# SSNP162 – Joints 2D and 3D for the laws `JOINT_MECA_RUPT` and `JOINT_MECA_FROT`

**Summary:**

This test has as an aim the validation of the hydro-mechanical elements of joints for the laws of behavior of the stoppings `JOINT_MECA_RUPT` and `JOINT_MECA_FROT`.

At the same time pure mechanical modelings are tested `PLAN_JOINT` in $2D$ (mesh `QUAD4`, `QUAD8`) and `3D_JOINT` in $3D$ (mesh `HEXA8`, `HEXA20` or `PENTA6`, `PENTA15`) with the two laws in the presence of imposed pressure. Then, always for same the quadratic elements coupled hydro-mechanical modelings `PLAN_JOINT_HYME` and `C3D_JOINT_HYME`, with the profile of uplift obtained in calculation.

The elements on which these modelings are pressed are the elements of standard cohesive joints.

Modeling $A$ : `PLAN_JOINT` mesh `QUAD4` with `JOINT_MECA_RUPT`
Modeling $B$ : `3D_JOINT` mesh `HEXA8` with `JOINT_MECA_RUPT`
Modeling $C$ : `3D_JOINT` mesh `PENTA6` with `JOINT_MECA_RUPT`

Modeling $D$ : `PLAN_JOINT` mesh `QUAD4` with `JOINT_MECA_FROT`
Modeling $E$ : `3D_JOINT` mesh `HEXA8` with `JOINT_MECA_FROT`
Modeling $F$ : `3D_JOINT` mesh `PENTA6` with `JOINT_MECA_FROT`

Modeling $G$ : `PLAN_JOINT_HYME` mesh `QUAD8` with `JOINT_MECA_RUPT`
Modeling $H$ : `3D_JOINT_HYME` mesh `HEXA20` with `JOINT_MECA_RUPT`
Modeling $I$ : `3D_JOINT_HYME` mesh `PENTA15` with `JOINT_MECA_RUPT`

Modeling $J$ : `PLAN_JOINT_HYME` mesh `QUAD8` with `JOINT_MECA_FROT`
Modeling $K$ : `3D_JOINT_HYME` mesh `HEXA20` with `JOINT_MECA_FROT`
Modeling $L$ : `3D_JOINT_HYME` mesh `PENTA15` with `JOINT_MECA_FROT`

Modeling $M$ : `PLAN_JOINT` mesh `QUAD8` with `JOINT_MECA_RUPT`
Modeling $N$ : `3D_JOINT` mesh `HEXA20` with `JOINT_MECA_RUPT`
Modeling $O$ : `3D_JOINT` mesh `PENTA15` with `JOINT_MECA_RUPT`

Modeling $P$ : `PLAN_JOINT` mesh `QUAD8` with `JOINT_MECA_FROT`
Modeling $Q$ : `3D_JOINT` mesh `HEXA20` with `JOINT_MECA_FROT`

Modeling $R$ : `3D_JOINT` mesh `PENTA15` with `JOINT_MECA_FROT`

# 1 Problem of reference

## 1.1 Geometry

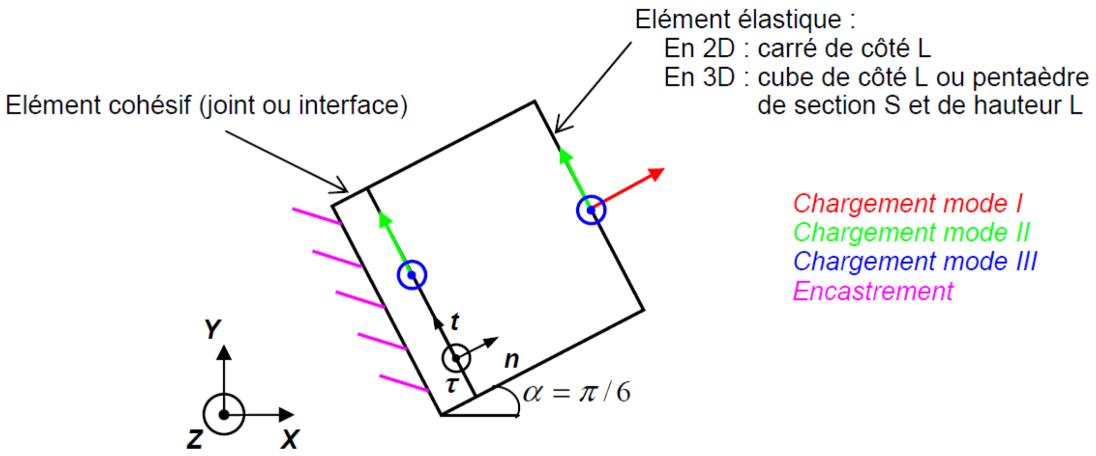

**Figure 1 : Representation of the system of two elements in the plan $(X, Y)$.**

One chooses $L=1\,mm$.

