–

$$u_{\theta}=0$$ éq 2.1

$$u_z=0$$

.

1.1

1.2

- 58
- 3
- 95607489060205
- 51103884487243
- 112
- (3.95607489060205E-01, 4.51103884487243E-01,
- 422
- (3.95607489060205E-01, 4.51103884487243E-01, 1.99999999999678E-01)

58
58
58
58
112
112
112

112
422
422
422
422

1.3

1.4 Why not of test on a group of nodes as for other modelings 3D?

1.5

onforme
S
E to connect it
, even if that accurately any more does not represent the geometry of a circle

2 Conclusion

SSNP167 - Inclusion of two crowns under nonuniform pressure

Summary:

This test is used to evaluate the performances of Aster with regard to the treatment of the contact between two structures with compatibility and geometrical incompatibility between surfaces Master and slave within the framework of the assumption of the small disturbances.

One considers a structure made up of two concentric crowns. The interface between the two crowns is the surface of contact. Their rigidity, represented by their Young modulus plays an important role in the evaluation of the value of the deformations and the fluctuations of the contact pressure. One also seeks to know which are the effects of the use of a grid of a higher nature, with the use of linear or quadratic grids.

An analytical solution was developed for this problem in order to validate the calculated digital results. The validation of this test relates to the values of the contact pressure and displacement.

3 Problem of reference

3.1 Geometry

The structure is made up of two concentric circular rings. The internal ray R_2 external crown is equal to the external ray of the interior crown (Figure 1.1-1).
Dimensions structural feature are:

$R_1 = 1{,}0\,m\,;\,R_2 = 0{,}6\,m\,;\,R_3 = 0{,}2\,m$

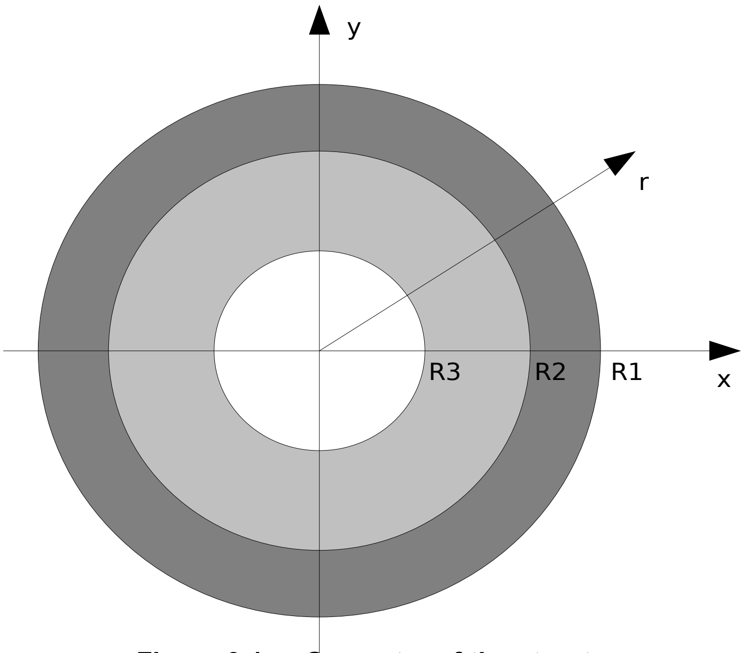

Figure 3.1-a: Geometry of the structure

3.2 Properties of materials

The Young modulus and the Poisson's ratio of material of the external crown are given by E_1 and ν_1 (respectively E_2 and ν_2 for the interior crown).

3.3 Boundary conditions and loadings

One will model in what follows the application of a nonuniform pressure on the edge $r = R_1$ external crown: $p = \alpha_0 + \alpha_1 . \cos(2\theta)$, θ being the polar angle describing the position of the point $\theta = \arctan(\frac{Y}{X})$.
For analytical calculation, one will solve in first part the problem for a pressure $p = \alpha_0$, then one will endeavour to solve the problem for a pressure of the form $p = \alpha_1 . \cos(2\theta)$. Then the principle of superposition will be applied.

3.3.1 Boundary conditions in the case of the uniform pressure

The edges of the crowns are subjected to displacements ($r = R_1$ and $r = R_3$) equivalents with the application of a pressure $p = \alpha_0$ on the edge of the external crown ($r = R_1$).

$$\xi_X(r)=f(r)\cos(\arctan(\frac{Y}{X}))$$
$$\xi_Y(r)=f(r)\sin(\arctan(\frac{Y}{X}))$$

The function $f(r)$ radial displacement is given according to the properties of materials and the pressure p . It is form $A_i r+\frac{B_i}{r};i=1,2$, in each field. In the case of plane deformations (MODELING = 'D_PLAN') one a:

$$f(R_1)=A_1R_1+\frac{B_1}{R_1}$$
$$f(R_3)=A_2R_3+\frac{B_2}{R_3}$$ éq 1.1

with:

$$A_1=\frac{(1+\nu_1)(1-2\nu_1)}{E_1}\frac{-pR_1^2+\lambda R_2^2}{R_1^2-R_2^2};B_1=\frac{1+\nu_1}{E_1}(-p+\lambda)\frac{R_1^2R_2^2}{R_1^2-R_2^2}$$
$$A_2=\frac{-(1+\nu_2)(1-2\nu_2)}{E_2}\lambda\frac{R_2^2}{R_2^2-R_3^2};B_2=\frac{-1+\nu_2}{E_2}\lambda\frac{R_2^2R_3^2}{R_2^2-R_3^2}$$ éq 1.2

where λ is the contact pressure whose analytical expression is:

$$\lambda=2p(1-\nu_1)\frac{\frac{R_1^2}{R_1^2-R_2^2}}{\frac{R_1^2+R_2^2(1-2\nu_1)}{R_1^2-R_2^2}+\frac{E_1}{E_2}\frac{(1+\nu_2)}{(1+\nu_1)}\frac{R_2^2(1-2\nu_2)+R_3^2}{R_2^2-R_3^2}}$$ éq. 1.3

3.3.2 Boundary conditions in the case of the variable pressure

The edges of the crowns are subjected to displacements ($r=R_1$ and $r=R_3$) equivalents with the application of a pressure $p=\alpha_1.\cos(2\theta)$ on the edge of the external crown ($r=R_1$).

$$\xi_X(r,\theta)=u_r(r,\theta)\cos(\theta)-u_\theta(r,\theta)\sin(\theta)$$
$$\xi_Y(r,\theta)=u_r(r,\theta)\sin(\theta)+u_\theta(r,\theta)\cos(\theta)$$

