SSNP144 – Use of a model at cohesive zones with method X-FEM

Summary

The purpose of this test is to validate the law of behavior cohesive put concerned during the cracking of elements $X-FEM$. This approach, described in [R7.02.12] allows to model the opening of an interface by taking account of a force of cohesion between the lips of this one.

Modelings A with K bring into play a parallelepipedic bar fissured on all its section (one speaks then about interface), subjected to an imposed displacement, which has as a consequence the separation of the two parts of the structure in mode I as well as in mode II . The influence of the refinement of the grid as of the type of elements used is studied.
Modeling L test the case of an interface which opens and is closed in mode I . One uses in a simultaneous way there penalized contact and piloting.

The validation is done by comparison of the values of Lagrangian of contact and friction with an analytical reference solution.

Contents

1 Problem of reference

1.1 Geometry 3D

The structure is a right parallelepiped at square base. Dimensions of the bar (see [Figure 1.1-a]) are: $LX=5\text{m}$, $LY=5\text{m}$ and $LZ=25\text{m}$.

The interface is introduced by functions of level (level sets) directly into the file orders using the operator DEFI_FISS_XFEM [U4.82.08]. The interface is present in the middle of the structure by the means of its representation by a level set LSN (see [Figure 1.1-a]) of equation:

$$\text{LSN (pour le plan de l'interface) : } Z-\frac{LZ}{2}$$

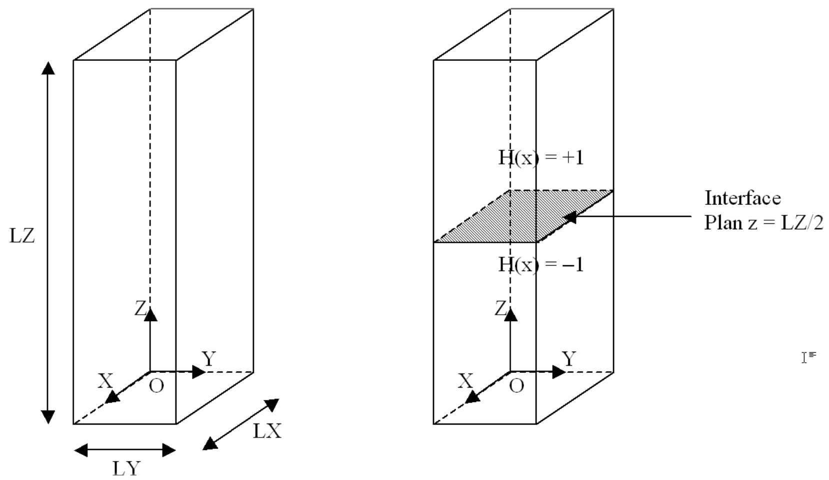

Figure 1.1-a : Geometry of the bar and positioning of the interface

1.2 Geometry 2D

The structure is a rectangle. Dimensions of the bar (see [Figure 1.2-a]) are: $\text{LX}=1\text{m}$, $\text{LY}=5\text{m}$.

The interface is introduced by a function of level (level set) directly into the file orders using the operator DEFI_FISS_XFEM [U4.82.08]. The interface is present in the middle of the structure by the means of its representation by a level set LSN (see [Figure 1.2-a]) of equation:

$$\text{LSN (pour le plan de l'interface) : } Y-\frac{LY}{2}$$

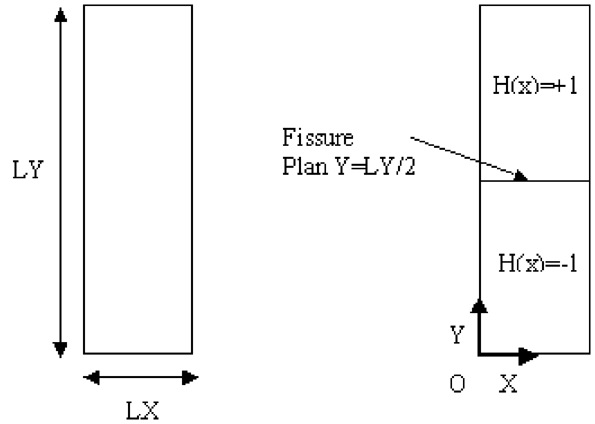

Figure 1.2-a : Geometry of the plate and positioning of the interface

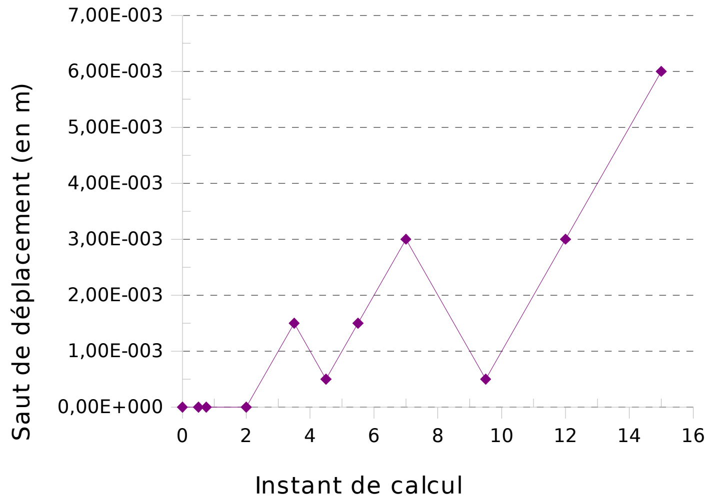

Figure 1.2-b: Evolution of the jump of displacement imposed by piloting according to the moment of calculation

1.3 Propriétés of material

Young modulus: $E = 0,5.E6Pa$

Poisson's ratio: $\nu = 0,0$

The presence of a cohesive law to the interface is indicated using the keyword RELATION=' CZM_EXP_REG' at the time of the definition of the zone of contact (order DEFI_CONTACT, keyword factor ZONE). The cohesive law then established has as parameters G_c, σ_c and κ_0, and is explained in detail in the documentatioN [R7.02.11]

Tenacity:	$G_c = 900$ N/m	(keyword: GC)
Constraint criticizes with the rupture:	$\sigma_c = 1,1.E6$ Pa	(keyword: SIGM_C)
Parameter of regularization of energy:	$\kappa_0 = 1,0E\text{-}3$	(keyword PENA_ADHERENCE)

Note:

The data materials do not have of course authority to represent a material in particular. They are only intended for digital tests of validation.

1.4 Boundary conditions and loadings

1.4.1 Degrees of freedom nouveau riches and boundary conditions

Let us recall that displacement under $X-FEM$ is the sum of a continuous displacement and a discontinuous displacement. In the case of an interface, bottomless of crack in the following way, the approximation of displacement is written:

$$u^h(x)=\sum_{i\in N_n(x)} a_i \Phi_i(x)+\sum_{j\in N_n(x)\cap K} b_j \Phi_j(x) H(lsn(x))$$

Where:

a_i and b_i are the degrees of freedom of displacement to the node i ,

Φ_i functions of form associated with the node i ,

$N_n(x)$ is the whole of the nodes whose support contains the point x ,

K is the whole of the nodes whose support is entirely cut by the crack,

$H(x)$ is the Heaviside function generalized defined by $H(x)=\begin{cases}-1 & \text{si } x<0\\ +1 & \text{si } x\geq 0\end{cases}$,

$lsn(x)$ is the normal value of the level-set at the point x .

