohesive constraint obtained.

2.2 In mode I pure

One presents the analytical solution of the total answer of the system written in the local reference mark $(\boldsymbol{n}, \boldsymbol{t}, \boldsymbol{\tau})$. One applies a loading colinéaire to the normal: $\boldsymbol{U}=U\,\boldsymbol{n}$, the cohesive element opens in mode I pure and the tangential constraints as well as the tangential jumps remain worthless. One thus brings back oneself to a scalar problem. One notes $\sigma=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}$ the single nonworthless component of the tensor of the constraint of the elastic element in the local reference mark. One presents the solution of the total answer for the cohesive laws:

- **`CZM_EXP_REG, CZM_EXP_MIX`**

The cohesive relation of behavior is given by (see Doc. [`R7.02.11`]):

$$\vec{\sigma}=\begin{pmatrix}\sigma_n\\ \sigma_t\\ \sigma_\tau\end{pmatrix}=\begin{pmatrix}\sigma_c\cdot e^{-\frac{\sigma_c}{G_c}\cdot\delta_n}\\ 0\\ 0\end{pmatrix}$$

with δ_n the jump of normal displacement. The elastic law of the voluminal element gives:

$$\sigma=E\,\varepsilon=F/S$$

where ε is the elastic strain and where F is the force corresponding to the displacement imposed on surface S. In the case of figure where the constraint threshold in the cohesive element is not reached, the solution is elastic, the total answer is linear, it is expressed in the following way:

$$U(F)=\frac{FL}{SE}$$

When the threshold of rupture is reached, the jump in the cohesive element is not null any more, the answer is not linear any more. The balance of the system is given by:

$$\sigma=\sigma_n$$

Moreover, in this simple case of loading, imposed displacement is equal to the sum of the jump of displacement and displacement related to the deformation ε elastic element:

$$U=\delta_n+L\,\varepsilon$$

One from of deduced the relation between the force and imposed displacement:

$$U(F)=-\frac{G_c}{\sigma_c}\log\left(\frac{F}{S\sigma_c}\right)+\frac{FL}{SE}$$

Notice : - according to the data material one can not have a back return of the total answer which one collects with the piloting of the loading.
- the piloting of the elements of interface is carried out on the percentage of energy dissipated.

- **CZM_LIN_REG, CZM_OUV_MIX, CZM_TAC_MIX, CZM_FAT_MIX**

The cohesive relation of behavior is given by:

$$\vec{\sigma}=\begin{pmatrix}\sigma_n\\\sigma_t\\\sigma_\tau\end{pmatrix}=\begin{pmatrix}\sigma_c\left(1-\delta_n\frac{\sigma_c}{2G_c}\right)\\0\\0\end{pmatrix}$$

The same reasoning is adopted as with the exponential law, the analytical solution of the total answer is expressed in the following way:

$$\boxed{U(F)=\frac{F}{S}\left(\frac{L}{E}-\frac{2G_c}{\sigma_c^2}\right)+2\frac{G_c}{\sigma_c}}$$

Notice : for the law CZM_FAT_MIX the preceding total answer is valid only if chargemement is monotonous. In most case this one is cyclic since this law is intended for tiredness. One proposes to refer to documentation [R7.02.11] cohesive laws for more information.

- **CZM_LIN_MIX**

Deformation being uniform in the block, in the same way that previously, we can connect imposed displacement U on vis-a-vis the jump of displacement through the cohesive element by $U=[\![u]\!]+L\epsilon$, deformation ϵ being given by $\epsilon=\frac{\sigma}{E}$. The constraint being uniform in the block, it is given by the cohesive law like $\sigma=\sigma_c\left(1-[\![u]\!]\frac{\sigma_c}{2G_c}\right)$. By combining these expressions, one obtains:

$$U=[\![u]\!]+L\frac{\sigma_c}{E}\left(1-[\![u]\!]\frac{\sigma_c}{2G_c}\right)$$

In this test, we validate the good implementation of the piloting of the loading in this test, besides validating the good implementation of the cohesive law. In a general way, when the piloting of the loading is used, the increment of displacement Δu is related to the step of time Δt by the relation (see documentation [R5.03.80]):

$$f(\Delta u)=\frac{\Delta t}{\mathrm{COEF\_MULT}}$$

where:
– f is the function of piloting, which depends on the quantity on one seeks to control,
– COEF_MULT is the value indicated under the keyword of the same name, the keyword factor PILOTING order STAT_NON_LINE.