## 1.2 Properties of material

### 1.2.1 Law `JOINT_MECA_RUPT`: material `JOINT_MECA_RUPT`

**Cubic** : rubber band
Young modulus: $E=3\times10^{12}\,Pa$
Poisson's ratio: $\nu=0$

| | | |
|---|---|---|
| normal rigidity: | $K_N=10^{12}\,Pa/m$ | (keyword: `K_N`) |
| tangential rigidity: | $K_T=2\times10^{12}\,Pa/m$ | (keyword: `K_T`) |
| tensile strength: | $\sigma_{max}=0.1\,MPa$ | (keyword: `SIGM_MAX`) |
| tangential regularization of damage | $\alpha=1.5$ | (keyword: `ALPHA`) |
| parameter of brittle smoothing of fracture | $P_{rup}=0.5$ | (keyword: `PENA_RUPTURE`) |
| penalization of the contact | $P_{cont}=3$ | (keyword: `PENA_CONTACT`) |
| Density of the fluid (only for modelings HM) | $\rho_{fluide}=1000\,kg/m^3$ | (keyword: `RHO_FLUIDE`) |
| Dynamic viscosity of the fluid (only for modelings HM) | $\mu_{fluide}=0.001\,Pa.s$ | (keyword: `VISC_FLUIDE`) |
| Opening of regularization of the flow (only for modelings HM) | $\delta_{min}=1.E-10\,m$ | keyword: `OUV_MIN`) |

NB: The data materials do not have of course authority to represent a material in particular. They are only intended for digital tests of validation.

### 1.2.2 Law `JOINT_MECA_FROT`: material `JOINT_MECA_FROT`

**Cubic** *:* rubber band
Young modulus: $E=3\times10^{12}\,Pa$
Poisson's ratio: $\nu=0$

| | | |
|---|---|---|
| normal rigidity: | $K_N=10^{12}\,Pa/m$ | (keyword: `K_N`) |
| tangential rigidity: | $K_T=2\times10^{12}\,Pa/m$ | (keyword: `K_T`) |
| coefficient of friction: | $\mu=0.5$ | (keyword: `DRIVEN`) |
| adhesion (friction with normal loading no one) | $c=10^5\,Pa$ | (keyword: `ADHESION`) |
| regularization of the tangent slope in slip | $\lambda=10^{-6}K_T$ | (keyword: `PENA_TANG`) |

NB: The data materials do not have of course authority to represent a material in particular. They are only intended for digital tests of validation.

## 1.3 Boundary conditions and loadings

**Embedding** : Imposed displacements are worthless on the FACE of the cohesive element Oposed with the elastic element.
The joint is tilted with 30 degrees compared to the plan horizontal, which gives the following directions of loading according to the mode of request:

**In mode** $I$ : An imposed displacement $U$ is applied to the face of the elastic element opposed to the joint (see figure 1).

$$DX=2.16506351 \quad DY=1.250 \quad DZ=0$$

**In mode** $II$ : Imposed displacement $U$ is applied to all the nodes of the voluminal element.

$$DX=-1.250 \quad DY=2.165063509461 \quad DZ=0$$

**In mode** $III$ : Imposed displacement $U$ is applied to all the nodes of the voluminal element.

$$DX=0.0 \quad DY=0.0 \quad DZ=2.5$$

In mode $I$ the joint follows a standard loading return ticket: initially to the partial opening ( $\delta_n>0$ ) , then one passes in compression until $\delta_n<0$ and finally it is discharged up to its point from balance $\delta_n=0$ .
In order to supplement the test in mixed mode one partially gives the responsibility in mode $I$ and then one makes a return ticket in mode $II$ (or $III$ according to modeling).

# 2 Reference solution

## 2.1 Case general

In this part, one details the analytical solution in mode $I$ pure in its form $3D$. For calculations $2D$ plans, the solution is identical. The component of the jump and the following vector forced $\tau$ do not intervene, and it is enough to replace surface $S$ by the length $L$ in the solution.

For the loadings in mode of shearing, the elastic element does not play a part.

Initially, the joint follows a loading of type return ticket in mode $I$ what makes it possible to test the profile of the curve forces displacement at various moments for this standard loading. In the second part, one charges in a sequential way in mode $I$, then in mode $II$ (or $III$ according to modeling). Then one modifies the loading in mode $I$ and discharges in mode $II$. One thus requests the coupling between the two load patterns (it is checked, for example, that the tangential slope evolves according to the normal loading). One tests also the profile of the curve forces displacement for the tangential stress.