The function $u_r(r,\theta)$ radial displacement, and that of tangential displacement $u_\theta(r,\theta)$ are given according to the properties of materials, pressure p , geometrical characteristics and square of the relationship between the rays: $f_1=(\frac{R_2}{R_1})^2;f_2=(\frac{R_3}{R_2})^2$. In the case of plane deformations (MODELING = 'D_PLAN') one a:

$$u_r(R_1,\theta)=\frac{1+\nu_1}{E_1}[(-2A_1R_1+2\frac{C_1}{R_1^3}+4\frac{D_1}{R_1})-\nu_1(4B_1R_1^3+4\frac{D_1}{R_1})]\cos(2\theta)$$
$$u_\theta(R_1,\theta)=\frac{1+\nu_1}{E_1}[(2A_1R_1+6B_1R_1^3+2\frac{C_1}{R_1^3}-2\frac{D_1}{R_1})-\nu_1(4B_1R_1^3-4\frac{D_1}{R_1})]\sin(2\theta)$$
$$u_r(R_3,\theta)=\frac{1+\nu_2}{E_2}[(-2A_2R_3+2\frac{C_2}{R_3^3}+4\frac{D_2}{R_3})-\nu_2(4B_2R_3^3+4\frac{D_2}{R_3})]\cos(2\theta)$$
$$u_\theta(R_3,\theta)=\frac{1+\nu_2}{E_2}[(2A_2R_3+6B_2R_3^3+2\frac{C_2}{R_3^3}-2\frac{D_2}{R_3})-\nu_2(4B_2R_3^3-4\frac{D_2}{R_3})]\sin(2\theta)$$

éq 1.4

with:

$$A_1=\frac{\alpha_1(2f_1^2+f_1+1)-\lambda(f_1^3+f_1^2+2f_1)}{2(1-f_1)^3};\ B_1=\frac{-1}{R_2^2}\frac{\alpha_1(3f_1^2+f_1)-\lambda(f_1^3+3f_1^2)}{6(1-f_1)^3};$$
$$C_1=R_2^4\frac{\alpha_1(f_1+3)-\lambda(3f_1+1)}{6(1-f_1)^3};\ D_1=-R_2^2\frac{\alpha_1(f_1^2+f_1+2)-\lambda(2f_1^2+f_1+1)}{2(1-f_1)^3}$$
$$A_2=\frac{\lambda(2f_2^2+f_2+1)}{2(1-f_2)^3};\ B_2=\frac{-1}{R_3^2}\frac{\lambda(3f_2^2+f_2)}{6(1-f_2)^3};$$
$$C_2=R_3^4\frac{\lambda(f_2+3)}{6(1-f_2)^3};\ D_2=-R_3^2\frac{\lambda(f_2^2+f_2+2)}{2(1-f_2)^3}$$

éq 1.5

where λ is the contact pressure whose analytical expression is:

$$\lambda=\frac{coef_1}{coef_2+coef_3}\alpha_1$$

éq 1.6

such as:

$$coef_1=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-12f_1^2-8f_1-12)-\nu_1(-12f_1^2-8f_1-12)]$$
$$coef_2=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-3f_1^3-15f_1^2-9f_1-5)-\nu_1(-2f_1^3-18f_1^2-6f_1-6)]$$
$$coef_3=\frac{1+\nu_2}{6E_2(1-f_2)^3}[(-5f_2^3-9f_2^2-15f_2-3)-\nu_2(-6f_2^3-6f_2^2-18f_2-2)]$$

éq 1.7

4 Reference solution

We develop here an analytical solution to the problem presented above. This solution will be developed with the assumption of small deformations by considering that the materials of the crowns isotropic, are governed by a linear elastic law without temperature variation.
The solution in displacement of the problem has the following generic form:

$$u=u_r(r,\theta,z).\underline{e_r}+u_\theta(r,\theta,z).\underline{e_\theta}+u_z(r,\theta,z).\underline{e_z}$$

One will solve the problem within the framework of the assumption of the plane deformations. Our loading being written in the form $p=\alpha_0+\alpha_1.\cos(2\theta)$, one will uncouple the resolution of the problem in a part where the pressure is uniform $p=\alpha_0$, and a part where the pressure is variable $p=\alpha_1.\cos(2\theta)$.

4.1.1 Uniform pressure

By using symmetries of the problem and the assumption of invariance according to Z of the plane deformations, the solution of the problem takes the following shape:

$$\begin{array}{c} u_r=u_r(r) \\ u_\theta=0 \\ u_z=0 \end{array} \quad \text{éq 2.1}$$

By using the equation of Lamé-Navier:

$$(\lambda+\mu)\underline{grad}(\nabla.(\underline{u}))+\mu\Delta\underline{u}+\underline{fd}=\underline{0} \quad \text{éq 2.2}$$

where $\underline{fd}=\underline{0}$ are here the worthless voluminal efforts, and the F ormule of the Laplacian:

$$\Delta\underline{u}=\underline{grad}(\nabla.(\underline{u}))+\underline{rot}\,\underline{rot}(\underline{u}) \quad \text{éq 2.3}$$

One can write éq 2.2 pennies the form:

$$(\lambda+2\mu)\underline{grad}(\nabla.(\underline{u}))+\mu\,\underline{rot}\,\underline{rot}(\underline{u})+\underline{fd}=\underline{0} \quad \text{éq 2.4}$$

that is to say still while using $\underline{rot}(\underline{u})=\vec{0}$ et $\underline{fd}=\vec{0}$ et $\underline{u}=u_r(r).\underline{e_r}$:

$$\begin{array}{c} \nabla.(\underline{u})=\frac{d}{dr}u_r(r)+\frac{1}{r}u_r(r) \\ \underline{grad}(\nabla.\underline{u})=\frac{d}{d_r}[\frac{1}{r}\frac{d}{d_r}(r\,u_r(r))].\underline{e}_r \\ \text{soit encore } (\lambda+2\mu)\frac{d}{d_r}[\frac{1}{r}\frac{d}{d_r}(r\,u_r(r))]=0 \end{array} \quad \text{éq 2.5}$$

By integrating the equation, one obtains for the crowns (external and interior) the following shape of the field of displacement:

$$u_r=C_i r+\frac{D_i}{r} \quad u_\theta=0 \quad u_z=0 \quad \text{éq 2.6}$$

To determine C_i and D_i, it remains us to impose the limiting conditions in pressure and displacement. For that, the deformations should initially be calculated then constraints associated with the field with displacement.

Déformations are the symmetrical part of the gradient of displacements. One obtains:

$$\epsilon_{rr}=C_i-\frac{D_i}{r^2}$$
$$\epsilon_{\theta\theta}=C_i+\frac{D_i}{r^2}$$
$$\epsilon_{zz}=\epsilon_{r\theta}=\epsilon_{rz}=\epsilon_{\theta z}=0$$
éq 2.7

By applying the law of Hooke:

$$\underline{\underline{\sigma}}=\lambda\, tr(\underline{\underline{\epsilon}})\underline{\underline{1}}+2\mu\underline{\underline{\epsilon}}$$
éq 2.8

one obtains the following general form for the constraints:

$$\sigma_{rr}=\frac{E}{1+\nu}\left(\frac{C_i}{1-2\nu}-\frac{D_i}{r^2}\right)$$
$$\sigma_{\theta\theta}=\frac{E}{1+\nu}\left(\frac{C_i}{1-2\nu}+\frac{D_i}{r^2}\right)$$
$$\sigma_{zz}=\frac{2\nu E C_i}{(1+\nu)(1-2\nu)}$$
$$\sigma_{r\theta}=\sigma_{rz}=\sigma_{\theta z}=\underline{0}$$
éq 2.9

