SSNP169 – Full disc crossed by an interface X-FEM under nonuniform pressure

Summary:

This test allows to validate treatment of the contact in X-FEM within the framework of the assumption of the small disturbances.

One consider initially a full disc half-compartment by a circular interface. Then, one consider one square full also crossed by a circular interface. In both cases, L'interface is the surface of contact. The rigidity of the solid, represented by the Young modulus plays an important role in the evaluation of the value of the deformations and the fluctuations of the contact pressure.

An analytical solution was developed for this problem in order to validate the calculated digital results. L is testedbe values of the displacement and contact pressure.

1 Problem of reference

1.1 Geometry

The structure is made up of a cylindrical crown crossed by a circular interface X-FEM. The ray R_2 the ray of the circular interface defines (Figure 1.1-1) where the contact will be defined.
Dimensions structural feature are:

$R_1=1{,}0\,m\,;\,R_2=0{,}6\,m$

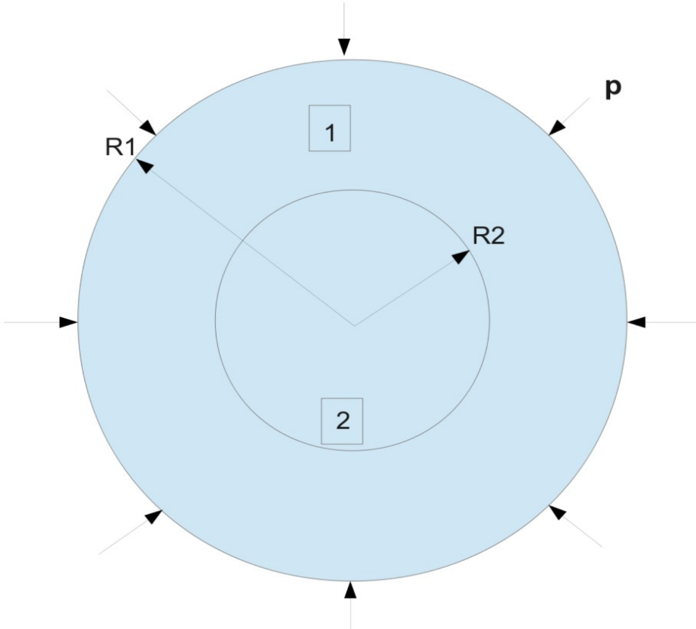

Figure 1.1-a: Geometry of the structure

1.2 Properties of materials

The Young modulus and the Poisson's ratio of material of the external crown are given by E_1 and ν_1 (respectively E_2 and ν_2 for the interior crown).

1.3 Boundary conditions and loadings

In what follows, one treats the case of a full disc subjected to a nonuniform external pressure. However, the boundary conditions (as well as the analytical solution) are generalizable with the case of the square surface crossed by a circular interface. The case of square surface enables us to treat a grid which would not have same symmetries as the interface: in the case of the disc, one uses a radiant regular grid, whereas in the case of the square, one uses a regular grid along the edges of the square.
One models in what follows the application of a nonuniform pressure on the edge $r=R_1$ external crown: $p=\alpha_0+\alpha_1.\cos(2\theta)$, θ being the polar angle describing the position of the point $\theta=\arctan(\frac{Y}{X})$. For analytical calculation, one solves in first part the problem for a pressure $p=\alpha_0$, then one solves the problem for a pressure of the form $p=\alpha_1.\cos(2\theta)$. For the total solution, the principle of superposition is applied.
One imposes boundary conditions of Dirichlet. Displacements are imposed on external surface in $r=R_1$. One will define also a ray $R_3=0.2\mathrm{m}$ where one imposes displacements, equivalent to those which would produce a full disc (Figure 1.3-1).

For surface squareE, one will impose displacements on the outsides, like on the disc of ray $R_3=0.2\text{m}$.
One carries out for each modeling two calculations: the first calculation with contact on the level of the interface, and the second calculation with contact pressure applied to the level of the interface.