## 2.2 In mode $I$ pure

One presents the analytical solution of the total answer of the system written in the local reference mark $(\boldsymbol{n},\boldsymbol{t},\boldsymbol{\tau})$. One applies a loading colinéaire to the normal: $\boldsymbol{U}=U\boldsymbol{n}$, the cohesive element opens in mode $I$ pure and the tangential constraints as well as the tangential jumps remain worthless. One thus brings back oneself to a scalar problem. One notes $\sigma=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}$ the single nonworthless component of the tensor of the constraint of the elastic element in the local reference mark:

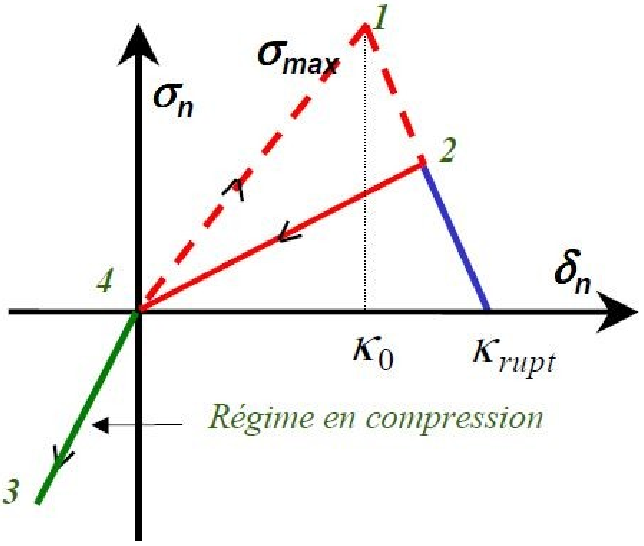

**Illustration 1: Loading return ticket in mode $I$ law of JOINT_MECA_RUPT**

- `JOINT_MECA_RUPT`

One presents the solution of the total answer of the system joint+cube (see Document [R7.01.25]).

The equality of the constraints gives:

$$\begin{cases}\sigma=K_N\delta_n\\ \sigma=E\,\varepsilon_{cube}=E(U-\delta_n)/L_{cube}\end{cases}\quad\text{in the elastic zone}$$

$$\begin{cases}\sigma=\sigma_{max}-K_N/P_{rup}(\delta_n-\sigma_{max}/K_N)\\ \sigma=E\,\varepsilon_{cube}=E(U-\delta_n)/L_{cube}\end{cases}\quad\text{in the zone of softening}$$

Where $L_{cube}=1$ is the length of the edge of the cube in our case. One notes the maximum opening of the joint until the rupture supplements by $U_{max}=\sigma_{max}(1+P_{rup})/K_N$ and the maximum elastic opening by $U_{el}=\sigma_{max}(K_N L_{cube}+E)/K_N E$ .

The solution is given by the linear function per pieces:

| | | |
|---|---|---|
| $\sigma=-K_N E/(K_N L_{cube}+E/P_{cont})U$ | si $U<0$ | in the compression zone |
| $\sigma=K_N E/(K_N L_{cube}+E)U$ | si $U<U_{el}$ | in the elastic zone in traction |
| $\sigma=K_N E/(-K_N L_{cube}+E*P_{rup})(U_{max}-U)$ | si $U<U_{max}$ | in the zone of softening |
| $\sigma=0$ | si $U>U_{max}$ | complete damage |

The force tested is thus given by $F=\sigma S$ where $S$ corresponds to the surface to which one applies the loading.
For hydro-mechanical modelings (modelings G, H, I) the value of the force is given by: $F=(\sigma-p_{fluide})S$ where $p_{fluide}$ indicate the pressure of the fluid contributing to the opening of the joint.

- **`JOINT_MECA_FROT`**

One presents the solution of the total answer of the system more cubic joint (see Doc. [`R7.01.25`]).

The equality of the constraints gives:

$$\begin{cases}\sigma=K_N\delta_n\\ \sigma=E\,\varepsilon_{cube}=E(U-\delta_n)/L_{cube}\end{cases}\quad\text{in the elastic zone}$$

$$\begin{cases}\sigma=c/\mu\\ \sigma=E\,\varepsilon_{cube}=E(U-\delta_n)/L_{cube}\end{cases}\quad\text{in the plastic zone}$$

Where $L_{cube}=1$ is the length of the edge of the cube in our case. Like the maximum constraint in mode $I$ for the law of Mohr-Coulomb is given by $\sigma_{max}=c/\mu$ , the maximum elastic opening (elastic threshold of traction) is equal to $U_{el}=\sigma_{max}(K_N L_{cube}+E)/K_N E$ . One advances up to this value of displacement one checks the value of $\sigma=\sigma_{max}=c/\mu$ , then one imposes double displacement is one checks that the value of constraint does not evolve. Then in the field of compression the solution is always elastic $\sigma=K_N\delta_n$ .
The force tested is given by $F=\sigma S$ where $S$ corresponds to the surface to which one applies the loading.

For hydro-mechanical modelings (modelings J, K, L) the value of the force is given by: $F=(\sigma-p_{fluide})S$ where $p_{fluide}$ indicate the pressure of the fluid contributing to the opening of the joint.