While posing:

$$A_i=\frac{E_i}{(1+\nu_i)(1-2\nu_i)}C_i \qquad B_i=\frac{E_i}{1+\nu_i}D_i$$
éq 2.10

the nonworthless constraints become:

$$\sigma_{rr}=A_i-\frac{B_i}{r^2}$$
$$\sigma_{\theta\theta}=A_i+\frac{B_i}{r^2}$$
$$\sigma_{zz}=2\nu A_i$$
éq 2.11

It any more but does not remain us to calculate the values of A_i and B_i for each crown. One will note λ_n the contact pressure between the two crowns such as:

$$\underline{\underline{\sigma}}_{1rr}(R_2).(-\underline{e}_r)=\lambda_n\underline{e}_r$$
$$\underline{\underline{\sigma}}_{2rr}(R_2).\underline{e}_r=-\lambda_n\underline{e}_r$$
éq 2.12

with the boundary conditions:

$$\underline{\underline{\sigma}}_{1rr}(R_1).\underline{e}_r=-p.\underline{e}_r$$
$$\underline{\underline{\sigma}}_{2rr}(R_3).(-\underline{e}_r)=\underline{0}$$
éq 2.13

The condition of continuity on displacement with the interface between the two groups of contact gives moreover:

$$u_{r;1}(R2)=u_{r;2}(R2)$$
éq 2.14

We thus have 5 equations for 5 unknown factors $A_1, B_1, A_2, B_2, \lambda_n$.

The system of the first 4 equations enables us to obtain:

$$A_1=\frac{-p R_1^2+\lambda_n R_2^2}{R_1^2-R_2^2};\ B_1=(-p+\lambda_n)\frac{R_1^2 R_2^2}{R_1^2-R_2^2}$$
$$A_2=-\lambda_n\frac{R_2^2}{R_2^2-R_3^2};\ B_2=-\lambda_n\frac{R_2^2 R_3^2}{R_2^2-R_3^2} \quad \text{éq 2.15}$$

and the equation of continuity on displacement finally makes it possible to have the contact pressure:

$$\lambda_n=\frac{2 p R_1^2(1-\nu_1)}{R_1^2+R_2^2(1-2\nu_1)+\frac{E_1}{E_2}\frac{1+\nu_2}{1+\nu_1}\frac{R_1^2-R_2^2}{R_2^2-R_3^2}(R_2^2(1-2\nu_2)+R_3^2)} \quad \text{éq 2.16}$$

4.1.2 Variable pressure

By using the assumption of invariance according to Z of the plane deformations, the solution of the problem takes the following shape:

$$u_r=u_r(r,\theta)$$
$$u_\theta=u_\theta(r,\theta) \quad \text{éq 2.17}$$
$$u_z=0$$

Subsequently, one will note the specific parameters to each solid by an index **I**, with i=1,2.

In the absence of forces of volume, one will use a form of the function of Airy proposed by Michel [1], in polar coordinates:

$$\chi(r,\theta)=A_{01}r^2+A_{02}r^2\log(r)+A_{03}\log(r)+A_{04}\theta$$
$$+(A_{11}r^3+A_{12}rlog(r)+A_{13}r^{-1})\cos(\theta)+A_{14}r\theta\sin(\theta)$$
$$+(B_{11}r^3+B_{12}r\log(r)+B_{13}r^{-1})\sin(\theta)+B_{14}r\theta\cos(\theta)$$
$$+\sum_{n=2}^{\infty}(A_{n1}r^{n+2}+A_{n2}r^{-n+2}+A_{n3}r^n+A_{n4}r^{-n})\cos(n\theta) \quad \text{éq 2.18}$$
$$+\sum_{n=2}^{\infty}(B_{n1}r^{n+2}+B_{n2}r^{-n+2}+B_{n3}r^n+B_{n4}r^{-n})\sin(n\theta)$$

The terms of the tensor of the nonworthless constraints of Cauchy are expressed according to the function of Airy as follows:

$$\sigma_{rr}=\frac{1}{r}\frac{\partial\chi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\chi}{\partial\theta^2}$$
$$\sigma_{\theta\theta}=\frac{\partial^2\chi}{\partial r^2} \quad \text{éq 2.19}$$
$$\sigma_{r\theta}=\frac{-\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\chi}{\partial\theta}\right)$$
$$\sigma_{zz}=\nu(\sigma_{rr}+\sigma_{\theta\theta})$$

Our pressure $p=\alpha_1.\cos(2\theta)$ varying in $\cos(2\theta)$, one will take only the part varying in $\cos(2\theta)$ in the function of Airy. The function of Airy will be written then:

$$\chi_i(r,\theta)=(A_i r^2+B_i r^4+\frac{C_i}{r^2}+D_i)\cos(2\theta) \quad \text{éq 2.20}$$

From there, one can express the nonworthless constraints in the polar reference mark:

$$\sigma^i_{rr}=(-2A_i-6\frac{C_i}{r^4}-4\frac{D_i}{r^2})\cos(2\theta)$$
$$\sigma^i_{\theta\theta}=(2A_i+12B_i r^2+6\frac{C_i}{r^4})\cos(2\theta) \quad \text{éq 2.21}$$
$$\sigma^i_{r\theta}=2(A_i+3B_i r^2-3\frac{C_i}{r^4}-\frac{D_i}{r^2})\sin(2\theta)$$
$$\sigma^i_{zz}=\nu(\sigma_{rr}+\sigma_{\theta\theta})$$

and terms of the tensor of deformations by using the law of Hooke:

$$\underline{\underline{\varepsilon}}=\frac{1}{E}((1+\nu)\underline{\underline{\sigma}}-\nu\, tr(\underline{\underline{\sigma}})\underline{\underline{I}}) \quad \text{éq 2.22}$$

The nonworthless terms of the tensor of the deformations are expressed then according to the constants of the problem:

$$\varepsilon^i_{rr}=\frac{1+\nu_i}{E_i}[(-2A_i-6\frac{C_i}{r^4}-4\frac{D_i}{r^2})-\nu_i(12B_i r^2-4\frac{D_i}{r^2})]\cos(2\theta)$$
$$\varepsilon^i_{\theta\theta}=\frac{1+\nu_i}{E_i}[(2A_i+12B_i r^2+6\frac{C_i}{r^4})-\nu_i(12B_i r^2-4\frac{D_i}{r^2})]\cos(2\theta) \quad \text{éq 2.23}$$
$$\varepsilon^i_{r\theta}=\frac{1+\nu_i}{E_i}[2(A_i+3B_i r^2-3\frac{C_i}{r^4}-\frac{D_i}{r^2})]\sin(2\theta)$$

One will use these fields to express displacements in the polar reference mark.
One a:

$$\frac{\partial u_r}{\partial r}=\varepsilon_{rr}$$
$$\frac{\partial u_\theta}{\partial\theta}=r\,\varepsilon_{\theta\theta}-u_r \quad \text{éq 2.24}$$
$$\frac{1}{2}(\frac{\partial u_\theta}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r})=\varepsilon_{r\theta}$$