For more details, to refer to the reference material $X-FEM$ [R7.02.12].

It is thus possible to want to impose limiting conditions on total displacement (or physical displacement), on its continuous component or its discontinuous component. We initially will give various useful relations between this physical displacement and the existing degrees of freedom in Code_Aster.

Let us take the example represented Figure 1.2-a: an edge of a grid $2D$ unspecified crossing by an interface:

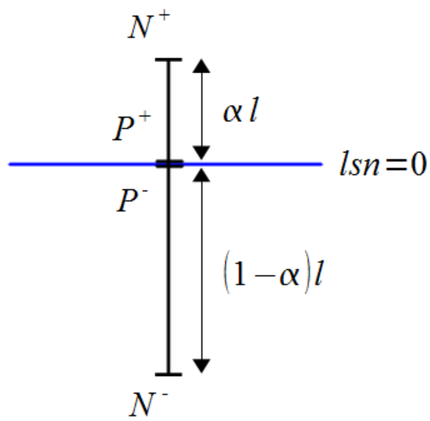

Figure 1.4.1-1: Edge intersected by an interface

Let us call P^+ and P^- the points geometrical belonging to the segment and located respectively just in lower part and to the top of the level set and let us consider without loss of general information of vertical displacements. We have:

$$\begin{cases} DY(N^+)=a_y^+ + b_y^+ \\ DY(N^-)=a_y^- - b_y^- \\ DY(P^+)=\alpha\left(a_y^+ + b_y^+\right)+(1-\alpha)\left(a_y^- + b_y^-\right) \\ DY(P^-)=\alpha\left(a_y^+ - b_y^+\right)+(1-\alpha)\left(a_y^- - b_y^-\right) \end{cases}$$

One can draw the expression from it from the jump of displacement for the edge:

$$[\![DY]\!]=DY(P^+)-DY(P^-)=\alpha b_y^+ + (1-\alpha) b_y^-$$

1.4.2 Loading

In mode I, the nodes of the lower face of the bar are embedded and a vertical displacement is imposed on those of the higher face, as explained on the figure 1.4.2-1.
In mode II of opening, the nodes located under the level set are embedded, and ON imposes a vertical displacement DY (DZ in $3D$) of $1{,}0E\text{-}13$ with all nodes located at the top of the level set. One thus ensures the nullity of the indicator of contact at the level of the interface. We also impose a pure shearing on the level of the cohesive zone what taking into account the notations of the figure 1.4.1-1 come down to be imposed for each intersected edge:

$$\begin{cases} DX(P^-)=DX(N^-)=0 \\ DX(P^+)=DX(N^+) \end{cases}$$

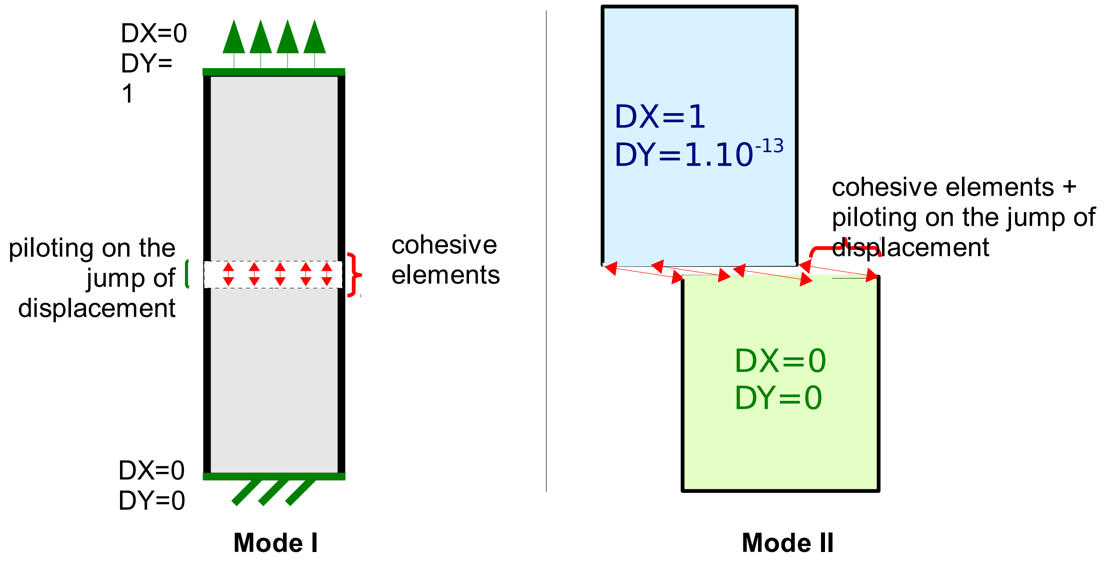

Figure 1.4.2-1: Loading applied, in mode I and in mode II

We see that this requires to impose connections between degrees of freedom on both sides of the edge i who depend on the coefficient α. In order not to weigh down the command file unnecessarily, we will not test in mode II that grids for which:

– that is to say all the intersected edges are it in their medium, i.e. for each one $\alpha=0{,}5$
– that is to say the interface is in conformity, i.e. $\alpha=1$

In the first case, where all the edges are intersected in their medium, we ensure the nullity of the indicator of contact while imposing $H1Y=5.\mathrm{E}-14$ on all the nodes ends of the edges intersected by the interface. With regard to shearing, we must impose the relations:

$$\begin{cases} DX(P^-)=DX(N^-)\Leftrightarrow \dfrac{a_x^+-b_x^+}{2}+\dfrac{a_x^--b_x^-}{2}=a_x^--b_x^-=0 \\ DX(P^+)=DX(N^+)\Leftrightarrow \dfrac{a_x^++b_x^+}{2}+\dfrac{a_x^-+b_x^-}{2}=a_x^++b_x^+ \end{cases}$$

We see whereas if we impose the relation $a_x^+=b_x^+$ then $DX(P^-)=0$.
If we impose the relation then $b_x^+=b_x^-$, we have moreover $DX(P^+)=DX(N^+)$.

In the second case, with interface in conformity with the grid, the jump is represented by the degree of Heaviside freedom at the point confused with the interface and we must thus impose $H1Y=1.\mathrm{E}-13$ on the nodes confused with the interface. With regard to shearing, if N^- corresponds to the point confused with the interface P^- , we must impose:

$$\begin{cases} DX(P^-)=a_x^--b_x^-=0 \\ DX(P^+)=DX(N^+)\Leftrightarrow a_x^-+b_x^-=a_x^+ \end{cases}$$

We see whereas if we impose the relation $a_x^-=b_x^-$ then $DX(N^-)=DX(P^-)=0$.
If we impose the relation then $a_x^-=\dfrac{a_x^+}{2}$, we have moreover $DX(P^+)=DX(N^+)$.
These two relations are imposed for all the nodes in with respect to the meshs crossed by the interface.