## 2.3 In mode $II$ and $III$ pure

The system written in the local reference mark $(\boldsymbol{n},\boldsymbol{t},\boldsymbol{\tau})$ . One applies a loading perpendicular to the normal directly to the joint: $\boldsymbol{U}=U_t\boldsymbol{\tau}$ . The cohesive element opens in mode $II$ . If the loading normal does not evolve one brings back oneself to a scalar problem. One notes $\sigma_n=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}$ , $\sigma_\tau=\boldsymbol{\tau}.\boldsymbol{\sigma}.\boldsymbol{n}$ constraints in the local reference mark.
The loading in mode $I$ being maintained, one charges in mode $II$ (or $III$ according to modeling). The coupling between the two modes is thus requested. The solution varies according to law of behavior:

- `JOINT_MECA_RUPT`
  When the joint is partially open ( $\delta_n>0$ ) the tangential slope evolves according to this opening normal like: $K_T^{evol}=K_T\times(1-\delta_n/L_{CT})$ (see Doc. [`R7.01.25`])
  where $L_{CT}=\sigma_{max}(1+P_{rup})/K_N\tan(\alpha\pi/4)$ is the critical length of tangential damage. The solution is given by: $\sigma_\tau=K_T^{evol}U_t$

- `JOINT_MECA_FROT`
  One tests curved force-displacement for the joint, whose normal loading evolves. The joint is partially closed ( $\delta_n>0$ ). The solution is the simple slip of Mohr-Coulomb. $\sigma_\tau=-\mu\sigma_n+c$ This curve is tested at various moments (see illustration):

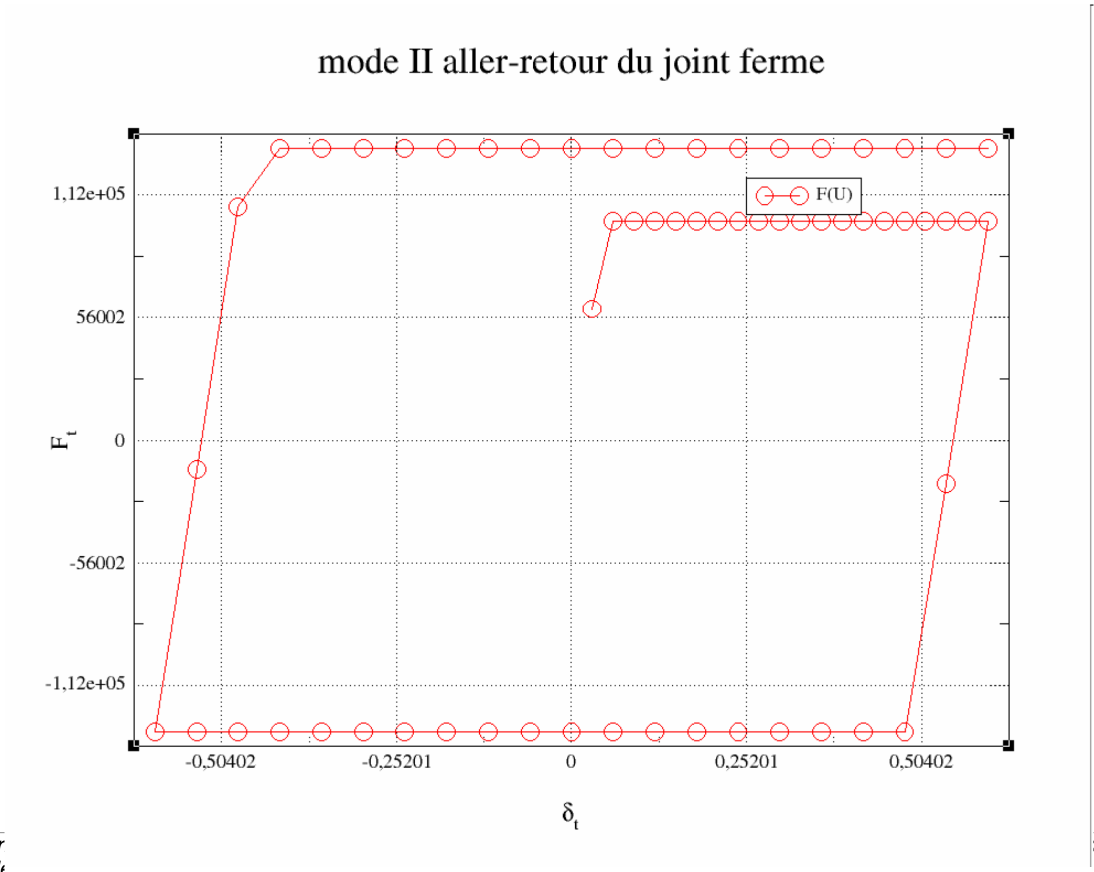

**Illustration 2: Curve forces (tangent) displacement for the law `JOINT_MECA_FROT`**

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|---|
| |

# 3 Modeling A

Validation of the joint $2D$ with the law JOINT_MECA_RUPT

## 3.1 Characteristics of modeling

Modeling in plane deformations D_PLAN for the elastic element.
Modeling plan for the element of joint (keyword PLAN_JOINT).