By integrating these relations, one can express displacements:

$$u^i_r=\frac{1+\nu_i}{E_i}[(-2A_i r+2\frac{C_i}{r^3}+4\frac{D_i}{r})-\nu_i(4B_i r^3+4\frac{D_i}{r})]\cos(2\theta)$$
$$u^i_\theta=\frac{1+\nu_i}{E_i}[(2A_i r+6B_i r^3+2\frac{C_i}{r^3}-2\frac{D_i}{r})-\nu_i(4B_i r^3-4\frac{D_i}{r})]\sin(2\theta) \quad \text{éq 2.25}$$

Now that one expressed all our fields according to the constants $A_i, B_i, C_i, D_i (i=1,2)$, one must calculate these last according to the geometrical characteristics and of the loading.
One will note λ the contact pressure between the two crowns.
The boundary conditions are:

$$\sigma^1_{rr}(R_1)=-\alpha\cos(2\theta): \text{pression externe appliquée}$$
$$\sigma^1_{r\theta}(R_1)=0: \text{pression tangentielle nulle sur le bord extérieur de la couronne 1}$$
$$\sigma^2_{rr}(R_2)=\lambda: \text{pression de contact appliquée par la couronne 1 sur la couronne 2}$$
$$\sigma^1_{rr}(R_2)=\lambda: \text{pression de contact appliquée par la couronne 2 sur la couronne 1}$$
$$\sigma^1_{r\theta}(R_2)=0: \text{pas de frottement entre les deux couronnes}$$
$$\sigma^2_{r\theta}(R_2)=0: \text{pas de frottement entre les deux couronnes}$$
$$\sigma^2_{rr}(R_3)=0: \text{bord extérieur de la couronne 2 libre}$$
$$\sigma^2_{r\theta}(R_3)=0: \text{bord extérieur de la couronne 2 libre}$$

We thus have 8 equations for the 8 unknown factors: $A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2$.
While posing:

$$f_1=\left(\frac{R_2}{R_1}\right)^2 \text{ et } f_2=\left(\frac{R_3}{R_2}\right)^2$$

The system of 8 equations enables us to have:

$$A_1=\frac{\alpha_1(2f_1^2+f_1+1)-\lambda(f_1^3+f_1^2+2f_1)}{2(1-f_1)^3};\ B_1=\frac{-1}{R_2^2}\frac{\alpha_1(3f_1^2+f_1)-\lambda(f_1^3+3f_1^2)}{6(1-f_1)^3};$$
$$C_1=R_2^4\frac{\alpha_1(f_1+3)-\lambda(3f_1+1)}{6(1-f_1)^3};\ D_1=-R_2^2\frac{\alpha_1(f_1^2+f_1+2)-\lambda(2f_1^2+f_1+1)}{2(1-f_1)^3}$$
$$A_2=\frac{\lambda(2f_2^2+f_2+1)}{2(1-f_2)^3};\ B_2=\frac{-1}{R_3^2}\frac{\lambda(3f_2^2+f_2)}{6(1-f_2)^3};$$
$$C_2=R_3^4\frac{\lambda(f_2+3)}{6(1-f_2)^3};\ D_2=-R_3^2\frac{\lambda(f_2^2+f_2+2)}{2(1-f_2)^3}$$ éq 2.26

One can express the contact pressure analytically
By using the continuity of radial displacement on the level of the interface of contact:

$$u_r^1(R_2)=u_r^2(R_2)$$ éq 2.27

one can express the contact pressure analytically:

$$\lambda=\frac{coef_1}{coef_2+coef_3}\alpha_1$$ éq 2.28

such as:

$$coef_1=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-12f_1^2-8f_1-12)-\nu_1(-12f_1^2-8f_1-12)]$$
$$coef_2=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-3f_1^3-15f_1^2-9f_1-5)-\nu_1(-2f_1^3-18f_1^2-6f_1-6)]$$
$$coef_3=\frac{1+\nu_2}{6E_2(1-f_2)^3}[(-5f_2^3-9f_2^2-15f_2-3)-\nu_2(-6f_2^3-6f_2^2-18f_2-2)]$$ éq 2.29

4.1.3 Values tested

One tests the contact pressure to the interface between the two crowns, on the level of surface slave, as well as displacements according to X and Y: u_x, u_y (and according to Z also for the models 3D), in plane deformations.

The value of the pressure applied to the edge external of crown in $r=R_1$ express yourself in the form: $p(\theta)=10^7+10^5\cos(2\theta)(Pa)$, with $\theta=\arctan(\frac{Y}{X})$.

One will test the values of the points located on the ray $r=R_2$, side of crown 2, with various polar angles: $\theta\in\{0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\pi,\frac{5\pi}{4},\frac{3\pi}{2},\frac{7\pi}{4}\}$ for models A, B, C, D.

With regard to the models E, G and H, one will make a test min-max on three groups of nodes. Each group of nodes contains three nodes with coordinates X and Y equal, which makes it possible to test the invariance of the got result following Z. the value of reference used in the test is the analytical minimal value (respectively maximum) of the component tested for each group of node. With regard to the model F, only three nodes with equal coordinates X and Y were tested, in the center of the structure, in order to avoid the effects edge due to the fact that the grids of surfaces of main contact and slave are not compatible.

4.2 Bibliography

[1] J.R. To bore, “Elasticity”, Kluwer Academic Publishers, 1982

5 Modeling A

5.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN). Surfaces mistress and slave are in conformity. Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)$ (Pa), θ being the polar angle.

The external crown defines main surface.

5.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:

- 480 meshs of the type SEG2;
- 2640 meshs of the type QUAD4.

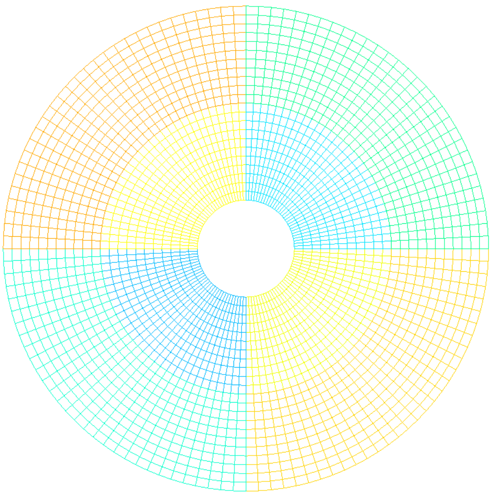

Figure 5.2-a: Grid of modeling A

5.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y) analytically calculated displacements.

If displacements are worthless, one defines an absolute tolerance equal to 2 % of the value of radial displacement maximum U_{rmax}=5.5E-03.