IL is to be noted that, in the command file, syntax to impose these various degrees of freedom differs according to the operators. The keyword factor `DDL_IMPO` of the operator `AFFE_CHAR_MECA` allows to impose total displacement (or physical displacement) thanks to the keywords `DX`, `DY`, and `DZ`, as well as the degrees of freedom discontinuous b_i thanks to the keywords `H1X`, `H1Y`, and `H1Z`. On the other hand, the keyword factor `LIAISON_GROUP` allows to control the continuous degrees of freedom a_i thanks to the keywords `DX`, `DY`, and `DZ` (and not total displacement) as well as the discontinuous degrees of freedom b_i thanks to the keywords`S H1X, H1Y, and H1Z.`

1.5 Ppatrolling the block of the loading

For the mode I piloting is carried out on the jump of vertical displacement. Vertical displacement DY (DZ in 3D) imposed on the nodes of the higher face of the bar is then unit. The real intensity of the displacement of these nodes is thus known during calculation via the parameter `ETA_PILOTAGE`. One uses one of pilotings `SAUT_IMPO or SAUT_LONG_ARC` **implemented especially for** $XFEM$, on the normal component $H1Y$ ($H1Z$ in 3D), corresponding to b_y (b_z in 3D) with the notations taken above, for all the edges intersected by the interface.

For the mode II piloting is carried out on the jump of horizontal displacement. Piloting is then carried out in a way identical to the mode I , but on the tangential component $H1X$.
Lastly, for each mode of opening, pilotings used make it possible to explore all the phases of the law of behavior: elastic linear load, damage and discharge.

Figures 1.5-1 and 1.5-2 represent the evolution of the jump of displacement according to the moment of calculation. In order to carry out discharges, several resumptions of calculation `(keyword reuse`

) are carried out to allow to change the value of the coefficient c_{mult} (keyword COEF_MULT) allowing to define the value of the increment of the jump of displacement $\Delta\delta_i$ to impose for a step of time Δt via the equation of piloting. As the problem is uniform, this relation is written in a simple way for SAUT_IMPO and SAUT_LONG_ARC : $\Delta\delta_i\ c_{mult}=\Delta t$. The piloting of the type PRED_ELAS is not tested yet for it case because it would require a restitution giving the possibility of controlling in discharge, discussed in [R5.03.80], with this kind of piloting.
The loading is carried out in ten phases, thanks to ten successive calls to STAT_NON_LINE :

Moment of final calculation	Phase	Jump of final displacement (in m)	C_{mult} DDL_IMPO LONG_ARC	C_{mult} PRED_ELAS
0.5	Beginning of initial elastic load	2,73E-7	1,833350000E+06	1,500013700E+03
0.75	Initial elastic discharge	1,36E-7	-1,833350000E+06	-1,500013700E+03
2	End of initial elastic load	8,18E-7	1,833350000E+06	1,500013700E+03
3	Damage 1	1,50E-3	1,000545747E+03	1,001000991E+00
3,5				8,001501926E-01
4	Elastic discharge 1	5,00E-4	-1,000000000E+03	-8,501501892E-01
4,5				-1,285714290E+00
5	Elastic load 1	1,50E-3	1,000000000E+03	1,285714286E+00
5,5				8,001501912E-01
7	Damage 2	3,00E-3	1,000000000E+03	2,089908926E+00
9	Elastic discharge 2	5,00E-4	-1,000000000E+03	-1,641475407E+00
9,5				-1,285714290E+00
10	Elastic load 2	3,00E-3	1,000000000E+03	1,285714286E+00
12				1,441475409E+00
15	Damage 3	6,00E-3	1,000000000E+03	4,179817852E+00

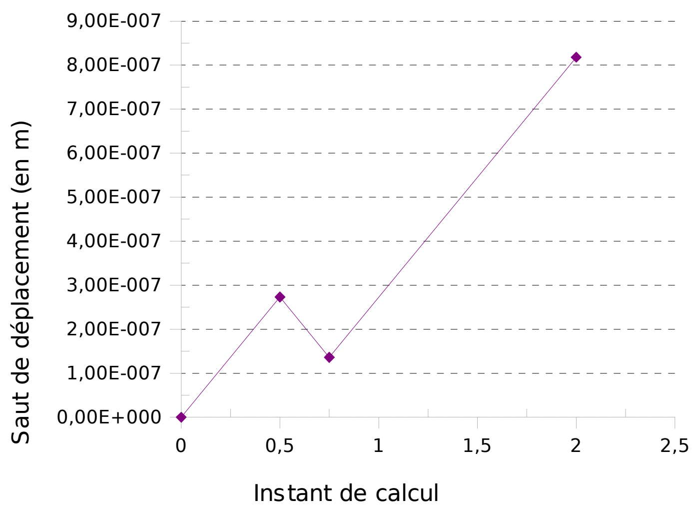

Figure 1.5-1: Evolution of the jump of displacement imposed by piloting until the moment of calculation 2.

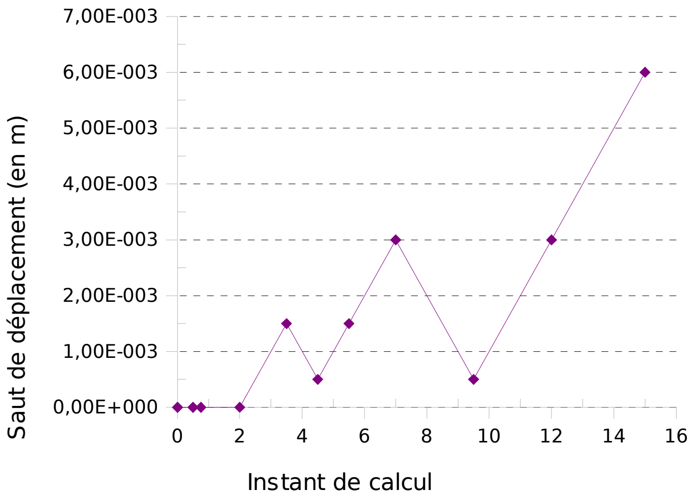

Figure 1.5-2: Evolution of the jump of displacement imposed by piloting according to the moment of calculation.

2 Reference solution

The reference solution for this case test is provided by the relation of behavior cohesive defined in [R7.02.12]:

$$t_c=\frac{\sigma_c}{\alpha}\exp\left(\frac{-\sigma_c}{G_c}\alpha\right)(\llbracket u\rrbracket_n+\beta^2\llbracket u\rrbracket_\tau)$$

Where:

- $\llbracket u\rrbracket$ is the jump of displacement
- $\llbracket u\rrbracket_n=n\otimes n.\llbracket u\rrbracket$ is the projection of the jump of displacement following the normal to the interface
- $\llbracket u\rrbracket_\tau=(\mathrm{Id}-n\otimes n).\llbracket u\rrbracket$ is the projection of the jump of displacement following the tangent plan to the interface
- β is a size obtained in experiments representing the relationship between the forces of opening in Mode I and in Mode II. For this test, this parameter is selected unit.
- $\llbracket u\rrbracket_{eq}=\sqrt{\|\llbracket u\rrbracket_n\|^2+\beta^2\|\llbracket u\rrbracket_\tau\|^2}$
- α is an internal variable the corresponding to greatest value of $\llbracket u\rrbracket_{eq}$ attack during the opening. This internal variable has as an initial value $\alpha_0=\frac{G_c}{\sigma_c}\text{PENA\_ADHERENCE}$. If the material leaves its field of elasticity, one has $\alpha=\llbracket u\rrbracket_{eq}$.