## 3.2 Characteristics of the grid

Many nodes: 6
The elastic element is one QUAD4.
The element of joint is one QUAD4 degenerated (confused nodes).

## 3.3 Sizes tested and results

### Mode $I$

One opens the joint until the partial damage[1]. $U=\beta\times U_{el}+(1-\beta)\times U_{max}$ Where $\beta=0.2$. Then it passes in the compression zone $U=-U_{el}$. Finally the joint is put in traction until the complete damage $U=U_{max}$. One tests the total answer (the resultant of the nodal force, FN) system (joined and cubic) in the local reference mark. The values of reference are analytical (see page 6).

### Mode $II$

The joint is open in mode $I$ until the partial damage, then he is requested in mode $II$. One tests the tangential slopes of rigidity to two values of normal opening. The incremental character of the evolution tangential makes modify the constraint tangential during the partial discharge in mode $I$ what returns complex its estimate analytical. Us it testont thus after the discharge by regression.

One notes by:

$$\sigma_{pena}=\sigma_{max}P_{cont}\times(E+L_{cube}\times K_N)/(E+L_{cube}\times K_N\times P_{cont})$$

$$\sigma_t(\delta_n)=K_T\times(1-\delta_n/L_{CT})U_t$$

| Size tested | Reference | Tolerance ( $\%$ ) |
|---|---|---|
| | **mode $I$** | |
| FN, value with the peak | $\sigma_{max}$, that is to say $1.D05$ | 0.10 |
| FN, damaged value | $\beta\,\sigma_{max}$, that is to say $2.D04$ | 0.10 |
| FN, value in compression | $\sigma_{pena}$, that is to say $-2.D05$ | 0.10 |
| | **mode $II$** | |
| FT, for two values of $\delta_n$ | $\sigma_t(\delta_n)$ | 0.10 |

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[1] to see the analytical solution for the notations

# 4 Modeling B

Validation of the joint $3D$ with the law `JOINT_MECA_RUPT`

## 4.1 Characteristics of modeling

Modeling `3D` for the element elastic element.
Modeling `3D_JOINT` for the element of joint

## 4.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of joint is one `HEXA8` degenerated (confused nodes).

## 4.3 Sizes tested and results

### Modes $I$ and $II$

Identical to modeling A.

### Mode $III$

The joint is open in mode $I$ until the partial damage, then he is requested in mode $III$. One tests the tangential slopes of rigidity to two values of normal opening.

# 5 Modeling C

Validation of the joint 3D with the law `JOINT_MECA_RUPT`

## 5.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 5.2 Characteristics of the grid

Many nodes: 9
The elastic element is one `PENTA6`.
The element of joint is one `PENTA6` degenerated (confused nodes).

## 5.3 Sizes tested and results

### Modes $I$ and $II$

Identical to modeling A. It is noted just that $FN=SIGN/2$ , because the surface of contact is worth $1/2$ .

### Mode $III$

The joint is open in mode $I$ until the partial damage, then he is requested in mode $III$ . One tests the tangential slopes of rigidity to two values of normal opening.

# 6 Modeling D

Validation of the joint $2\text{D}$ with the law `JOINT_MECA_FROT`

## 6.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT`).

## 6.2 Characteristics of the grid

Many nodes: 6
The elastic element is one `QUAD4`.
The element of joint is one `QUAD4` degenerated (confused nodes).

## 6.3 Sizes tested and results

### Mode I

One opens the joint until his threshold of tensile strength[2]. $U=U_{el}$ Then one draws from advantage to arrive at $U=2U_{el}$ . It passes in the compression zone $U=-U_{el}/3$ . Finally the joint is reloaded up to its point of balance $U=0$ . One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are analytically obtained (see page 6).