More particularly for the cohesive law CZM_LIN_MIX, the function of piloting is:

$f(\Delta u)=\frac{[\![u]\!]}{w_c}$ where $w_c=\frac{2G_c}{\sigma_c}$ is the critical jump of displacement.

For récaptuler, the increment of jump of displacement $[\![\Delta u]\!]$ applied at the time of a step of time Δt is written:

$$[\![\Delta u]\!]=\frac{\Delta t}{\mathrm{COEF\_MULT}}\frac{2G_c}{\sigma_c}$$

For this test $\mathrm{COEF\_MULT}=10$. The load of reference to be controlled is defined by an imposed displacement $U_0=2.5$. The parameter `ETA_PILO` giving the intensity of the load will thus be given by $\mathrm{ETA\_PILO}=\frac{U}{U_0}$.

2.3 In mode II and III pure

One tests only the law of behavior (see Doc. [`R7.02.11`] and [`R7.02.13`]):

- **`CZM_EXP_REG`**

$\boxed{\sigma_T=\sigma_c.e^{-\frac{\sigma_c}{G_c}.\delta_T}}$ T indicating respectively t in mode II and τ in mode III

- **`CZM_LIN_REG, CZM_OUV_MIX, CZM_TAC_MIX`**

$\boxed{\sigma_T=\sigma_c\left(1-\delta_T\frac{\sigma_c}{2G_c}\right)}$, T indicating respectively t in mode II and τ in mode III

3 Modeling A

Validation of the joint $2D$ with the cohesive law `CZM_EXP_REG`

3.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT`).

3.2 Characteristics of the grid

Many nodes: 6
The elastic element is one `QUAD4`.
The element of joint is one `QUAD4` degenerated (confused nodes).

3.3 Results of modeling A

The piloting of the loading is tested in mode I. Indeed, one places figure in the case of where the total answer has a back return (see internal note H-T64-2007-03420-FR for more details on this point).

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	4.49911D-02	0.10
`SIGN`	1.56379D-01	0.10
`SITX`	0.D+00	0.10

Mode II

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	1.56379D-01	0.10

4 Modeling B

Validation of the joint `3D HEXA8` with the cohesive law `CZM_EXP_REG`

4.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

4.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of joint is one `HEXA8` degenerated (confused nodes).

4.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.

Mode I

Size tested	Reference	Tolerance ($\%$)
DX on Node 2	2.16506D-08	0.10
`SIXX`	4.49911D-02	0.10
`SIGN`	1.56379D-01	0.10
`SITX`	0.D+00	0.10
`SITY`	0.D+00	

Mode II

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	1.56379D-01	0.10
`SITY`	0.D+00	0.10

Mode III

Size tested	Reference	Tolerance ($\%$)
`SIG NR`	0.D+00	0.10
`SITX`	0.D+00	0.10
`SITY`	1.56379D-01	0.10

5 Modeling C

Validation of the joint `3D PENTA6` with the cohesive law `CZM_EXP_REG`

5.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

5.2 Characteristics of the grid

Many nodes: 9
The elastic element is one `PENTA6`.
The element of joint is one `PENTA6` degenerated (confused nodes).

5.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.

Mode I

Size tested	Reference	Tolerance ($\%$)
DX on Node 2	2.16506D-08	0.10
`SIXX`	4.49911D-02	0.10
`SIGN`	1.56379D-01	0.10
`SITX`	0.D+00	0.10
`SITY`	0.D+00	0.10

Mode II

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	1.56379D-01	0.10
`SITY`	0.D+00	0.10

Mode III

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	0.D+00	0.10
`SITY`	1.56379D-01	0.10

6 Modeling D

Validation of the joint $2D$ with the cohesive law `CZM_LIN_REG`

6.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT`).