One thus knows explicitly the force of cohesion[1] according to the jump of displacement.

Moment of final calculation	Phase	Jump of final displacement (in m)	$\|t_c\|$ (out of Pa)
0.5	Beginning of initial elastic load	2,73E-7	3.66296853301E+05
0.75	Initial elastic discharge	1,36E-7	1.8314842665E+05
2	End of initial elastic load	8,18E-7	1.098890559903E+06
3.5	Damage 1	1,50E-3	1.75867720687844E+05
4,5	Elastic discharge 1	5,00E-4	2.49999999999582E-04
5.5	Elastic load 1	1,50E-3	1.75867720687844E+05
7	Damage 2	3,00E-3	2.8117686527187E+04
9.5	Elastic discharge 2	5,00E-4	4.68628108785798E+03
12	Elastic load 2	3,00E-3	2.8117686527187E+04
15	Damage 3	6,00E-3	7.18731177854856E+02

This value will thus be compared with the values of Lagrangian of respectively named contact and friction LAGS_C and LAGS_F1.

Mode I :

For reasons of invariance, let us consider a unidimensional problem. We will also suppose that the structure is with a grid with only three elements, which does not change anything with the validity results stated thereafter. The interface is present in the middle of the central element as shows it

1 Force per unit of area, homogeneous with a constraint

[Figure 2-a]. Since we control by DDL_IMPO the degree of freedom $H1Y$ node N2 (also noted b_{y2}), let us solve the problem according to this parameter.

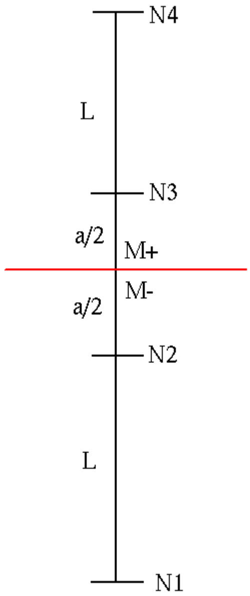

Figure 2-a: Unidimensional representation of the problem of reference

Let us express displacements of the various points structure:

$$\begin{cases} u^1 = 0 \\ u^2 = L\epsilon \\ u^+ = (L+\frac{a}{2})\epsilon \\ u^- = (L+\frac{a}{2})\epsilon + [[u]]_n \\ u^3 = (L+a)\epsilon + [[u]]_n \\ u^4 = (2L+a)\epsilon + [[u]]_n \end{cases}$$

Let us express displacements of the points now N2 , M+ , M- and N3 according to the degrees of freedom $X-FEM$.

$$\begin{cases} u^2 = a_{y2} - b_{y2} \\ u^3 = a_{y2} + b_{y3} \\ u^+ = \frac{a_{y2}+b_{y2}}{2} + \frac{a_{y3}+b_{y3}}{2} \\ u^- = \frac{a_{y2}-b_{y2}}{2} + \frac{a_{y3}-b_{y3}}{2} \end{cases}$$

By reversing this system, one obtains:

$$\begin{cases} a_{y2}=\frac{u_2}{2}-\frac{u_3}{2}+u^+ \\ b_{y2}=-\frac{u_2}{2}-\frac{u_3}{2}+u^+ \\ a_{y3}=-\frac{u_2}{2}+\frac{u_3}{2}+u^- \\ b_{y3}=\frac{u_2}{2}+\frac{u_3}{2}-u^- \end{cases}$$

The elastic law of the structure gives: $\sigma=E\epsilon$.
The balance of the system gives $\sigma=t_c$.
Lastly, the cohesive law of behavior provides a relation between constraint and jump of displacement. This relation is different according to whether the joint remains in its elastic range or that it is damaged.

If the joint remains in the elastic range, the internal variable α remain constant. One has then $t_c=\frac{\sigma_c}{\alpha}\exp(\frac{-\sigma_c}{G_c}\alpha)[\![u]\!]_n$.

Displacements of the various points of the structure are written then:

$$\begin{cases} u^1=0 \\ u^2=\frac{L}{E}\frac{\sigma_c}{\alpha}\exp(\frac{-\sigma_c}{G_c}\alpha)[\![u]\!]_n \\ u^+=\frac{L+\frac{a}{2}}{E}\frac{\sigma_c}{\alpha}\exp(\frac{-\sigma_c}{G_c}\alpha)[\![u]\!]_n \\ u^-=\left(1+\frac{L+\frac{a}{2}}{E}\frac{\sigma_c}{\alpha}\exp(\frac{-\sigma_c}{G_c}\alpha)\right)[\![u]\!]_n \\ u^3=\left(1+\frac{L+a}{E}\frac{\sigma_c}{\alpha}\exp(\frac{-\sigma_c}{G_c}\alpha)\right)[\![u]\!]_n \\ u^4=\left(1+\frac{2L+a}{E}\frac{\sigma_c}{\alpha}\exp(\frac{-\sigma_c}{G_c}\alpha)\right)[\![u]\!]_n \end{cases}$$

By injecting these expressions in the formulas giving b_{y2} and b_{y3} , it is noted that $b_{y2}=b_{y3}$.
One has thus $[[u]]_n=u^+-u^-=b_{y2}+b_{y3}=2.b_{y2}$. The problem is thus entirely given for gives degree of freedom b_{y2} , corresponding to the half jump of displacement, and known at any moment thanks to piloting.

If the joint remains in phase of damage, the internal variable α becomes equal to the jump of displacement $[[u]]_n$. One has then $t_c=\sigma_c\exp(\frac{-\sigma_c}{G_c}[\![u]\!]_n)$.

Displacements of the various points of the structure are written then:

$$
\begin{cases}
u^1=0 \\
u^2=\frac{L}{E}\sigma_c\exp(\frac{-\sigma_c}{G_c}[\![u]\!]_n) \\
u^+=\frac{L+\frac{a}{2}}{E}\sigma_c\exp(\frac{-\sigma_c}{G_c}[\![u]\!]_n) \\
u^-=\frac{(L+\frac{a}{2})}{E}\sigma_c\exp(\frac{-\sigma_c}{G_c}[\![u]\!]_n)+[[u]]_n \\
u^3=\frac{(L+a)}{E}\sigma_c\exp(\frac{-\sigma_c}{G_c}[\![u]\!]_n)+[[u]]_n \\
u^4=\frac{(2L+a)}{E}\sigma_c\exp(\frac{-\sigma_c}{G_c}[\![u]\!]_n)+[[u]]_n
\end{cases}
$$

By injecting these expressions in the formulas giving b_{y2} and b_{y3}, it is noted that $b_{y2}=b_{y3}$.
One has thus $[[u]]_n=u^+-u^-=b_{y2}+b_{y3}=2.\mathrm{b}_{y2}$. The problem is thus entirely given for gives degree of freedom b_{y2}, corresponding to the half jump of displacement, and known at any moment thanks to piloting.