### Mode $II$

The joint is requested in slip in mode $II$ . One test E then the value of adhesion $c$ . Then one modifies the normal opening in mode $I$ and one makes the slip return ticket in mode $II$ for $U_t=\pm U_{el}/3$ .

| Size tested | Reference | Tolerance ( $\%$ ) |
|---|---|---|
| | **mode** $I$ | |
| `FN`, value with the peak | $\sigma_{max}$ that is to say 2.D05 | 0.10 |
| `FN`, not variation of the value to the peak | $\sigma_{max}$ that is to say 2.D05 | 0.10 |
| `FN`, value in compression | $\sigma_{max}/3$ that is to say -6.66666667D04 | 0.10 |
| `FN`, value at the point of balance | 0.0 | 0.10 |
| | **mode** $II$ | |
| `FT`, value of adhesion | $C$ that is to say 1.D05 | 0.10 |
| `FT`, value in slip and adhesion | $-K_N U_t \mu - c$ that is to say -1.2445666D+06 | 0.10 |
| `FT`, value in slip and adhesion | $K_N U_t \mu - c$ that is to say 1.244647D+06 | 0.10 |

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[2] to see the analytical solution for the notations

# 7 Modeling E

Validation of the joint $3D$ with the law `JOINT_MECA_FROT`

## 7.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 7.2 Characteristics of the grid

Many nodes: 12
The elastic element is one `HEXA8`.
The element of joint is one `HEXA8` degenerated (confused nodes).

## 7.3 Sizes tested and results

**Mixed mode $I$ , $II$ , $III$**
One makes only tests of nonregression, by mixing the load patterns. One applies the same loading as in modeling D, but one adds there the loading in the third direction.

# 8 Modeling F

Validation of the joint $3D$ with the law `JOINT_MECA_FROT`

## 8.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 8.2 Characteristics of the grid

Many nodes: 9
The elastic element is one `PENTA6`.
The element of joint is one `PENTA6` degenerated (confused nodes).

## 8.3 Sizes tested and results

### Mixed mode $I$ , $II$ , $III$

One makes only tests of nonregression, by mixing the load patterns. One applies the same loading as in modeling D, but one adds there the loading in the third direction.

# 9 Modeling G

Validation of the joint $2\text{D}$ $\text{HYME}$ with the law `JOINT_MECA_RUPT`

## 9.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT_HYME`).

## 9.2 Characteristics of the grid

Many nodes: 13 (including 3 nodes with degrees of freedom of pressure)
The elastic element is one `QUAD8`.
The element of joint is one `QUAD8` degenerated (confused nodes).

## 9.3 Sizes tested and results

To test the hydraulic part, a pressure is forced $p_{fluide}=500\,Pa$ on the node $NS5$ and one tests the same value on the node $NS7$ .

One tests mechanics in mode $I$ pure. One opens the joint until the partial damage[3]. $U=\beta\times\%U_{el}+(1-\beta)\times\%U_{max}$ where $\beta=0.2$ . Then it passes in the compression zone $U=-U_{el}$ . Finally the joint is put in traction until the complete damage $U=U_{max}$ . One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are obtained analytically by taking account of the pressure of fluid (see page 6).

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[3] to see the analytical solution for the notations

# 10 Modeling H

Validation of the joint $3\text{D}\quad\text{HYME}$ `HEXA20` with the law `JOINT_MECA_RUPT`

## 10.1 Characteristics of modeling

Modeling in `3D` for the elastic element.
Modeling `3D_JOINT_HYME` for the element of joint

## 10.2 Characteristics of the grid

Many nodes: 32 (including 8 nodes with degrees of freedom of pressure)
The elastic element is one `HEXA20`.
The element of joint is one `HEXA20` degenerated (confused nodes).

## 10.3 Sizes tested and results

To test the hydraulic part, a pressure is forced $p_{fluide}=500\,Pa$ on the nodes $NS19$ and $NS20$ and one tests the same value on nodes $NS17$ and $NS18$ .

One tests mechanics in mode $I$ pure. One opens the joint until the partial damage[4]. $U=\beta\times U_{el}+(1-\beta)\times U_{max}$ where $\beta=0.2$ . Then it passes in the compression zone $U=-U_{el}$ . Finally the joint is put in traction until the complete damage $U=U_{max}$ . One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are obtained analytically by taking account of the pressure of fluid (see page 6).

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[4] to see the analytical solution for the notations

# 11 Modeling I

Validation of the joint $3\mathrm{D}$ $\mathrm{HYME}$ `PENTA15` with the law `JOINT_MECA_RUPT`

## 11.1 Characteristics of modeling

Modeling in `3D` for the elastic element.
Modeling `3D_JOINT_HYME` for the element of joint

## 11.2 Characteristics of the grid

Many nodes: 24 (including 6 nodes with degrees of freedom of pressure)
The elastic element is one `PENTA15`.
The element of joint is one `PENTA15` degenerated (confused nodes).

## 11.3 Sizes tested and results

To test the hydraulic part, a pressure is forced $p_{fluide}=500\,Pa$ on the node $NS13$ and one tests the same value on the nodes $NS14$ and $NS15$ .

One tests mechanics in mode $I$ pure. One opens the joint until the partial damage[5]. $U=\beta\times U_{el}+(1-\beta)\times U_{max}$ where $\beta=0.2$ . Then it passes in the compression zone $U=-U_{el}$ . Finally the joint is put in traction until the complete damage $U=U_{max}$ . One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are obtained analytically by taking account of the pressure of fluid (see page 6).