One calculates the contact pressure (LAGS_C) and components of displacement in the plan (X, Y) (DX, DY) for the nodes of following coordinates:

- N9 (0.6, 0.0)
- N10 (0.0, 0.6)
- N11 (- 0.6, 0 .0)
- N12 (0.0, - 0.6)
- N303 (4.24264068712436E-01, 4.24264068711421E-01)
- N371 (- 4.24264068711421E-01, 4.24264068712436E-01)
- N439 (- 4.24264068712436E-01, - 4.24264068711421E-01)
- N507 (4.24264068711421E-01, - 4.24264068712436E-01)

Identification	Reference	Aster	tolerance
LAGS_C with node 9	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 9	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 9	$U_y(x,y)$	Analytical	0,02.Vmax
LAGS_C with node 10	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 10	$U_x(x,y)$	Analytical	0,02.Vmax
DY with node 10	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 11	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 11	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 11	$U_y(x,y)$	Analytical	0,02.Vmax
LAGS_C with node 12	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 12	$U_x(x,y)$	Analytical	0,02.Vmax
DY with node 12	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 303	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 303	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 303	$U_y(x,y)$	Analytical	0,02.Vmax
LAGS_C with node 371	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 371	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 371	$U_y(x,y)$	Analytical	0,02.Vmax
LAGS_C with node 439	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 439	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 439	$U_y(x,y)$	Analytical	0,02.Vmax
LAGS_C with node 507	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 507	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 507	$U_y(x,y)$	Analytical	0,02.Vmax

Table 5.3-1

6 Modeling B

6.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN). Surfaces mistress and slave are not in conformity.
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0\text{E}9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0\text{E}7+10\text{E}5.\cos(2\theta)\ (Pa)$, θ being the polar angle.

The external crown defines main surface.

6.2 Characteristics of the grid

The grid (Figure 4.2-1) comprises:
- 480 meshs of the type SEG2;
- 2640 meshs of the type QUAD4.

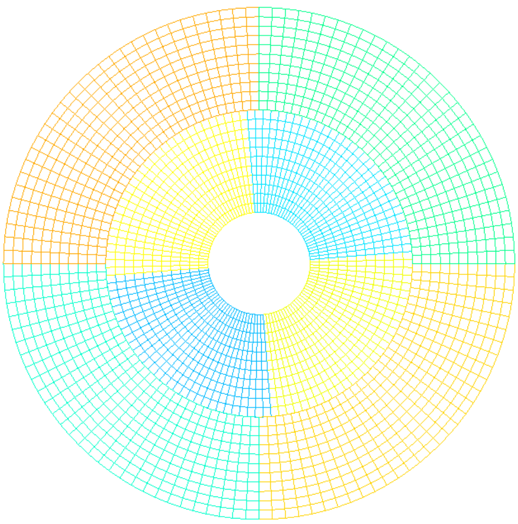

Figure 6.2-a: Grid of modeling B

6.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y) analytically calculated displacements.

If displacements are worthless, one defines an absolute tolerance equal to 2 % of the value of radial displacement maximum U_{rmax}=5.5E-03.

One calculates the contact pressure (LAGS_C) and components of displacement in the plan (X, Y) (DX, DY) for the nodes of following coordinates:

- N9 (5.98150400239877E-01, 4.70754574367070E-02)
- N10 (- 4.70754574367070E-02, 5.98150400239877E-01)
- N11 (- 5.98150400239877E-01, - 4.70754574367070E-02)
- N12 (4.70754574367070E-02, - 5.98150400239877E-01)
- N303 (3.89668829003112E-01, 4.56243579355747E-01)

- N371 (- 4.56243579355747E-01, 3.89668829003112E-01)
- N439 (- 3.89668829003112E-01, - 4.56243579355747E-01)
- N507 (4.56243579355747E-01, - 3.89668829003112E-01)

Identification	Reference	Aster	tolerance
LAGS_C with node 9	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 9	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 9	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 10	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 10	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 10	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 11	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 11	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 11	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 12	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 12	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 12	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 303	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 303	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 303	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 371	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 371	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 371	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 439	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 439	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 439	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 507	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 507	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 507	$U_y(x,y)$	Analytical	2.10^{-2}

Table 6.3-1

7 Modeling C

7.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN). Surfaces mistress and slave are in conformity. Young moduli $E_1 = E_2$ and Poisson's ratios $\nu_1 = \nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)$ (Pa), θ being the polar angle.

The external crown defines main surface.

7.2 Characteristics of the grid

The grid (Figure 4.2-1) comprises:

- 480 meshs of the type SEG3;
- 2640 meshs of the type QUAD8.

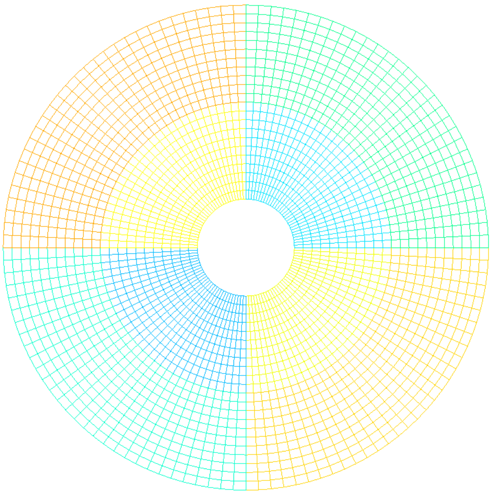

Figure 7.2-a: Grid of modeling C

7.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y) analytically calculated displacements.

If displacements are worthless, one defines an absolute tolerance equal to 2 % of the value of radial displacement maximum Urmax=5.5E-03.

One calculates the contact pressure (LAGS_C) and components of displacement in the plan (X, Y) (DX, DY) for the nodes of following coordinates:

- N9 (0.6, 0.0)
- N10 (0.0, 0.6)
- N11 (- 0.6, 0 .0)
- N12 (0.0, - 0.6)
- N587 (4.24264068712436E-01, 4.24264068711421E-01)
- N726 (- 4.24264068711421E-01, 4.24264068712436E-01)
- N865 (- 4.24264068712436E-01, - 4.24264068711421E-01)
- N1004 (4.24264068711421E-01, - 4.24264068712436E-01)

Identification	Reference	Aster	tolerance
LAGS_C with node 9	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 9	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 9	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 10	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 10	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 10	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 11	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 11	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 11	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 12	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 12	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 12	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 587	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 587	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 587	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 726	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 726	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 726	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with node 865	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with node 865	$U_x(x,y)$	Analytical	2.10^{-2}
DY with node 865	$U_y(x,y)$	Analytical	2.10^{-2}
LAGS_C with the node 1004	$\lambda(x,y)$	Analytical	2.10^{-2}
DX with the node 1004	$U_x(x,y)$	Analytical	2.10^{-2}
DY with the node 1004	$U_y(x,y)$	Analytical	2.10^{-2}

Table 7.3-1

8 Modeling D

8.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN). Surfaces mistress and slave are not in conformity.
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.

The external crown defines main surface.

8.2 Characteristics of the grid

The grid (Figure 4.2-1) comprises:
- 480 meshs of the type SEG3;
- 2640 meshs of the type QUAD8.

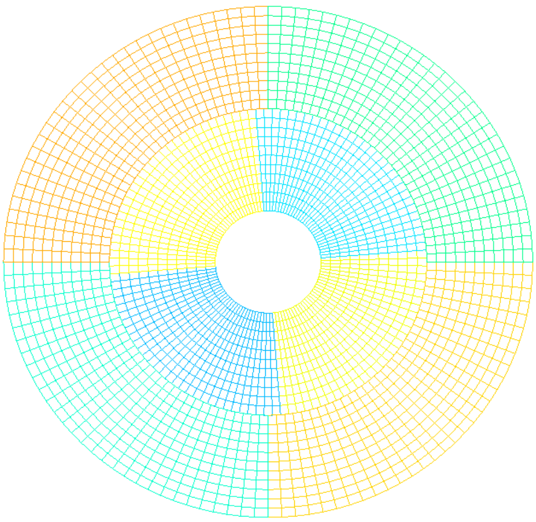

Figure 8.2-a: Grid of modeling D

8.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y) analytically calculated displacements.