<u>Mode II :</u>

Boundary conditions and relations imposed thanks to the keyword factor `LIAISON_GROUP` ensure a perfect shearing at the level of the interface. The displacement of all the points located under the level set is thus null and the displacement of all the points located at the top of the level set is equal to the jump of displacement.
The force of cohesion in the joint is thus known like an explicit function of the jump of displacement itself known at any moment thanks to piloting.

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling `XFEM`, in plane constraints, the interface of discontinuity being defined by a function of level (level set noted LN for the level set normal) directly in the command file using the operator `DEFI_FISS_XFEM` [`U4.82.08`].

The equation of the function of level for the interface is the following one:

$$\mathrm{LN}=Y-2.5$$

No level set tangential is necessary since the keyword is used `TYPE_DISCONTINUITE=' INTERFACE'`, which makes it possible to have the structure completely cut in two parts.

The cohesive law is introduced via the operator `DEFI_CONT`, while specifying `ALGO_CONT=' CZM'`, and the loI of behavior cohesive is activated thanks to the keyword `RELATION=' CZM_EXP_REG'`.

The modes here are tested I and II of opening.

3.2 Characteristics of the grid

One discretizes the structure in 1×5 finite elements `QUAD4`. The interface is thus present within the central element by the means as of level sets.

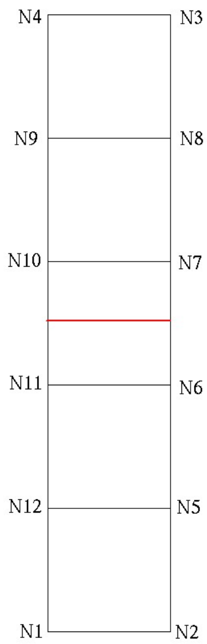

Figure 3.2-a: Grid of modeling A

3.3 Piloting

A specific piloting is used $X-FEM$ `JUMP _IMPO` on the whole of the edges intersected by the crack.

3.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Y for all the nodes	1.36362396705485E-07	1.0E-6
0,51.0E-6	LAGS_C for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1Y for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-6
2	H1Y for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-6
5,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
7	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_C for all the nodes	28117.686527187	1.0E-6
9,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-6
12	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_C for all the nodes	28117.686527187	1.0E-6
15	H1Y for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_C for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6

5.5	`LAGS_F1` for all the nodes	1.75867720687844E+05	1.0E-6
7	`H1X` for all the nodes	1.49999999999958E-03	1.0E-6
7	`LAGS_F1` for all the nodes	28117.686527187	1.0E-6
9,5	`H1X` for all the nodes	2.49999999999582E-04	1.0E-6
9.5	`LAGS_F1` for all the nodes	4686.28108785798	1.0E-6
12	`H1X` for all the nodes	1.49999999999958E-03	1.0E-6
12	`LAGS_F1` for all the nodes	28117.686527187	1.0E-6
15	`H1X` for all the nodes	2.99999999999958E-03	1.0E-6
15	`LAGS_F1` for all the nodes	718.731177854856	1.0E-6

3.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

4 Modeling B

4.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling A.

4.2 Characteristics of the grid

One discretizes the structure in 2×5 finite elements TRIA3. The interface is thus present within the central elements by the means as of level sets.

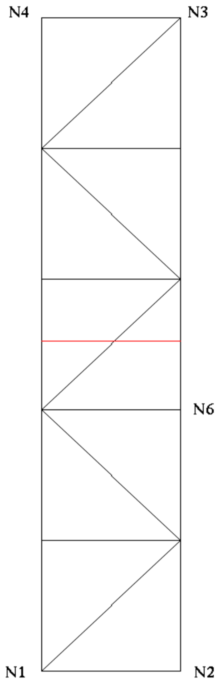

Figure 4.2-a: Grid of modeling B

4.3 Piloting

A specific piloting is used $XFEM$ SAUT_IMPO on the whole of the edges intersected by the crack.

4.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Y for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1Y for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-6

2	H1Y for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-6
5,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
7	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_C for all the nodes	28117.686527187	1.0E-6
9,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-6
12	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_C for all the nodes	28117.686527187	1.0E-6
15	H1Y for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_C for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
7	H1X for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-6
12	H1X for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
15	H1X for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-6

4.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

5 Modeling C

5.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling A .

5.2 Characteristics of the grid

The grid is refined here compared to that of modeling A . One discretizes the structure in 11×55 finite elements QUAD4. The interface is thus present within the central elements by the means of level sets.

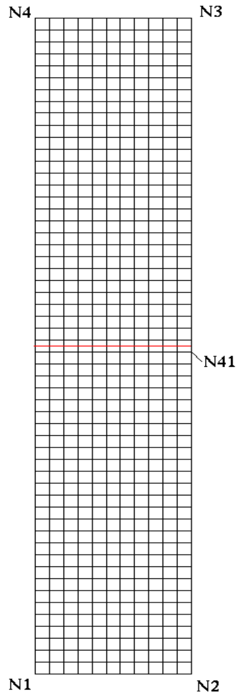

Figure 5.2-a: Grid of modeling C

5.3 Piloting

A specific piloting is used $X-FEM$ SAUT_LONG_ARC on the whole of the edges intersected by the crack.

5.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Y for all the nodes	1.36362396705485E-07	1.0E-6
0,51.0 E-6	LAGS_C for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1Y for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-6
2	H1Y for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-6
5,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
7	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_C for all the nodes	28117.686527187	1.0E-6
9,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-6
12	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_C for all the nodes	28117.686527187	1.0E-6
15	H1Y for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_C for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
7	H1X for all the nodes	1.49999999999958E-03	1.0E-6

7	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-6
12	H1X for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
15	H1X for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-6

5.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

6 Modeling D

6.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling A .

6.2 Characteristics of the grid

The grid is refined here compared to that of modeling C . One discretizes the structure in $2\times(11\times55)$ finite elements TRIA3. The interface is thus present within the central elements by the means of level sets.

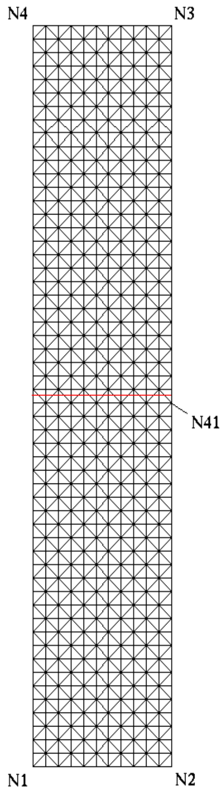

Figure 6.2-a: Grid of modeling D

6.3 Piloting

A specific piloting is used $XFEM$ SAUT_LONG_ARC on the whole of the edges intersected by the crack.