6.2 Characteristics of the grid

Many nodes: 6
The elastic element is one `QUAD4`.
The element of joint is one `QUAD4` degenerated (confused nodes).

6.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	9.27629539D-02	0.10
`SIGN`	5.4887555D-01	0.10
`SITX`	0.D+00	0.10

Mode II

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	4.3314872D-01	0.10

7 Modeling E

Validation of the joint 3D `HEXA8` with the cohesive law `CZM_LIN_REG`

7.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

7.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of joint is one `HEXA8` degenerated (confused nodes).

7.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	9.27629539D-02	0.10
`SIGN`	5.4887555D-01	0.10
`SITX`	0.D+00	0.10
`SITY`	0.D+00	0.10

Mode II

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	4.3314616D-01	0.10
`SITY`	0.D+00	0.10

Mode III

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	0.D+00	0.10
`SITY`	2.931186D-01	0.10

8 Modeling F

Validation of the joint 3D `PENTA6` with the cohesive law `CZM_LIN_REG`

8.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

8.2 Characteristics of the grid

Many nodes: 9
The elastic element is one `PENTA6`.
The element of joint is one `PENTA6` degenerated (confused nodes).

8.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	9.27629539D-02	0.10
`SIGN`	5.4887555D-01	0.10
`SITX`	0.D+00	0.10
`SITY`	0.D+00	0.10

Mode II

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	4.3314616D-01	0.10
`SITY`	0.D+00	0.10

Mode III

Size tested	Reference	Tolerance ($\%$)
`SIGN`	0.D+00	0.10
`SITX`	0.D+00	0.10
`SITY`	2.931186D-01	0.10

9 Modeling G

Validation of the element of interface `HEXA8` with the cohesive laws `CZM_OUV_MIX` and `CZM_TAC_MIX` in mode of opening.

9.1 Characteristics of modeling

Modeling `D_PLAN` for the elastic element.
Modeling `PLAN_INTERFACE_S` for the element of interface

9.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of interface is one `HEXA8` degenerated (confused nodes).

9.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
We present only those obtained with `CZM_OUV_MIX` :

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	7.37899D-01	0.10
`SIGN`	3.47849D-01	0.10
`SITX`	0.D+00	0.10
`V1`	2.6729D-01	0.10
`V4`	7.00003D-01	0.10

10 Modeling H

Validation of the element of interface `HEXA20` with the cohesive laws `CZM_OUV_MIX` and `CZM_TAC_MIX` in mode of opening.

10.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

10.2 Characteristics of the grid

Many nodes: 32
The elastic element is one `HEXA20`.
The element of interface is one `HEXA20` degenerated (confused nodes).

10.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
The results are identical in mode I for the two laws, we present only those obtained with `CZM_OUV_MIX` :

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	7.37899D-01	0.10
`SIGN`	3.47849D-01	0.10
`SITX`	0.D+00	0.10
`V1`	2.6729D-01	0.10
`V4`	7.00003D-01	0.10

11 Modeling I

Validation of the element of interface `PENTA15` with the cohesive laws `CZM_OUV_MIX` and `CZM_TAC_MIX` in mode of opening.

11.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

11.2 Characteristics of the grid

Many nodes: 24
The elastic element is one `PENTA15`.
The element of interface is one `PENTA15` degenerated (confused nodes).

11.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
The results are identical in mode I for the two laws, we present only those obtained with `CZM_OUV_MIX` :

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	2.16506D-08	0.10
`SIXX`	7.37899D-01	0.10
`SIGN`	3.47849D-01	0.10
`SITX`	0.D+00	0.10
`V1`	2.6729D-01	0.10
`V4`	7.00003D-01	0.10

12 Modeling J

Validation of the element of interface `HEXA8` with the cohesive law for tiredness `CZM_FAT_MIX` in mode of opening. The loading here is cyclic (see 5) in teeth of saw. The odd moments are the tops and the even moments correspond to the hollows.