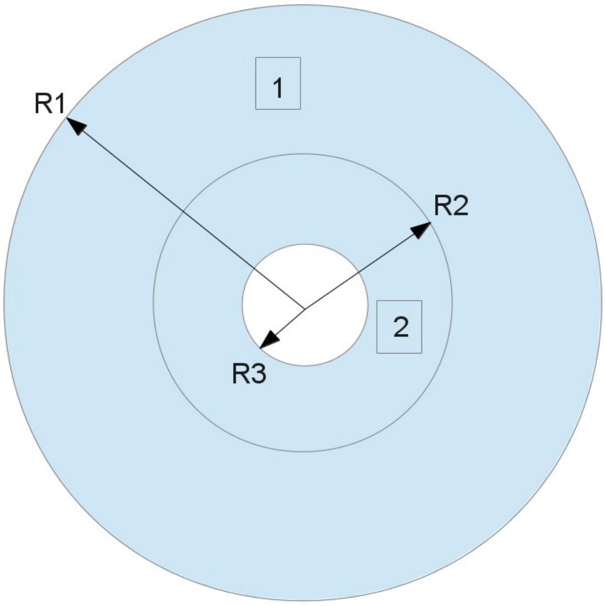

Figure 1.3-a: Geometry of the structure

1.3.1 Boundary conditions in the case of the uniform pressure

The edges of the solids are subjected to displacements ($r=R_1$ and $r=R_3$) equivalents with the application of a pressure $p=\alpha_0$ on the edge of the external crown ($r=R_1$).

$$\xi_X(r)=f(r)\cos(\arctan(\frac{Y}{X}))$$
$$\xi_Y(r)=f(r)\sin(\arctan(\frac{Y}{X}))$$

The function $f(r)$ radial displacement is given according to the properties of materials and the pressure p . In the case of plane deformations (`MODELING = 'D_PLAN'`) one a:

$$f(R_1)=A_1 R_1+\frac{B_1}{R_1}$$
$$f(R_3)=A_2 R_3$$ éq 1.1

with:

$$A_1=\frac{(1+\nu_1)(1-2\nu_1)}{E_1}\frac{-pR_1^2+\lambda R_2^2}{R_1^2-R_2^2};B_1=\frac{1+\nu_1}{E_1}(-p+\lambda)\frac{R_1^2 R_2^2}{R_1^2-R_2^2}$$
$$A_2=\frac{-(1+\nu_2)(1-2\nu_2)}{E_2}\lambda$$ éq 1.2

where λ is the contact pressure whose analytical expression is:

$$\lambda=2p\frac{(1-\nu_1^2)}{E_1}\frac{\frac{R_1^2}{R_1^2-R_2^2}}{\frac{1+\nu_1}{E_1}\frac{R_1^2+R_2^2(1-2\nu_1)}{R_1^2-R_2^2}+\frac{1+\nu_2}{E_2}(1-2\nu_2)}$$ éq. 1.3

1.3.2 Boundary conditions in the case of the variable pressure

The edges of the crowns are subjected to displacements ($r=R_1$ and $r=R_3$) equivalents with the application of a pressure $p=\alpha_1.\cos(2\theta)$ on the edge of the external crown ($r=R_1$).

$$\xi_X(r,\theta)=u_r(r,\theta)\cos(\theta)-u_\theta(r,\theta)\sin(\theta)$$
$$\xi_Y(r,\theta)=u_r(r,\theta)\sin(\theta)+u_\theta(r,\theta)\cos(\theta)$$

The function $u_r(r,\theta)$ radial displacement, and that of tangential displacement $u_\theta(r,\theta)$ are given according to the properties of materials, pressure p , geometrical characteristics and square of the relationship between the rays: $f_1=(\frac{R_2}{R_1})^2 ; f_2=(\frac{R_3}{R_2})^2$. In the case of plane deformations (MODELING = 'D_PLAN') one a:

$$u_r(R_1,\theta)=\frac{1+\nu_1}{E_1}[(-2A_1R_1+2\frac{C_1}{R_1^3}+4\frac{D_1}{R_1})-\nu_1(4B_1R_1^3+4\frac{D_1}{R_1})]\cos(2\theta)$$
$$u_\theta(R_1,\theta)=\frac{1+\nu_1}{E_1}[(2A_1R_1+6B_1R_1^3+2\frac{C_1}{R_1^3}-2\frac{D_1}{R_1})-\nu_1(4B_1R_1^3-4\frac{D_1}{R_1})]\sin(2\theta)$$
$$u_r(R_3,\theta)=\frac{1+\nu_2}{E_2}[(-2A_2R_3)-\nu_2(4B_2R_3^3)]\cos(2\theta)$$
$$u_\theta(R_3,\theta)=\frac{1+\nu_2}{E_2}[(2A_2R_3+6B_2R_3^3)-\nu_2(4B_2R_3^3)]\sin(2\theta)$$ éq 1.4