6.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	*Tolerance (%)*
0,5	H1Y for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1Y for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-6
2	H1Y for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-6
5,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
7	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_C for all the nodes	28117.686527187	1.0E-6
9,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-6
12	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_C for all the nodes	28117.686527187	1.0E-6
15	H1Y for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_C for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	*Tolerance (%)*
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
7	H1X for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-6

9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-6
12	H1X for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
15	H1X for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-6

6.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

7 Modélisation E

7.1 Characteristics of modeling

It is about a modeling $X-FEM$, in three dimensions, with definition of contact on the interface of discontinuity defined by a function of level (level set noted LN for the level set normal) directly in the command file using the operator `DEFI_FISS_XFEM` [`U4.82.08`].

The equation of the function of level for the interface is the following one:

$$\mathrm{LN}=Y-12.5$$

No level set tangential is necessary since the keyword is used `TYPE_DISCONTINUITE=' INTERFACE'`, which makes it possible to have the structure completely cut in two parts.

L has cohesive law is introduced via the operator `DEFI_CONT` , while specifying `ALGO_CONT=' CZM'` , and the lo I of behavior cohesive is activated thanks to the keyword `RELATION=' CZM_EXP_REG'` .

The modes here are tested I and II of opening.

7.2 Characteristics of the grid

One discretizes the structure in $1\times1\times5$ finite elements `HEXA8`. The interface is thus present within the central element by the means as of level sets.

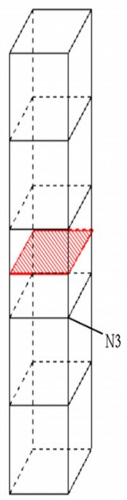

Figure 7.2-a : grid with 5 HEXA8

7.3 Piloting

Specific piloting of type `SAUT_IMPO`, one uses the group of the nodes located immediately at the top of the crack .

7.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Z for all the nodes	1.36362396705485E-07	1.0E-5
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-5
0,75	H1Z for all the nodes	6.818119835274E-08	1.0E-5
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-5
2	H1Z for all the nodes	4.0908719011645E-07	1.0E-5
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-5
3,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-5
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-5
4,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-5
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-5
5,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-5
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-5
7	H1Z for all the nodes	1.49999999999958E-03	1.0E-5
7	LAGS_C for all the nodes	28117.686527187	1.0E-5
9,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-5
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-5
12	H1Z for all the nodes	1.49999999999958E-03	1.0E-5
12	LAGS_C for all the nodes	28117.686527187	1.0E-5
15	H1Z for all the nodes	2.99999999999958E-03	1.0E-5
15	LAGS_C for all the nodes	718.731177854856	1.0E-5

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolérance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-5
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-5
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-5
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-5
2	H1X for all the nodes	4.0908719011645E-07	1.0E-5
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-5
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-5
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-5
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-5
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-5
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-5
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-5
7	H1X for all the nodes	1.49999999999958E-03	1.0E-5
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-5
9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-5

9.5	`LAGS_F1` for all the nodes	4686.28108785798	1.0E-5
12	`H1X` for all the nodes	1.49999999999958E-03	1.0E-5
12	`LAGS_F1` for all the nodes	28117.686527187	1.0E-5
15	`H1X` for all the nodes	2.99999999999958E-03	1.0E-5
15	`LAGS_F1` for all the nodes	718.731177854856	1.0E-5

7.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

8 MODélisation F

8.1 Characteristics of modeling

The characteristics of modeling are them same as for modeling E .

8.2 Characteristics of the grid

One discretizes the structure in $2\times(1\times1\times5)$ finite elements PENTA6. The interface is thus present within the central elements by the means as of level sets.

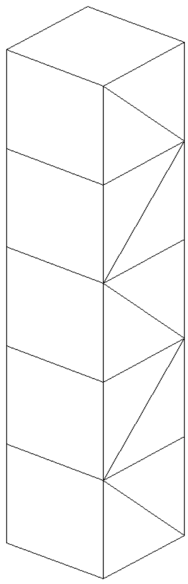

Figure 8.2-a: Grid of modeling F

8.3 Piloting

A specific piloting is used $X-FEM$ SAUT_IMPO using the group of nodes located immediately at the top of the crack.

8.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Z for all the nodes	1.36362396705485E-07	1.0E-5
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-5
0,75	H1Z for all the nodes	6.818119835274E-08	1.0E-5
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-5
2	H1Z for all the nodes	4.0908719011645E-07	1.0E-5
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-5
3,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-5
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-5
4,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-5
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-5
5,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-5
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-5
7	H1Z for all the nodes	1.49999999999958E-03	1.0E-5
7	LAGS_C for all the nodes	28117.686527187	1.0E-5
9,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-5
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-5
12	H1Z for all the nodes	1.49999999999958E-03	1.0E-5
12	LAGS_C for all the nodes	28117.686527187	1.0E-5
15	H1Z for all the nodes	2.99999999999958E-03	1.0E-5
15	LAGS_C for all the nodes	718.731177854856	1.0E-5

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-5
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-5
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-5
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-5
2	H1X for all the nodes	4.0908719011645E-07	1.0E-5
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-5
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-5
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-5
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-5
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-5
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-5
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-5
7	H1X for all the nodes	1.49999999999958E-03	1.0E-5
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-5

9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-5
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-5
12	H1X for all the nodes	1.49999999999958E-03	1.0E-5
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-5
15	H1X for all the nodes	2.99999999999958E-03	1.0E-5
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-5

8.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

9 ModeLisation G

9.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling E.

9.2 Characteristics of the grid

One discretizes the structure in $4\times(1\times1\times15)$ finite elements TETRA4. The interface is thus present within the central elements by the means of level sets.

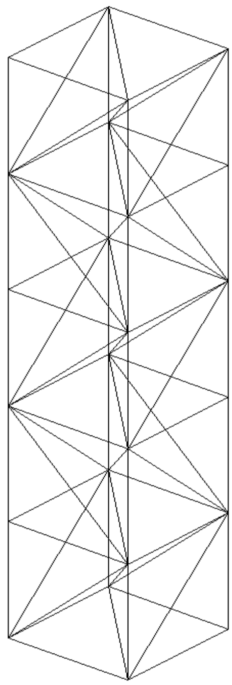

Figure 9.2-a: Grid of modeling G

9.3 Piloting

A specific piloting is used $X-FEM$ SAUT_LONG_ARC using the group of nodes located immediately at the top of the crack.

9.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Z for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1Z for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-6
2	H1Z for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-6
5,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
7	H1Z for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_C for all the nodes	28117.686527187	1.0E-6
9,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-6
12	H1Z for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_C for all the nodes	28117.686527187	1.0E-6
15	H1Z for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_C for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
7	H1X for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-6

9,5	`H1X` for all the nodes	2.49999999999582E-04	1.0E-6
9.5	`LAGS_F1` for all the nodes	4686.28108785798	1.0E-6
12	`H1X` for all the nodes	1.49999999999958E-03	1.0E-5
12	`LAGS_F1` for all the nodes	28117.686527187	1.0E-5
15	`H1X` for all the nodes	2.99999999999958E-03	1.0E-5
15	`LAGS_F1` for all the nodes	718.731177854856	1.0E-5

9.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

10 Modeling H

10.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling E.