If displacements are worthless, one defines an absolute tolerance equal to 2 % of the value of radial displacement maximum U_{rmax}=5.5E-03.

One calculates the contact pressure (LAGS_C) and components of displacement in the plan (X, Y) (DX, DY) for the nodes of following coordinates:

- N9 (5.98150400239877E-01, 4.70754574367070E-02)
- N10 (- 4.70754574367070E-02, 5.98150400239877E-01)
- N11 (- 5.98150400239877E-01, - 4.70754574367070E-02)
- N12 (4.70754574367070E-02, - 5.98150400239877E-01)
- N587 (3.89668829003112E-01, 4.56243579355747E-01)
- N726 (- 4.56243579355747E-01, 3.89668829003112E-01)
- N865 (- 3.89668829003112E-01, - 4.56243579355747E-01)

- N1004 (4.56243579355747E-01, - 3.89668829003112E-01)

Identification	Reference	Aster	tolerance
LAGS_C with node 9	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 9	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 9	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with node 10	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 10	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 10	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with node 11	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 11	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 11	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with node 12	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 12	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 12	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with node 587	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 587	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 587	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with node 726	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 726	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 726	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with node 865	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with node 865	$U_x(x, y)$	Analytical	2.10^{-2}
DY with node 865	$U_y(x, y)$	Analytical	2.10^{-2}
LAGS_C with the node 1004	$\lambda(x, y)$	Analytical	2.10^{-2}
DX with the node 1004	$U_x(x, y)$	Analytical	2.10^{-2}
DY with the node 1004	$U_y(x, y)$	Analytical	2.10^{-2}

Table 8.3-1

9 Modeling E

9.1 Characteristics of modeling

It is about a modeling 3D. Surfaces mistress and slave are in opposite with compatible grids. To avoid having too slow calculations, only the quarter of the geometry will be taken.
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)\;(Pa)$, θ being the polar angle.
The pressure does not vary in the longitudinal direction. To have a model in conformity with the plane deformations, one will force a longitudinal displacement no one on the two upper surfaces (in red on figure 7.2-1). One will apply like boundary conditions to other surfaces (in blue on figure 7.2-1) solutions displacements.

The external crown defines main surface.

9.2 Characteristics of the grid

The grid (Figure 7.2-1) comprises:
- 448 meshs of the type QUAD4;
- 384 meshs of the type HEXA8.

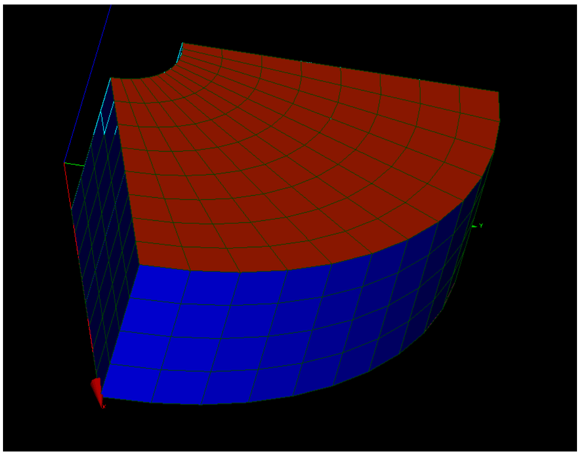

Figure 9.2-a: Grid of modeling E

9.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y, U_z) analytically calculated displacements.
If displacements are worthless, one defines an absolute terminal equal to 2% the value of maximum displacement Vmax=5.5E-03.
One calculates the minimal value and the maximum value of the contact pressure (LAGS_C) and components of displacement (DX, DY, DZ) for the groups of following nodes:
- GROUP 1: N47 (5.54327719507606E-01, 2.29610059417040E-01, 0.00000000000000E+00),
  N103 (5.54327719507606E-01, 2.29610059417040E-01, 4.00000000000000E-01),
  N412 (5.54327719507606E-0, 2.29610059417040E-01, 1.99999999999678E-01)
- GROUP 2: N50 (4.24264068712436E-01, 4.24264068711421E-01, 0.00000000000000E+00),
  N106 (4.24264068712436E-01, 4.24264068711421E-01, 4.00000000000000E-01),
  N403 (4.24264068712436E-01, 4.24264068711421E-01, 1.99999999999678E-01)

- GROUP 3: N53 (2.29610059417922E-01, 5.54327719507241E-01, 0.00000000000000E+00),
N109 (2.29610059417922E-01, 5.54327719507241E-01, 4.00000000000000E-01),
N394 (2.29610059417922E-01, 5.54327719507241E-01, 1.99999999999678E-01)

Identification	Reference	Aster	tolerance
`LAGS_C min` group 1	$\lambda(x,y,z)$	-9355226.31353	2.10^{-2}
`DX min` group 1	$U_x(x,y,z)$	-0.0050526512252	2.10^{-2}
`DY min` group 1	$U_y(x,y,z)$	-0.0020785420526	2.10^{-2}
`DZ min` group 1	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C max` group 1	$\lambda(x,y,z)$	-9355226.31353	2.10^{-2}
`DX max` group 1	$U_x(x,y,z)$	-0.0050526512252	2.10^{-2}
`DY max` group 1	$U_y(x,y,z)$	-0.0020785420526	2.10^{-2}
`DZ max` group 1	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C min` group 2	$\lambda(x,y,z)$	-9259259.25926	2.10^{-2}
`DX min` group 2	$U_x(x,y,z)$	-0.0037844796198	2.10^{-2}
`DY min` group 2	$U_y(x,y,z)$	-0.0037579927128	2.10^{-2}
`DZ min` group 2	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C max` group 2	$\lambda(x,y,z)$	-9259259.25926	2.10^{-2}
`DX max` group 2	$U_x(x,y,z)$	-0.0037844796198	2.10^{-2}
`DY max` group 2	$U_y(x,y,z)$	-0.0037579927128	2.10^{-2}
`DZ max` group 2	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C min` group 3	$\lambda(x,y,z)$	-9163292.20499	2.10^{-2}
`DX min` group 3	$U_x(x,y,z)$	-0.0020034145592	2.10^{-2}
`DY min` group 3	$U_y(x,y,z)$	-0.0048020637882	2.10^{-2}
`DZ min` group 3	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C max` group 3	$\lambda(x,y,z)$	-9163292.20499	2.10^{-2}
`DX max` group 3	$U_x(x,y,z)$	-0.0020034145592	2.10^{-2}
`DY max` group 3	$U_y(x,y,z)$	-0.0048020637882	2.10^{-2}
`DZ max` group 3	$U_z(x,y,,z)$	0.0	1.E-8

Table 9.3-1

10 Modeling F

10.1 Characteristics of modeling

It is about a modeling 3D. Surfaces mistress and slave in opposite are such as their grid is not compatible. To avoid having too slow calculations, only the quarter of the geometry will be taken.
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.
The pressure does not vary in the longitudinal direction. To have a model in conformity with the plane deformations, one will force a longitudinal displacement no one on the two upper surfaces (in red on figure 8.2-1). One will apply like boundary conditions to other surfaces (in blue on figure 8.2-1) solutions displacements.