12.1 Characteristics of modeling

Modeling `D_PLAN` for the elastic element.
Modeling `PLAN_INTERFACE_S` for the element of interface

12.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of interface is one `HEXA8` degenerated (confused nodes).

12.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.

Size tested	**Reference**	**Tolerance ($\%$)**
`SIXX inst. 3`	6.05826	0.10
`SIGN ( SIEF_ELGA ) inst. 9`	1.45221	0.10
`SIGN (DEPL) inst. 9`	1.45221	0.10
`SITX (SIEF_ELGA) inst. 7`	0.0	0.10
`V1 inst. 3`	2.04959E-02	0.10
`V4 inst. 3`	1.94457E-01	0.10

Remarks :
–`SIXX` is tested on the elastic voluminal element, the other tests are realized on the element of interface.
–One tests the normal constraint on a point of gauss: `SIGN (SIEF_ELGA)` as well as the multiplier of Lagrange on a node medium: `SIGN (DEPL)`.

13 Modeling K

Validation of the element of interface `HEXA20` with the cohesive law for tiredness `CZM_FAT_MIX` in mode of opening. The loading here is cyclic (see 5) in teeth of saw. The odd moments are the tops and the even moments correspond to the hollows.

13.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

13.2 Characteristics of the grid

Many nodes: 32
The elastic element is one `HEXA20`.
The element of interface is one `HEXA20` degenerated (confused nodes).

13.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
The sizes tested and the results are identical to those carried out in $2D$ (see modeling J paragraph 12.3).

Size tested	Reference	Tolerance (%)
`SIXX inst. 3`	6.05826	0.10
`SIGN ( SIEF_ELGA ) inst. 9`	1.45221	0.10
`SIGN (DEPL) inst. 9`	1.45221	0.10
`SITX (SIEF_ELGA) inst. 7`	0.0	0.10
`V1 inst. 3`	2.04959E-02	0.10
`V4 inst. 3`	1.94457E-01	0.10

14 Modeling L

Validation of the element of interface `PENTA15` with the cohesive law for tiredness `CZM_FAT_MIX` in mode of opening. The loading here is cyclic (see 5) in teeth of saw. The odd moments are the tops and the even moments correspond to the hollows.

14.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

14.2 Characteristics of the grid

Many nodes: 24
The elastic element is one `PENTA15`.
The element of interface is one `PENTA15` degenerated (confused nodes).

14.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
The sizes tested and the results are identical to those carried out in $2D$ (see modeling J paragraph 12.3).

Size tested	Reference	Tolerance ($\%$)
`SIXX inst. 3`	6.05826	0.10
`SIGN ( SIEF_ELGA ) inst. 9`	1.45221	0.10
`SIGN (DEPL) inst. 9`	1.45221	0.10
`SITX (SIEF_ELGA) inst. 7`	0.0	0.10
`V1 inst. 3`	2.04959E-02	0.10
`V4 inst. 3`	1.94457E-01	0.10

15 Modeling M

Validation of the element of interface `HEXA8` with the cohesive law for tiredness `CZM_TRA_MIX` in mode of opening. The loading here is cyclic to traverse all the states of the law.

15.1 Characteristics of modeling

Modeling `D_PLAN` for the elastic element.
Modeling `PLAN_INTERFACE_S` for the element of interface

15.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of interface is one `HEXA8` degenerated (confused nodes).