10.2 Characteristics of the grid

The grid is refined here compared to that of modeling F. One discretizes the structure in $5\times1\times13$ finite elements HEXA8. The interface is thus present within the central elements by the means of level sets.

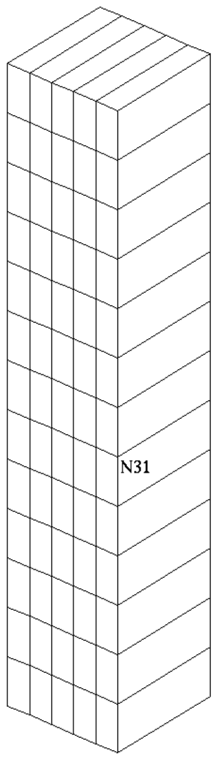

Figure 10.2-a: Grid of modeling H

10.3 Piloting

A specific piloting is used $X-FEM$ SAUT_LONG_ARC using the group of nodes located immediately at the top of the crack.

10.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Z for all the nodes	1.36362396705485E-07	1.0E-5
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-5
0,75	H1Z for all the nodes	6.818119835274E-08	1.0E-5
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-5
2	H1Z for all the nodes	4.0908719011645E-07	1.0E-5
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-5
3,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-5
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-5
4,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-5
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-5
5,5	H1Z for all the nodes	7.49999999999582E-04	1.0E-5
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-5
7	H1Z for all the nodes	1.49999999999958E-03	1.0E-5
7	LAGS_C for all the nodes	28117.686527187	1.0E-5
9,5	H1Z for all the nodes	2.49999999999582E-04	1.0E-5
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-5
12	H1Z for all the nodes	1.49999999999958E-03	1.0E-5
12	LAGS_C for all the nodes	28117.686527187	1.0E-5
15	H1Z for all the nodes	2.99999999999958E-03	1.0E-5
15	LAGS_C for all the nodes	718.731177854856	1.0E-5

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-5
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-5
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-5
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-5
2	H1X for all the nodes	4.0908719011645E-07	1.0E-5
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-5
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-5
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-5
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-5
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-5
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-5
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-5
7	H1X for all the nodes	1.49999999999958E-03	1.0E-5
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-5

9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-5
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-5
12	H1X for all the nodes	1.49999999999958E-03	1.0E-5
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-5
15	H1X for all the nodes	2.99999999999958E-03	1.0E-5
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-5

10.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

11 Modeling I

11.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling E.

11.2 Characteristics of the grid

The grid is refined here compared to that of modeling F. One discretizes the structure in $2\times(5\times1\times13)$ finite elements PENTA6. The interface is thus present within the central elements by the means of level sets.

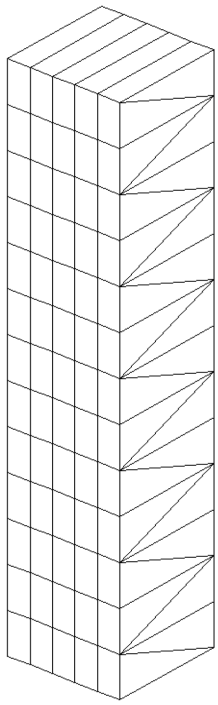

Figure 11.2-a: Grid of modeling I

11.3 Piloting

A specific piloting is used $X-FEM$ SAUT_IMPO using the group of nodes located immediately at the top of the crack.

11.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Z for all the nodes	1.36362396705485E-07	2.0E-6
0.5	LAGS_C for all the nodes	3.66296853301E+05	2.0E-6
0,75	H1Z for all the nodes	6.818119835274E-08	2.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	2.0E-6
2	H1Z for all the nodes	4.0908719011645E-07	2.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	2.0E-6
3,5	H1Z for all the nodes	7.49999999999582E-04	2.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	2.0E-6
4,5	H1Z for all the nodes	2.49999999999582E-04	2.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	2.0E-6
5,5	H1Z for all the nodes	7.49999999999582E-04	2.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	2.0E-6
7	H1Z for all the nodes	1.49999999999958E-03	2.0E-6
7	LAGS_C for all the nodes	28117.686527187	2.0E-6
9,5	H1Z for all the nodes	2.49999999999582E-04	2.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	2.0E-6
12	H1Z for all the nodes	1.49999999999958E-03	2.0E-6
12	LAGS_C for all the nodes	28117.686527187	2.0E-6
15	H1Z for all the nodes	2.99999999999958E-03	2.0E-6
15	LAGS_C for all the nodes	718.731177854856	2.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	2.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	2.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	2.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	2.0E-6
2	H1X for all the nodes	4.0908719011645E-07	2.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	2.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	2.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	2.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	2.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	2.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	2.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	2.0E-6
7	H1X for all the nodes	1.49999999999958E-03	2.0E-6
7	LAGS_F1 for all the nodes	28117.686527187	2.0E-6

9,5	`H1X` for all the nodes	2.49999999999582E-04	2.0E-6
9.5	`LAGS_F1` for all the nodes	4686.28108785798	2.0E-6
12	`H1X` for all the nodes	1.49999999999958E-03	2.0E-6
12	`LAGS_F1` for all the nodes	28117.686527187	2.0E-6
15	`H1X` for all the nodes	2.99999999999958E-03	2.0E-6
15	`LAGS_F1` for all the nodes	718.731177854856	2.0E-6

11.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

12 Modeling J

12.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling A , put except for one changes the position of the level set in order to position it with the interface between two elements.
The equation of the function of levels for the interface is the following one:

$$\mathrm{LN}=Y-2$$

12.2 Characteristics of the grid

One discretizes the structure in 2×5 finite elements TRIA3. The interface is thus present between two elements by the means of level sets.

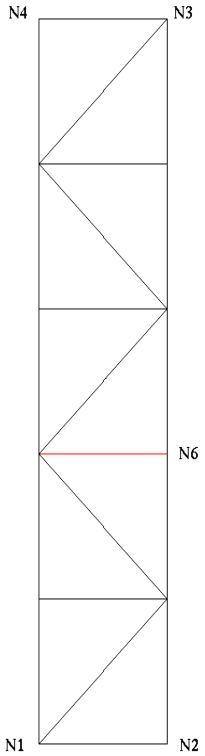

Figure 12.2-a: Grid of modeling J

12.3 Piloting

One uses a specific piloting XFEM SAUT_IMPO on all the intersected edges.

12.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniforms on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Y for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1Y for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_C for all the nodes	1.8314842665E+05	1.0E-6
2	H1Y for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_C for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_C for all the nodes	58622.573562549	1.0E-6
5,5	H1Y for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_C for all the nodes	1.75867720687844E+05	1.0E-6
7	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_C for all the nodes	28117.686527187	1.0E-6
9,5	H1Y for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_C for all the nodes	4686.28108785798	1.0E-6
12	H1Y for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_C for all the nodes	28117.686527187	1.0E-6
15	H1Y for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_C for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction LAGS_F1 in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
7	H1X for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-6
12	H1X for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-6

15	H1X for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-6

12.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

13 Modeling K

13.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling G, put except for one changes the position of the level set in order to position it with the interface between two elements.
The equation of the function of levels for the interface is the following one:

$$\mathrm{LN}=Y-10$$

13.2 Characteristics of the grid

The grid is the same one as that of modeling G.