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[5] to see the analytical solution for the notations

# 12 Modeling J

Validation of the joint $2D$ $HYME$ with the law `JOINT_MECA_FROT`

## 12.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT_HYME`).

## 12.2 Characteristics of the grid

Many nodes: 13 (including 3 nodes with degrees of freedom of pressure)
The elastic element is one `QUAD8`.
The element of joint is one `QUAD8` degenerated (confused nodes).

## 12.3 Sizes tested and results

To test the hydraulic part, a pressure is forced $p_{fluide}=500\,Pa$ on the node $NS5$ and one tests the same value on the node $NS7$ .

### Mode $I$

One opens the joint until his threshold of tensile strength[6]. $U=U_{el}$ Then one draws from advantage to arrive at $U=2U_{el}$ . It passes in the compression zone $U=-U_{el}/3$ . Finally the joint is reloaded up to its point of balance $U=0$ . One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are obtained analytically by taking account of the pressure of fluid what makes shifted the normal curve to the bottom (see page 6).

### Mode $II$

The joint is requested in slip mode $II$ . One test then the value of adhesion $c$ . Then one modifies the normal opening in mode $I$ and one makes the slip return ticket in mode $II$ for $U_t=\pm U_{el}/3$ . The values of reference are obtained analytically by taking account of the pressure of fluid what does not influence the tangential curve (see page 6).

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[6] to see the analytical solution for the notations

# 13 Modeling K

Validation of the joint $3D$ $\mathrm{HYME}$ HEXA20 with the law JOINT_MECA_FROT

## 13.1 Characteristics of modeling

Modeling in 3D for the elastic element.
Modeling 3D_JOINT_HYME for the element of joint

## 13.2 Characteristics of the grid

Many nodes: 32 (including 8 nodes with degrees of freedom of pressure)
The elastic element is one HEXA20.
The element of joint is one HEXA20 degenerated (confused nodes).

## 13.3 Sizes tested and results

To test the hydraulic part, a pressure is forced $p_{fluide}=500\,Pa$ on the nodes $NS19$ and $NS20$ and one tests the same value on nodes $NS17$ and $NS18$ .

One makes only tests of nonregression, by mixing the load patterns. One applies the same loading as in modeling E, but one adds there the loading in the third direction.

# 14 Modeling L

Validation of the joint $3D$ $HYME$ `PENTA15` with the law `JOINT_MECA_FROT`

## 14.1 Characteristics of modeling

Modeling in `3D` for the elastic element.
Modeling `3D_JOINT_HYME` for the element of joint

## 14.2 Characteristics of the grid

Many nodes: 24 (including 6 nodes with degrees of freedom of pressure)
The elastic element is one `PENTA15`.
The element of joint is one `PENTA15` degenerated (confused nodes).

## 14.3 Sizes tested and results

To test the hydraulic part, a pressure is forced $p_{fluide}=500\,Pa$ on the node $NS13$ and one tests the same value on the nodes $NS14$ and $NS15$ .

One makes only tests of nonregression, by mixing the load patterns. One applies the same loading as in modeling E, but one adds there the loading in the third direction.

# 15 Modeling M

Validation of the joint $2D$ into quadratic with the law `JOINT_MECA_RUPT`

## 15.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT`).

## 15.2 Characteristics of the grid

Many nodes: 13
The elastic element is one `QUAD8`.
The element of joint is one `QUAD8` degenerated (confused nodes).

## 15.3 Sizes tested and results

### Mode $I$

One opens the joint until the partial damage[7]. $U=\beta\times U_{el}+(1-\beta)\times U_{max}$ Where $\beta=0.2$. Then it passes in the compression zone $U=-U_{el}$. Finally the joint is put in traction until the complete damage $U=U_{max}$. One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are analytical (see page 6).

### Mode $II$

The joint is open in mode $I$ until the partial damage, then he is requested in mode $II$. One tests the tangential slopes of rigidity to two values of normal opening. The incremental character of the evolution tangential makes modify the constraint tangential during the partial discharge in mode $I$ what returns complex its estimate analytical. Us it testont thus after the discharge by regression.

One notes by:

$$\sigma_{pena}=\sigma_{max}P_{cont}\times(E+L_{cube}\times K_N)/(E+L_{cube}\times K_N\times P_{cont})$$
$$\sigma_t(\delta_n)=K_T\times(1-\delta_n/L_{CT})U_t$$

| Size tested | Reference | Tolerance ( $\%$ ) |
|---|---|---|
| | **mode** $I$ | |
| `FN`, value with the peak | $\sigma_{max}$, that is to say $1.D05$ | 0.10 |
| `FN`, damaged value | $\beta\,\sigma_{max}$, that is to say $2.D04$ | 0.10 |
| `FN`, value in compression | $\sigma_{pena}$, that is to say $-2.D05$ | 0.10 |
| | **mode** $II$ | |
| `FT`, for two values of $\delta_n$ | $\sigma_t(\delta_n)$ | `0.10` |

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[7] to see the analytical solution for the notations

# 16 Modeling NR

Validation of the joint $3D$ into quadratic with the law `JOINT_MECA_RUPT`

## 16.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 16.2 Characteristics of the grid

Many nodes: 32
The elastic element is one `HEXA20`.
The element of joint is one `HEXA20` degenerated (confused nodes).