15.3 Sizes tested and results

Size tested	Reference	Tolerance ($\%$)
Total answer F inst 9.6	1.03932	0.10
Total answer U inst 9.6	0.06235	0.10
`SIGN (SIEF_ELGA)` inst. 9	9	0.10
`SIGN (DEPL)` inst. 9	9	0.10
`SITX` inst. 9.6	0.0	0.10
V1 inst. 2.5 (contact)	0.01	0.10
V4 inst. 2.5 (contact)	0	0.10
V1 inst. 5.4 (plate)	0.017991	0.10
V4 inst. 5.4 (plate)	0.079910	0.10
V1 inst. 7.4 (discharge)	0.029991	0.10
V4 inst. 7.4 (discharge)	0.199910	0.10
V1 inst. 12.6 (endo)	0.108662	0.10
V4 inst. 12.6 (endo)	0.869312	0.10
V1 inst. 23 (rupture)	0.16	0.10
V4 inst. 23 (rupture)	1	0.10

One tests the normal constraint on a point of gauss: `SIGN (SIEF_ELGA)` as well as the multiplier of Lagrange on a node medium: `SIGN (DEPL)`.

16 Modeling NR

Validation of the element of interface `HEXA20` with the cohesive law for tiredness `CZM_TRA_MIX` in mode of opening. The loading here is cyclic to traverse all the states of the law.

16.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

16.2 Characteristics of the grid

Many nodes: 32
The elastic element is one `HEXA20`.
The element of interface is one `HEXA20` degenerated (confused nodes).

16.3 Sizes tested and results

The sizes tested and the results are identical to those carried out in $2\mathrm{D}$ (see modeling M).

Size tested	**Reference**	**Tolerance ($\%$)**
SIGN (SIEF_ELGA) inst. 9	9	0.10
SIGN (DEPL) inst. 9	9	0.10
SITY inst. 9.6	0.0	0.10
V1 inst. 2.5 (contact)	0.01	0.10
V4 inst. 2.5 (contact)	0	0.10
V1 inst. 5.4 (plate)	0.017991	0.10
V4 inst. 5.4 (plate)	0.079910	0.10
V1 inst. 7.4 (discharge)	0.029991	0.10
V4 inst. 7.4 (discharge)	0.199910	0.10
V1 inst. 12.6 (endo)	0.108662	0.10
V4 inst. 12.6 (endo)	0.869312	0.10
V1 inst. 23 (rupture)	0.16	0.10
V4 inst. 23 (rupture)	1	0.10

One tests the normal constraint on a point of gauss: `SIGN (SIEF_ELGA)` as well as the multiplier of Lagrange on a node medium: `SIGN (DEPL)`.
Unlike $2\mathrm{D}$, one tests `SITY` rather than `SITX`

17 Modeling O

Validation of the element of interface `PENTA15` with the cohesive law for tiredness `CZM_TRA_MIX` in mode of opening. The loading here is cyclic to traverse all the states of the law.

17.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

17.2 Characteristics of the grid

Many nodes: 24
The elastic element is one `PENTA15`.
The element of interface is one `PENTA15` degenerated (confused nodes).

17.3 Sizes tested and results

The sizes tested and the results are identical to those carried out in $2\mathrm{D}$ (see modeling M).

Size tested	**Reference**	**Tolerance ($\%$)**
SIGN (SIEF_ELGA) inst. 9	9	0.10
SIGN (DEPL) inst. 9	9	0.10
SITY inst. 9.6	0.0	0.10
V1 inst. 2.5 (contact)	0.01	0.10
V4 inst. 2.5 (contact)	0	0.10
V1 inst. 5.4 (plate)	0.017991	0.10
V4 inst. 5.4 (plate)	0.079910	0.10
V1 inst. 7.4 (discharge)	0.029991	0.10
V4 inst. 7.4 (discharge)	0.199910	0.10
V1 inst. 12.6 (endo)	0.108662	0.10
V4 inst. 12.6 (endo)	0.869312	0.10
V1 inst. 23 (rupture)	0.16	0.10
V4 inst. 23 (rupture)	1	0.10

One tests the normal constraint on a point of gauss: `SIGN (SIEF_ELGA)` as well as the multiplier of Lagrange on a node medium: `SIGN (DEPL)`.
Unlike $2\mathrm{D}$ one tests `SITY` rather than `SITX`.

18 Modeling P

Validation of the implementation of the law CZM_TAC_MIX in formulation X-FEM. The mode of opening is tested. This modeling is an adaptation to X-FEM of modeling G .

