SSNP159 – Elastic energy in great plastic deformations of a tensile test bar

Summary:

This quasi-static mechanical test consists to subject to a simple traction a bar of rectangular section ($3D$) or cylindrical ($2D$ axisymmetric). The object is to validate the calculation of elastic energies in three formalisms of deformation: ‘SMALL‘,’GDEF_LOG‘and’SIMO_MIEHE‘, with the order POST_ELEM.
Moreover, modeling B of this case test, proposes to test the keyword factor ETAT_INIT proposed by the order STAT_NON_LINE, with GDEF_LOG. The user wishing to impose an initial state of stress with the formalismGDEF_LOG will find the necessary information.

The bar is modelled by a voluminal element (HEXA20, modeling A) or quadrangular (QUAD4, for an axisymmetric modeling, modeling B).

The solution is analytical.

1 Problem of reference

1.1 Geometry

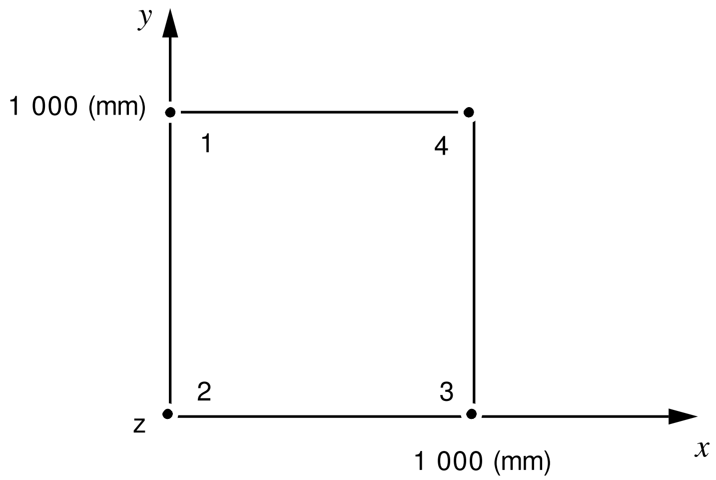

Figure 1.1-1: Geometry of reference

1.2 Properties of material

The material obeys a law of behavior in great deformations figure with linear isotropic work hardening. The traction diagram is given in the plan deformation logarithmic curve - rational constraint.

$$\sigma=\frac{F}{S}=\frac{F}{S_0}.\frac{l}{l_0}$$

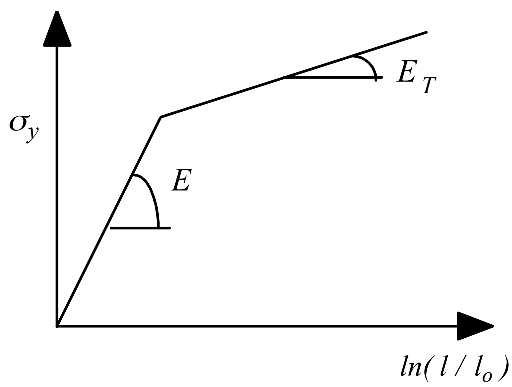

Figure 1.2-1: Traction diagram

$$\nu=0,3$$
$$E=200000\text{MPa}$$
$$E_T=2000\text{MPa}$$
$$\sigma_y=1000\text{MPa}$$

l_0 and l are, respectively, the initial length and the current length of the useful part of the test-tube.

S_0 and S are, respectively, initial and current surface.

1.3 Boundary conditions and loadings

The bar, initial length l_0 , blocked in the direction Ox on the face [1.2] with a mechanical displacement of traction u^{meca} varying linearly in time on the face $[3,4]$:

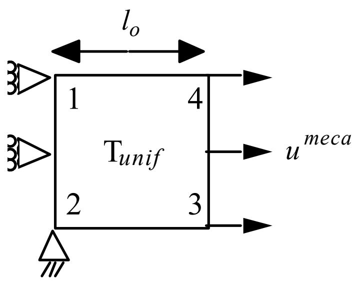

Figure 1.3-1: Limiting conditions and loading

2 Reference solution

The analytical solution of `SIMO_MIEHE` allows to define the loading to be applied to obtain a constraint of Kirchhoff of $1500\mathrm{MPa}$. This loading is then applied to the other models.