## 16.3 Sizes tested and results

### Modes $I$ and $II$

Identical to modeling Mr.

### Mode $III$

The joint is open in mode $I$ until the partial damage, then he is requested in mode $III$ . One tests the tangential slopes of rigidity to two values of normal opening.

# 17 Modeling O

Validation of the joint 3D into quadratic with the law `JOINT_MECA_RUPT`

## 17.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 17.2 Characteristics of the grid

Many nodes: 24
The elastic element is one `PENTA15`.
The element of joint is one `PENTA15` degenerated (confused nodes).

## 17.3 Sizes tested and results

### Modes $I$ and $II$

Identical to modeling Mr. It is noted just that $FN=SIGN/2$ , because the surface of contact is worth $1/2$ .

### Mode $III$

The joint is open in mode $I$ until the partial damage, then he is requested in mode $III$ . One tests the tangential slopes of rigidity to two values of normal opening.

# 18 Modeling P

Validation of the joint $2D$ into quadratic with the law `JOINT_MECA_FROT`

## 18.1 Characteristics of modeling

Modeling in plane deformations `D_PLAN` for the elastic element.
Modeling plan for the element of joint (keyword `PLAN_JOINT`).

## 18.2 Characteristics of the grid

Many nodes: 13
The elastic element is one `QUAD8`.
The element of joint is one `QUAD8` degenerated (confused nodes).

## 18.3 Sizes tested and results

### Mode I

One opens the joint until his threshold of tensile strength[8]. $U=U_{el}$ Then one draws from advantage to arrive at $U=2U_{el}$ . It passes in the compression zone $U=-U_{el}/3$ . Finally the joint is reloaded up to its point of balance $U=0$ . One tests the total answer (the resultant of the nodal force, `FN`) system (joined and cubic) in the local reference mark. The values of reference are analytically obtained (see page 6).

### Mode $II$

The joint is requested in slip in mode $II$ . One test E then the value of adhesion $c$ . Then one modifies the normal opening in mode $I$ and one makes the slip return ticket in mode $II$ for $U_t=\pm U_{el}/3$ .

| Size tested | Reference | Tolerance ( $\%$ ) |
|---|---|---|
| | **mode** $I$ | |
| `FN`, value with the peak | $\sigma_{max}$ that is to say 2.D05 | 0.10 |
| `FN`, not variation of the value to the peak | $\sigma_{max}$ that is to say 2.D05 | 0.10 |
| `FN`, value in compression | $\sigma_{max}/3$ that is to say -6.66666667D04 | 0.10 |
| `FN`, value at the point of balance | 0.0 | 0.10 |
| | **mode** $II$ | |
| `FT`, value of adhesion | $C$ that is to say 1.D05 | 0.10 |
| `FT`, value in slip and adhesion | $-K_N U_t \mu - c$ that is to say -1.2445666D+06 | 0.10 |
| `FT`, value in slip and adhesion | $K_N U_t \mu - c$ that is to say 1.244647D+06 | 0.10 |

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[8] to see the analytical solution for the notations

# 19 Modeling R

Validation of the joint $3D$ into quadratic with the law `JOINT_MECA_FROT`

## 19.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 19.2 Characteristics of the grid

Many nodes: 32
The elastic element is one `HEXA20`.
The element of joint is one `HEXA20` degenerated (confused nodes).

## 19.3 Sizes tested and results

### Mixed mode $I$ , $II$ , $III$

One makes only tests of nonregression, by mixing the load patterns. One applies the same loading as in modeling P, but one adds there the loading in the third direction.

# 20 Modeling S

Validation of the joint $3D$ into quadratic with the law `JOINT_MECA_FROT`

## 20.1 Characteristics of modeling

Modeling `3D` for the elastic element.
Modeling `3D_JOINT` for the element of joint

## 20.2 Characteristics of the grid

Many nodes: 24
The elastic element is one `PENTA6`.
The element of joint is one `PENTA6` degenerated (confused nodes).

## 20.3 Sizes tested and results

### Mixed mode $I$ , $II$, $III$

One makes only tests of nonregression, by mixing the load patterns. One applies the same loading as in modeling P, but one adds there the loading in the third direction.

# 21 Summary of the results

The digital results are in agreement with the analytical solution. These tests make it possible to validate the elements of joint, the elements of joint $HM$ in $2\mathrm{D}$ and $3\mathrm{D}$ in the various modes of opening.