18.1 Characteristics of modeling

The line of discontinuity is modelled by an interface X-FEM, which is introduced into the model by the operator DEFI_FISS_XFEM, with TYPE_DISCONTINUITE=' INTERFACES'. This line crosses the block right through. It is at a distance $0{,}4$ left edge and crosses the elements (see fig.2). It is said that the interface is nonin conformity.
The elements of contact are introduced by the discretization CONTACT=' STANDARD' in the operator MODI_MODELE_XFEM.
The cohesive law is then defined in the operator DEFI_CONTACT, by the keyword ALGO_CONT=' CZM' and RELATION=' CZM_TAC_MIX'.
The surface elements are of type D_PLAN.

18.2 Characteristics of the grid

The square is discretized at a rate of 4 elements by side. Consequently:
Many elements, of type HEXA8 : 16
Many nodes: 65.

18.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling. Not having more elements of interface strictly speaking in the model, one replaces the tests on SIGN and SIGTX by tests on the multipliers of contact X-FEM LAGS_C and LAGS_F1 respectively.

Mode I

Size tested	Reference	Tolerance (%)
ETA_PILO	8.29181D-01	0.10
DX on Nœud 2	2.16506D-08	0.10
SIXX on mesh 32	7.37899D-01	0.10
LAGS_C on node 9	3.47849D-01	0.10
LAGS_F1 on node 9	0.D+00	0.10

19 Modeling Q

Validation of the element of interface `HEXA8` with the cohesive laws `CZM_EXP_MIX` in mode of opening.

19.1 Characteristics of modeling

Modeling `D_PLAN` for the elastic element.
Modeling `PLAN_INTERFACE_S` for the element of interface

19.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of interface is one `HEXA8` degenerated (confused nodes).

19.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
One checks the analytical expression of displacement according to the force imposed as well as the internal constraints and variables which result from this for a percentage from energy dissipated from 0.3

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	1.5D-00	0.10
`SIXX`	4.89D-01	0.10
`SIGN`	6.52D-01	0.10
`SITX`	0.D+00	0.10
`V1`	4.276D-01	0.10
`V4`	7.D-01	0.10

20 Modeling R

Validation of the element of interface `HEXA20` with the cohesive laws `CZM_EXP_MIX` in mode of opening.

20.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

20.2 Characteristics of the grid

Many nodes: 32
The elastic element is one `HEXA20`.
The element of interface is one `HEXA20` degenerated (confused nodes).

20.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
One checks the analytical expression of displacement according to the force imposed as well as the internal constraints and variables which result from this, for a percentage of dissipated energy of 0.3

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	1.5D-00	0.10
`SIXX`	4.89D-01	0.10
`SIGN`	6.52D-01	0.10
`SITX`	0.D+00	0.10
`V1`	4.276D-01	0.10
`V4`	7.D-01	0.10

21 Modeling S

Validation of the element of interface `PENTA15` with the cohesive laws `CZM_EXP_MIX` in mode of opening.

21.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_INTERFACE_S` for the element of interface

21.2 Characteristics of the grid

Many nodes: 24
The elastic element is one `PENTA15`.
The element of interface is one `PENTA15` degenerated (confused nodes).

21.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling.
One checks the analytical expression of displacement according to the force imposed as well as the internal constraints and variables which result from this for a percentage from energy dissipated from 0.3

Mode I

Size tested	Reference	Tolerance ($\%$)
`DX` on Node 2	1.5D-00	0.10
`SIXX`	4.89D-01	0.10
`SIGN`	6.52D-01	0.10
`SITX`	0.D+00	0.10
`V1`	4.276D-01	0.10
`V4`	7.D-01	0.10

22 Modeling T

Validation of the implementation of the law `CZM_LIN_MIX` in formulation X-FEM. The mode of opening is tested. This modeling is an adaptation to X-FEM of modeling G.