2.1 Generic result with the formalisms

For a uniaxial tensile test according to the direction $\boldsymbol{x}$, tensors of constraint of Kirchhoff $\boldsymbol{\tau}$ and of Cauchy $\boldsymbol{\sigma}$ are form:

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\boldsymbol{\tau}=J\,\boldsymbol{\sigma}=\begin{bmatrix}\tau & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

Tensor gradients of the transformation $\mathbf{F}$ and $\bar{\mathbf{F}}$ express themselves:

$$\boldsymbol{F}=\begin{bmatrix}F & 0 & 0\\ 0 & F_y & 0\\ 0 & 0 & F_y\end{bmatrix},\bar{\boldsymbol{F}}=J^{\frac{-1}{3}}=\begin{bmatrix}\bar{F} & 0 & 0\\ 0 & \bar{F}_y & 0\\ 0 & 0 & \bar{F}_y\end{bmatrix}$$

Displacement checks:

$$F=1+\frac{u^{meca}}{l_0}$$

The function of isotropic work hardening linear is written:

$$R(p)=\frac{E\,E_T}{E-E_T}$$

2.2 Results for `Simo_Miehe`

The isochoric tensor of plastic deformations G^P is form:

$$\boldsymbol{G^P}=\begin{bmatrix}G^P & 0 & 0\\ 0 & G_y^P & 0\\ 0 & 0 & G_y^P\end{bmatrix}$$, with $G_y^P=\frac{1}{\sqrt{G^P}}$ because $det(\boldsymbol{G^P})=1$

The law of behavior (left hydrostatic) is written:

$$tr(\boldsymbol{\tau})=\frac{3\mathrm{K}}{2}(J^2-1)\Rightarrow J^2=\frac{2\tau}{3\mathrm{K}}+1$$

The function threshold of plasticity is written:

$$f=0=\tau-R(p)-\sigma_y\Rightarrow p=\frac{E-E_T}{E\,E_T}(\tau-\sigma_y)$$

The plastic law of flow is written:

$$\bar{\boldsymbol{F}}.\dot{\boldsymbol{G}}^{\boldsymbol{P}}.\bar{\boldsymbol{F}}^{\boldsymbol{T}}=-3\,\dot{p}\frac{1}{\tau_{eq}}dev(\boldsymbol{\tau}).(\bar{\boldsymbol{F}}.\boldsymbol{G^P}.\bar{\boldsymbol{F}}^{\boldsymbol{T}})$$

By taking the first component, one obtains:

$\frac{\dot{G}^P}{G^P}=-2\,\dot{p}\Rightarrow G^p=e^{-2p}$ because $G^P(0)=1$

To conclude the problem, one uses the first component of the deviatoric part of the constraint:

$$dev(\boldsymbol{\tau})=\mu\, dev(\bar{\boldsymbol{b}}^e)\Rightarrow\tau=\mu\left(\bar{F}^2 G^P-\bar{F}_y^2 G_y^P\right)\Rightarrow\bar{F}^3-\bar{F}\frac{\tau}{\mu G^P}-\frac{1}{\left(G^P\right)^{(3/2)}}=0\ ,$$

because $\bar{\boldsymbol{b}}^e=\bar{F}.\boldsymbol{G}^P.\bar{\boldsymbol{F}}^T$

Elastic energy is written then:

$$\psi_{SM}^{elas}=\frac{K}{2}\left[\frac{J^2-1}{2}-\ln J\right]+\frac{\mu}{2}\left[tr\,\bar{\boldsymbol{b}}^e-3\right]=\frac{K}{2}\left[\frac{J^2-1}{2}-\ln J\right]+\frac{\mu}{2}\left[\bar{F}^2 G^P+\frac{2}{\bar{F}\sqrt{G^P}}-3\right]$$

For a constraint of Kirchhoff of 1500MPa, can then successively determine:

- $J=1{,}003$
- $\sigma=1495\,MPa$
- $p=0{,}2475$
- $G^p=0{,}6096$
- $\bar{F}=1{,}289$
- $F=1{,}290$
- $u^{meca}=290\,mm$
- $\psi_{SM}^{elas}=5{,}63\,MPa$ at the material point.

The displacement applied for two modelings and the 3 formalisms will be thus $u^{meca}=290\,mm$.