13.3 Piloting

One uses a specific piloting XFEM `SAUT_IMPO` on all the intersected edges.

13.4 Sizes tested and results

Mode I :

One tests the values of Lagrangian of contact `LAGS_C` in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator `STAT_NON_LINE`, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	`H1Z` for all the nodes	1.36362396705485E-07	1.0E-6
0.5	`LAGS_C` for all the nodes	3.66296853301E+05	1.0E-6
0,75	`H1Z` for all the nodes	6.818119835274E-08	1.0E-6
0.75	`LAGS_C` for all the nodes	1.8314842665E+05	1.0E-6
2	`H1Z` for all the nodes	4.0908719011645E-07	1.0E-6
2	`LAGS_C` for all the nodes	1.098890559903E+06	1.0E-6
3,5	`H1Z` for all the nodes	7.49999999999582E-04	1.0E-6
3.5	`LAGS_C` for all the nodes	1.75867720687844E+05	1.0E-6
4,5	`H1Z` for all the nodes	2.49999999999582E-04	1.0E-6
4.5	`LAGS_C` for all the nodes	58622.573562549	1.0E-6
5,5	`H1Z` for all the nodes	7.49999999999582E-04	1.0E-6
5.5	`LAGS_C` for all the nodes	1.75867720687844E+05	1.0E-6
7	`H1Z` for all the nodes	1.49999999999958E-03	1.0E-6
7	`LAGS_C` for all the nodes	28117.686527187	1.0E-6
9,5	`H1Z` for all the nodes	2.49999999999582E-04	1.0E-6
9.5	`LAGS_C` for all the nodes	4686.28108785798	1.0E-6
12	`H1Z` for all the nodes	1.49999999999958E-03	1.0E-6
12	`LAGS_C` for all the nodes	28117.686527187	1.0E-6
15	`H1Z` for all the nodes	2.99999999999958E-03	1.0E-6
15	`LAGS_C` for all the nodes	718.731177854856	1.0E-6

Mode II :

One tests the values of Lagrangian of friction `LAGS_F1` in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator `STAT_NON_LINE`, these values being uniform on the interface. To test all the values in once, one tests the minimum and the maximum of Lagrangian of friction.

Not	Identification	Reference	Tolerance (%)
0,5	H1X for all the nodes	1.36362396705485E-07	1.0E-6
0.5	LAGS_F1 for all the nodes	3.66296853301E+05	1.0E-6
0,75	H1X for all the nodes	6.818119835274E-08	1.0E-6
0.75	LAGS_F1 for all the nodes	1.8314842665E+05	1.0E-6
2	H1X for all the nodes	4.0908719011645E-07	1.0E-6
2	LAGS_F1 for all the nodes	1.098890559903E+06	1.0E-6
3,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
3.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
4,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
4.5	LAGS_F1 for all the nodes	58622.573562549	1.0E-6
5,5	H1X for all the nodes	7.49999999999582E-04	1.0E-6
5.5	LAGS_F1 for all the nodes	1.75867720687844E+05	1.0E-6
7	H1X for all the nodes	1.49999999999958E-03	1.0E-6
7	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
9,5	H1X for all the nodes	2.49999999999582E-04	1.0E-6
9.5	LAGS_F1 for all the nodes	4686.28108785798	1.0E-6
12	H1X for all the nodes	1.49999999999958E-03	1.0E-6
12	LAGS_F1 for all the nodes	28117.686527187	1.0E-6
15	H1X for all the nodes	2.99999999999958E-03	1.0E-6
15	LAGS_F1 for all the nodes	718.731177854856	1.0E-6

13.5 Comments

The values of Lagrangian of contact and friction are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

14 Modelisation L

14.1 Characteristics of modeling

It is about a modeling $X-FEM$, in plane constraints, with definition of contact on the interface of discontinuity defined by a function of level (level set noted LN for the level set normal) directly in the command file using the operator DEFI_FISS_XFEM [U4.82.08].
The statute main slave/for a surface of contact $X-FEM$ is given by the sign of the normal function of level LN : surface slave is negative side while surface Master is positive side.

The equation of the function of level for the interface is the following one:

$$\mathrm{LN}=Y-2.5$$

No level set tangential is necessary since the keyword is used TYPE_DISCONTINUITE=' INTERFACE', this Qui makes it possible to have the structure completely cut in two parts.

This tests models an interface in opening and closing for which the jump of displacement is controlled by a piloting by SAUT_IMPO.

The cohesive law is introduced via the operator DEFI_CONT, while specifying ALGO_CONT=' CZM', and the lol of behavior cohesive is activated thanks to the keyword RELATION=' CZM_EXP_REG'. In closing, the contact is managed by a term of penalization understood in the cohesive law.

14.2 Characteristics of the grid

The grid is the same one as that of modeling A. The interface is thus present within the central element by the means as of level sets.

14.3 Boundary conditions

The limiting conditions are the same ones as those of the mode I of opening of modelings $2\mathrm{D}$. Values of COEF_MULT a step of time in opening then a step of time in closing is modified to carry out.

Moment of final calculation	Phase	Jump of final displacement (in m)	c_{mult}	$H1Y$ final
0.5	Traction	2,73E-7	3.6667E6	1.36362396705485E-07
1.0	Compression	-2,73E-7	-9.16675E5	-1.36362396705485E-07

14.4 Reference solution

In opening, the solution is given by the cohesive law to paragraph 2.
In closing, the penalized solution is equal to the product of the jump of displacement by the coefficient of penalization which is the same one as the adhesion coefficient since we entered PENA_CONTACT=1. (see documentation [R7.02.11]).

14.5 Sizes tested and results

One tests the values of Lagrangian of contact LAGS_C in all the nodes of the mesh crossed by the interface after convergence of the iterations of each operator STAT_NON_LINE. To test all the values in once, one tests the minimum and the maximum of Lagrangian of contact.

Not	Identification	Reference	Tolerance (%)
0,5	H1Y for all the nodes	1.36362396705485E-07	1.0E-10
0.5	LAGS_C for all the nodes	3.66296853301E+05	1.0E-10
1	H1Y for all the nodes	-1.36362396705485E-07	1.0E-10
1	LAGS_C for all the nodes	-3.66296853301E+05	1.0E-10

14.6 Comments

This watch tests which the term of penalization includes in the cohesive law of behaviour for the treatment of the contact makes it possible to raise the incompatibility between contact of continuous method and piloting. Remain with the user to check that the interpenetration obtained remains physical, and to increase PENA_CONTACT if such is not the case.

The values of Lagrangian of contact are explicitly calculated according to the jump of displacement that one controls. It is thus natural to have quasi-worthless errors.

15 Summary of the results

The digital results are in agreement with the analytical solution. These tests make it possible to validate the cohesive law of contact implemented for the method $X-FEM$ in $2\mathrm{D}$ and $3\mathrm{D}$ in the various modes of opening.
Modeling L makes it possible of more than raise the incompatibility between contact of continuous method and piloting by using a term of penalization in the cohesive law.