22.1 Characteristics of modeling

The line of discontinuity is modelled by an interface X-FEM, which is introduced into the model by the operator `DEFI_FISS_XFEM` , with `TYPE_DISCONTINUITE=' INTERFACES` '. This line crosses the block right through. It is at a distance from $0{,}4\,mm$ left edge and crosses the elements (see fig.2). It is said that the interface is nonin conformity.
The elements of contact are introduced by the discretization `CONTACT=' MORTAR'` in the operator `MODI_MODELE_XFEM`.
The cohesive law is then defined in the operator `DEFI_CONTACT`, by the keyword `ALGO_CONT=' CZM'` and `RELATION=' CZM_LIN_MIX'`.
The surface elements are of type `C_PLAN`.

22.2 Characteristics of the grid

The square is discretized at a rate of 4 elements by side. Consequently:
Many elements, of type `HEXA4` : 16
Many nodes: 65.

22.3 Sizes tested and results

The remark on piloting evoked for modeling A is also true for this modeling. Not having more elements of interface strictly speaking in the model, one replaces the tests on `SIGN` and `SIGTX` by tests on the multipliers of contact X-FEM `LAGS_C` and `LAGS_F1` respectively.

Mode I

Size tested	Pas de time	Reference	Tolerance (%)
`DX` on node 2	4	1.71003596	0.10
`ETA_PILO`	4	7.898168291D-01	0.10
`SIXX` on mesh 32	2	6.599934D-01	0.10
`LAGS_C` on node 9	9	1.099989D-01	0.10
`LAGS_F1` on node 9	8	0.D+00	0.10

Obtaining these values is done according to the explanations of the part 2.2.

The jump of displacement at one moment t is: $[\![u]\!]=\frac{t}{\mathrm{COEF\_MULT}}\frac{2G_c}{\sigma_c}$. Thus, $[\![u]\!]=0.163636\ t$ (in mm).

One can then obtain the standard of the displacement applied by $U=[\![u]\!]+L\frac{\sigma_c}{E}\left(1-[\![u]\!]\frac{\sigma_c}{2G_c}\right)$. For $t=4$, one obtains $U=1.97\,mm$, from where $\mathrm{DX}=U\cos(\frac{\pi}{6})=1.71\,mm$ and $\mathrm{ETA\_PILO}=\frac{U}{U_0}=0.78\,mm$

From $[\![u]\!]$, one obtains the constraint by $\sigma=\sigma_c\left(1-[\![u]\!]\frac{\sigma_c}{2G_c}\right)$. For $t=9$, $\sigma=0.11\,Mpa$, which, taking into account the loading in pure mode I, can be tested by the multiplier of lagrange `LAGS_C`. For $t=2$, one finds $\sigma=0.88\,Mpa$, and one has $\sigma_{xx}=\sigma\cos^2(\frac{\pi}{6})=0.66\,MPa$, which is the value tested.

23 Modeling U

Validation of the implementation of the law `CZM_LIN_MIX` in formulation X-FEM. The mode of opening is tested. This modeling is identical to modeling T , but in plane deformations (`D_PLAN`) instead of plane constraints (`C_PLAN`).

23.1 Characteristics of modeling

Strictly identical to modeling T , except that the surface elements are of type `D_PLAN`.

23.2 Characteristics of the grid

The square is discretized at a rate of 4 elements by side. Consequently:
Many elements, of type `HEXA4` : 16
Many nodes: 65.

23.3 Sizes tested and results

The sizes tested are rigorously the same ones as in modeling T , and are thus obtained by same calculations, according to the explanations of the part 2.2.

Mode I

Size tested	Pas de time	Reference	Tolerance ($\%$)
`DX` on node 2	4	1.71003596	0.10
`ETA_PILO`	4	7.898168291D-01	0.10
`SIXX` on mesh 32	2	6.599934D-01	0.10
`LAGS_C` on node 9	9	1.099989D-01	0.10
`LAGS_F1` on node 9	8	0.D+00	0.10

24 Summary of the results

The digital results are in agreement with the analytical solution. These tests make it possible to validate the elements of joint, the elements of interface in $2D$ and $3D$, in the various modes of opening and XFEM $2D$ in opening.