2.3 Results for GDEF_LOG

The deformation logarithmic curve is written:

$$\boldsymbol{E}_{\log}=\frac{1}{2}\ln[\boldsymbol{C}]=\frac{1}{2}\ln[\boldsymbol{F}^T.\boldsymbol{F}]=\begin{bmatrix}\ln F & 0 & 0\\ 0 & \ln F_y & 0\\ 0 & 0 & \ln F_y\end{bmatrix}$$

The left higher quarter of the projector lagragien from of deduced, in notation of Voigt:

$$\boldsymbol{P}=2\frac{\partial\boldsymbol{E}_{\log}}{\partial\boldsymbol{C}}=\frac{1}{2}\begin{bmatrix}\frac{2}{F^2} & 0 & 0\\ 0 & \frac{2}{F_y^2} & 0\\ 0 & 0 & \frac{2}{F_y^2}\end{bmatrix}$$

And because of expression of the second constraint of Piola-Kirchhoff:

$\boldsymbol{S}=\boldsymbol{T}:\boldsymbol{P}=\boldsymbol{F}^{-1}.\boldsymbol{\tau}.\boldsymbol{F}^{-T}\Rightarrow\boldsymbol{T}=\boldsymbol{\tau}$

The law of behavior is written:

$T=E(\ln F-p)$

Because of the threshold of plasticity:

$$f=0=T-R(p)-\sigma_y\Rightarrow p=\frac{E-E_T}{E\,E_T}(T-\sigma_y)=\frac{E\ln F-\sigma_y}{E+\frac{E\,E_T}{E-E_T}}\ ,$$

The elastic energy of this formalism is thus written:

$$\psi_{\log}^{elas}=\frac{1}{2}\frac{T^2}{E}$$

Imposed displacement $u^{meca}=290\,mm$, one deduces:

- $F=1{,}290$
- $\ln F=0{,}255$
- $p=0{,}2475$
- $T=1500\mathrm{MPa}$
- $\sigma=1495\mathrm{MPa}$
- $\psi_{\log}^{elas}=5{,}625\,MPa$ at the material point.

2.4 Results in small deformations

In small deformations, the result is classical.
Axial deformation:

$$\varepsilon_x=\frac{u^{meca}}{l_0}$$

Behavior:

$$\sigma=E\left(\varepsilon_x-p\right)$$

Function threshold:

$$\sigma-R(p)-\sigma_y=0$$

From where:

$$p=\frac{E\,\varepsilon_x-\sigma_y}{E+\frac{E\,E_T}{E-E_T}}$$
$$\sigma=E\left(\varepsilon_x-p\right)$$

Elastic energy:

$$\psi_{HPP}^{elas}=\frac{\sigma^2}{2E}$$

Imposed displacement $u^{meca}=290\,mm$, one deduces:

- $\varepsilon_x=0{,}029$
- $p=0{,}281$
- $\sigma=1570\mathrm{MPa}$
- $\psi_{\log}^{elas}=6{,}16\,MPa$ at the material point.

2.5 Tests carried out

For each of the formalisms and each modeling, one tests the values of imposed displacement, the constraint of Cauchy σ , cumulated plastic deformation p and of elastic energy.

Attention, elastic energy tested is the value for the bar (not of the material point). For axisymmetric modeling, it is thus necessary multiplied the value of the elastic energy of the material point by $\frac{\pi R^2}{2\pi}$

.

3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling: 1 mesh HEXA20
1 mesh QUAD8

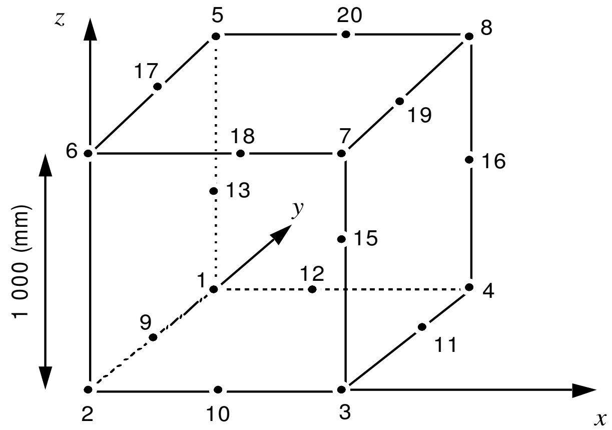

Figure 3.1-1: Grid of modeling A

Boundary conditions:

$N2$:	$U_x=U_y=U_z=0$	$N9$, $N13$, $N14$, $N5$, $N17$: $U_x=0$
$N1$:	$U_x=U_z=0$	
$N6$:	$U_x=U_y=0$	

Table 3.1-1: Limiting conditions modeling A

Load: Traction on the face $[3,4,8,7,11,16,19,15]$

The full number of increments is of 20 (20 increments enters $t=0\text{s}$ and 2s)
Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to 10^{-6}.