The external crown defines main surface.

10.2 Characteristics of the grid

The grid (Figure 8.2-1) comprises:
- 464 meshs of the type `QUAD4`;
- 400 meshs of the type `HEXA8`.

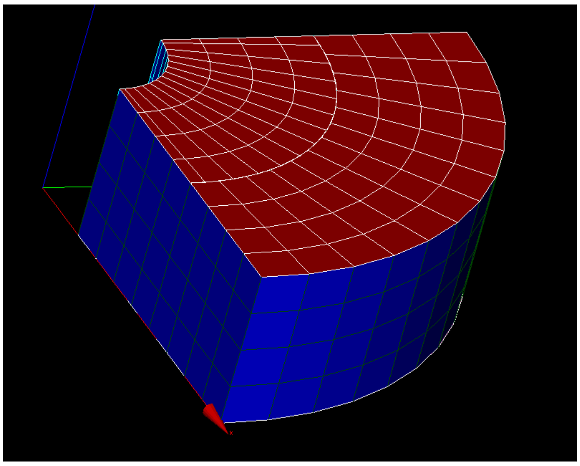

Figure 10.2-a: Grid of modeling F

10.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y, U_z) analytically calculated displacements.
If displacements are worthless, one defines an equal absolute terminal has 2% the value of maximum displacement Vmax=5.5E-03.
One calculates the contact pressure (`LAGS_C`) and components of displacement (`DX, DY, DZ`) for the following nodes:
- NR125(0.421224E-01, 4.427282E-01, 0.00000000000000E+00)
- NR285(0.421224E-01, 4.427282E-01, 4.00000000000000E-01)
- NR790(0.421224E-01, 4.427282E-01, 2.00000000000000E-01)

Identification	Reference	Aster	tolerance
LAGS_C with the node 125	$\lambda(x,y,z)$	Analytical	2.10^{-2}
DX with the node 125	$U_x(x,y,z)$	Analytical	2.10^{-2}
DY with the node 125	$U_y(x,y,z)$	Analytical	2.10^{-2}
DZ with the node 125	$U_z(x,y,,z$	Analytical	1.E-8
LAGS_C with the node 285	$\lambda(x,y,z)$	Analytical	2.10^{-2}
DX with the node 285	$U_x(x,y,z)$	Analytical	2.10^{-2}
DY with the node 285	$U_y(x,y,z)$	Analytical	2.10^{-2}
DZ with the node 285	$U_z(x,y,,z$	Analytical	1.E-8
LAGS_C with the node 790	$\lambda(x,y,z)$	Analytical	2.10^{-2}
DX with the node 790	$U_x(x,y,z)$	Analytical	2.10^{-2}
DY with the node 790	$U_y(x,y,z)$	Analytical	2.10^{-2}
DZ with the node 790	$U_z(x,y,,z$	Analytical	1.E-8

Table 10.3-1

10.4 Comments

The results relative to this model are to be considered carefully: need for creating a grid not Compatible for surfaces of main contact and slave, representing only the quarter of the geometry, and which does not generate positive pressure at the edges, D imposesE to confuse them geometrically nodes at the edges grids of surfaces of contact mistress and slave. This generates a profile of pressure relatively incorrect and oscillating at the edges of the surface of contact. This effect decreases by refining the grid.

11 Modeling G

11.1 Characteristics of modeling

It is about a modeling 3D. Surfaces mistress and slave in opposite have compatible grids. To avoid having too slow calculations, only the quarter of the geometry will be taken.
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.
The pressure does not vary in the longitudinal direction. To have a model in conformity with the plane deformations, one will force a longitudinal displacement no one on the two upper surfaces (in red on figure 9.2-1). One will apply like boundary conditions to other surfaces (in blue on figure 9.2-1) solutions displacements.

The external crown defines main surface.

11.2 Characteristics of the grid

The grid (Figure 9.2-1) comprises:
- 448 meshs of the type `QUAD8`;
- 384 meshs of the type `HEXA20`.

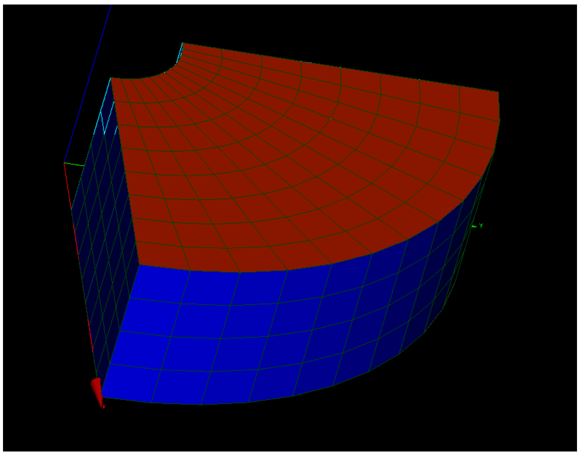

Figure 11.2-a: Grid of modeling G

11.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y, U_z) analytically calculated displacements.
If displacements are worthless, one defines an equal absolute terminal has 2% the value of maximum displacement Vmax=5.5E-03.
One calculates the minimal value and the maximum value of the contact pressure (`LAGS_C`) and components of displacement (`DX, DY, DZ`) for the groups of following nodes:

- GROUP 1: N78 (5.79555495773817E-01, 1.55291427060109E-01, 0.00000000000000E+00),
  N198 (5.79555495773817E-01, 1.55291427060109E-01, 4.00000000000000E-01),
  N1151 (5.79555495773817E-01, 1.55291427060109E-01, 1.99999999999678E-01)
- GROUP 2: N82 (4.24264068712436E-01, 4.24264068711421E-01, 0.00000000000000E+00),
  N202 (4.24264068712436E-01, 4.24264068711421E-01, 4.00000000000000E-01),
  N1139 (4.24264068712436E-01, 4.24264068711421E-01, 1.99999999999678E-01)

- GROUP 3: N85 (2.29610059417922E-01, 5.54327719507241E-01, 0.00000000000000E+00),
  N205 (2.29610059417922E-01, 5.54327719507241E-01, 4.00000000000000E-01),
  N1130 (2.29610059417922E-01, 5.54327719507241E-01, 1.99999999999678E-01)