3.2 Characteristics of the grid

Many nodes: 20

Many meshs: 2

1 HEXA20
1 QUAD8

3.3 Sizes tested and results

Identification	Reference			Tolerance
	SIMO_MIEHE	GDEF_LOG	HP	
$t=2$ Displacement DX ($N8$)	290	290	290	1,00%
$t=2$ Constraints $SIGXX$ ($PG1$)	1495	1495	1570	1,00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	0.2475	0.282	1,50%
$t=2$ ENER ELAS, TOTAL	5,63E+009	5,625E9	6,16E9	5,00%

Table 3.3-1: Results of modeling A

4 Modeling B

4.1 Characteristics of modeling

Modeling $2D$ axisymmetric: 1 mesh QUAD4
1 mesh SEG2

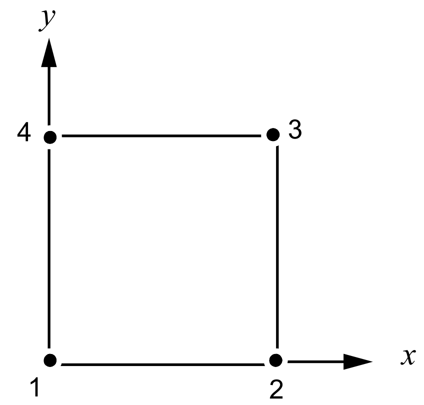

Figure 4.1-1: Grid of modeling B

Boundary conditions:

$N1$: $U_Y=0$

$N2$: $U_Y=0$

Table 4.1-1: Limiting conditions of modeling B

Loading:

Traction on the face $[3,4]$ (mesh SEG2)
The full number of increments is of 20 (20 increments enters $t=0\text{s}$ and 2s)

Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to 10^{-6}.

4.2 Characteristics of the grid

Many nodes: 4
Many meshs: 2

1 QUAD4
1 SEG2

4.3 Sizes tested and results

Identification	Reference			Tolerance
	SIMO_MIEHE	**GDEF_LOG**	**HP**	
$t=2$ Displacement DX ($N8$)	290	290	290	1,00%
$t=2$ Constraints $SIGXX$ ($PG1$)	1495	1495	1570	1,00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	0.2475	0.282	1,50%
$t=2$ ENER ELAS, TOTAL	2,82E+009	2.81 E9	3.08 E9	5,00%

Table 4.3-1: Results of modeling B

4.4 GDEF_LOG with ETAT_INIT

To impose an initial stress field in great deformations with the formalism GDEF_LOG, the user must give as starter the tensor of constraint defined in space logarithmic curve $\boldsymbol{T}$ (and not that of Cauchy $\boldsymbol{\sigma}$). The components of this last, being stored as internal variables, the operands should be used VARI and DEPL keyword factor ETAT_INIT order STAT_NON_LINE as indicated (these fields below can for example be obtained by the order CREA_CHAMP).

One recovers the field of internal variables (to obtain $\boldsymbol{T}$) and also the field of corresponding displacement:

```
VAR_LOG1=CRÉA_CHAMP (INFO=2,
            TYPE_CHAM=' ELGA_VARI_R',
            OPERATION=' EXTR',
            RESULTAT=LOG1,
            NOM_CHAM=' VARI_ELGA',
            INST=1.0,);
```

```
DEP_LOG1=CRÉA_CHAMP (INFO=2,
            TYPE_CHAM=' NOEU_DEPL_R',
            OPERATION=' EXTR',
            RESULTAT=LOG1,
            NOM_CHAM=' DEPL',
            INST=1.0,);
```

then one informs in STAT_NON_LINE, the initial state of stress:

```
ETAT_INIT=_F (VARI=VAR_LOG1,
               DEPL=DEP_LOG1).
```

5 Summary of the results

Results found in elastic term of energy with *Code_Aster* are very satisfactory in small deformations and deformations logarithmic curves, with variations with analytical the lower than 0.1%, and correct in deformation of SIMO_MIEHE, with a variation with analytical of approximately 3%, due at the end $tr\left(\bar{\boldsymbol{b}}^e\right)$, on which the fifth decimal plays a significant part.