Identification	Reference	Aster	tolerance
LAGS_C min group 1	$\lambda(x,y,z)$	-9376794.4168	2.10^{-2}
DX min group 1	$U_x(x,y,z)$	-0.0053079751381	2.10^{-2}
DY min group 1	$U_y(x,y,z)$	-0.0014125727708	2.10^{-2}
DZ min group 1	$U_z(x,y,,z)$	0.0	1.E-8
LAGS_C max group 1	$\lambda(x,y,z)$	-9376794.4168	2.10^{-2}
DX max group 1	$U_x(x,y,z)$	-0.0053079751381	2.10^{-2}
DY max group 1	$U_y(x,y,z)$	-0.0014125727708	2.10^{-2}
DZ max group 1	$U_z(x,y,,z)$	0.0	1.E-8
LAGS_C min group 2	$\lambda(x,y,z)$	-9259259.25926	2.10^{-2}
DX min group 2	$U_x(x,y,z)$	-0.0037844796198	2.10^{-2}
DY min group 2	$U_y(x,y,z)$	-0.0037579927128	2.10^{-2}
DZ min group 2	$U_z(x,y,,z)$	0.0	1.E-8
LAGS_C max group 2	$\lambda(x,y,z)$	-9259259.25926	2.10^{-2}
DX max group 2	$U_x(x,y,z)$	-0.0037844796198	2.10^{-2}
DY max group 2	$U_y(x,y,z)$	-0.0037579927128	2.10^{-2}
DZ max group 2	$U_z(x,y,,z)$	0.0	1.E-8
LAGS_C min group 3	$\lambda(x,y,z)$	-9163292.20499	2.10^{-2}
DX min group 3	$U_x(x,y,z)$	-0.0020034145592	2.10^{-2}
DY min group 3	$U_y(x,y,z)$	-0.0048020637882	2.10^{-2}
DZ min group 3	$U_z(x,y,,z)$	0.0	1.E-8
LAGS_C max group 3	$\lambda(x,y,z)$	-9163292.20499	2.10^{-2}
DX max group 3	$U_x(x,y,z)$	-0.0020034145592	2.10^{-2}
DY max group 3	$U_y(x,y,z)$	-0.0048020637882	2.10^{-2}
DZ max group 3	$U_z(x,y,,z)$	0.0	1.E-8

Table 11.3-1

12 Modeling H

12.1 Characteristics of modeling

It is about a modeling 3D. Surfaces mistress and slave in opposite are such as their grids are not compatible. To avoid having too slow calculations, only the quarter of the geometry will be taken.
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the edge of the external crown is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.
The pressure does not vary in the longitudinal direction. To have a model in conformity with the plane deformations, one will force a longitudinal displacement no one on the two upper surfaces (in red on figure 10.2-1). One will apply like boundary conditions to other surfaces (in blue on figure 10.2-1) solutions displacements.

The external crown defines main surface.

12.2 Characteristics of the grid

The grid (Figure 8.2-1) comprises:
- 464 meshs of the type `QUAD8`;
- 400 meshs of the type `HEXA20`.

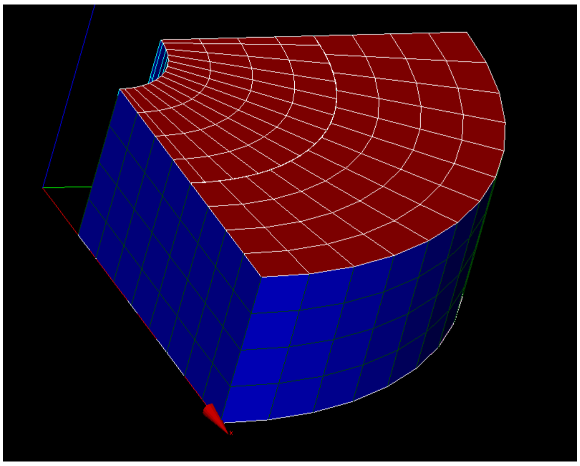

Figure 12.2-a: Grid of modeling H

12.3 Sizes tested and results

One will note λ the contact pressure calculated analytically, (U_x, U_y, U_z) analytically calculated displacements.
If displacements are worthless, one defines an equal absolute terminal has 2% the value of maximum displacement Vmax=5.5E-03.
One calculates the minimal value and the maximum value of the contact pressure (`LAGS_C`) and components of displacement (`DX, DY, DZ`) for the groups of following nodes:

- GROUP 1: N97 (5.54327719506785E-01, 2.29610059419021E-01, 0.00000000000000E+00),
  N213 (5.54327719506785E-01, 2.29610059419021E-01, 4.00000000000000E-01),
  N1256 (5.54327719506760E-01, 2.29610059419083E-01, 1.99999999999678E-01)
- GROUP 2: N100 (4.24264068711920E-01, 4.24264068711937E-01, 0.00000000000000E+00),
  N216 (4.24264068711920E-01, 4.24264068711937E-01, 4.00000000000000E-01),
  N1253 (5.68158077697052E-01, 1.92863679181931E-01, 3.49999999999963E-01)

- GROUP 3: N103 (2.29610059419306E-01, 5.54327719506668E-01, 0.00000000000000E+00),
  N219 (2.29610059419306E-01, 5.54327719506668E-01, 4.00000000000000E-01),
  N1214 (2.29610059419285E-01, 5.54327719506676E-01, 1.99999999999678E-01)

Identification	Reference	Aster	tolerance
`LAGS_C min` group 1	$\lambda(x,y,z)$	-9355226.31353	2.10^{-2}
`DX min` group 1	$U_x(x,y,z)$	-0.0050526512252	2.10^{-2}
`DY min` group 1	$U_y(x,y,z)$	-0.0020785420526	2.10^{-2}
`DZ min` group 1	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C max` group 1	$\lambda(x,y,z)$	-9355226.31353	2.10^{-2}
`DX max` group 1	$U_x(x,y,z)$	-0.0050526512252	2.10^{-2}
`DY max` group 1	$U_y(x,y,z)$	-0.0020785420526	2.10^{-2}
`DZ max` group 1	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C min` group 2	$\lambda(x,y,z)$	-9366931.51625	2.10^{-2}
`DX min` group 2	$U_x(x,y,z)$	-0.0051922141170	2.10^{-2}
`DY min` group 2	$U_y(x,y,z)$	-0.0037579927128	2.10^{-2}
`DZ min` group 2	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C max` group 2	$\lambda(x,y,z)$	-9366931.51625	2.10^{-2}
`DX max` group 2	$U_x(x,y,z)$	-0.0051922141170	2.10^{-2}
`DY max` group 2	$U_y(x,y,z)$	-0.0037579927128	2.10^{-2}
`DZ max` group 2	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C min` group 3	$\lambda(x,y,z)$	-9163292.20499	2.10^{-2}
`DX min` group 3	$U_x(x,y,z)$	-0.0020034145592	2.10^{-2}
`DY min` group 3	$U_y(x,y,z)$	-0.0048020637882	2.10^{-2}
`DZ min` group 3	$U_z(x,y,,z)$	0.0	1.E-8
`LAGS_C max` group 3	$\lambda(x,y,z)$	-9163292.20499	2.10^{-2}
`DX max` group 3	$U_x(x,y,z)$	-0.0020034145592	2.10^{-2}
`DY max` group 3	$U_y(x,y,z)$	-0.0048020637882	2.10^{-2}
`DZ max` group 3	$U_z(x,y,,z)$	0.0	1.E-8

Table 12.3-1

13 Summary of the results

This case test makes it possible to validate the continuous contact formulation in the presence of curved surfaces of contact with linear and quadratic grids, in 2D and 3D. It in particular could make it possible to establish the orders of convergence for the continuous method of contact and to check a convergence in energy of 1 and displacement of 2 for the linear grids, 2 in energy and 3 in displacement for the quadratic grids, for sufficiently rich diagrams of integration.