with:

$$A_1=\frac{\alpha_1(2f_1^2+f_1+1)-\lambda(f_1^3+f_1^2+2f_1)}{2(1-f_1)^3};B_1=\frac{-1}{R_2^2}\frac{\alpha_1(3f_1^2+f_1)-\lambda(f_1^3+3f_1^2)}{6(1-f_1)^3};$$
$$C_1=R_2^4\frac{\alpha_1(f_1+3)-\lambda(3f_1+1)}{6(1-f_1)^3};D_1=-R_2^2\frac{\alpha_1(f_1^2+f_1+2)-\lambda(2f_1^2+f_1+1)}{2(1-f_1)^3}$$
$$A_2=\frac{\lambda}{2};B_2=\frac{-\lambda}{6R_2^2};$$ éq 1.5

where λ is the contact pressure whose analytical expression is:

$$\lambda=\frac{coef_1}{coef_2+coef_3}\alpha_1$$ éq 1.6

such as:

$$coef_1=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-12f_1^2-8f_1-12)+\nu_1(12f_1^2+8f_1+12)]$$
$$coef_2=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-3f_1^3-15f_1^2-9f_1-5)+\nu_1(2f_1^3+18f_1^2+6f_1+6)]$$
$$coef_3=\frac{1+\nu_2}{6E_2}(-3+2\nu_2)$$

éq 1.7

2 Reference solution

We develop here an analytical solution to the problem presented above. This solution will be developed within the framework of the assumption of small deformations by considering that the materials of the crowns isotropic, are governed by a linear elastic law without temperature variation. The solution in displacement of the problem has the following generic form:

$$u = u_r(r,\theta,z).\underline{e_r} + u_\theta(r,\theta,z).\underline{e_\theta} + u_z(r,\theta,z).\underline{e_z}$$

One will solve the problem within the framework of the assumption of the plane deformations. Our loading being written in the form $p=\alpha_0+\alpha_1.\cos(2\theta)$, one will uncouple the resolution of the problem in a part where the pressure is uniform $p=\alpha_0$, and a part where the pressure is variable $p=\alpha_1.\cos(2\theta)$.

2.1.1 Uniform pressure

By using symmetries of the problem and the assumption of invariance according to Z of the plane deformations, the solution of the problem takes the following shape:

$$u_r = u_r(r) \quad u_\theta = 0 \quad u_z = 0 \qquad \text{éq 2.1}$$

By using the equation of Lamé-Navier:

$$(\lambda+\mu)\underline{grad}(\nabla.(\underline{u})) + \mu\Delta\underline{u} + \underline{fd} = \underline{0} \qquad \text{éq 2.2}$$

where $\underline{fd}=\underline{0}$ are here the worthless voluminal efforts, and the F ormule of the Laplacian:

$$\Delta\underline{u} = \underline{grad}(\nabla.(\underline{u})) + \underline{rot}\,\underline{rot}(\underline{u}) \qquad \text{éq 2.3}$$

One can write éq 2.2 pennies the form:

$$(\lambda+2\mu)\underline{grad}(\nabla.(\underline{u})) + \mu\,\underline{rot}\,\underline{rot}(\underline{u}) + \underline{fd} = \underline{0} \qquad \text{éq 2.4}$$

that is to say still while using $\underline{rot}(\underline{u})=\vec{0}$ et $\underline{fd}=\vec{0}$ et $\underline{u}=u_r(r).\underline{e_r}$:

$$\nabla.(\underline{u}) = \frac{d}{dr}u_r(r) + \frac{1}{r}u_r(r)$$
$$\underline{grad}(\nabla.\underline{u}) = \frac{d}{d_r}\left[\frac{1}{r}\frac{d}{d_r}(r\,u_r(r))\right].\underline{e_r} \qquad \text{éq 2.5}$$
$$\text{soit encore } (\lambda+2\mu)\frac{d}{d_r}\left[\frac{1}{r}\frac{d}{d_r}(r\,u_r(r))\right] = 0$$

By integrating the equation, one obtains for solid 1 the following shape of the field of displacement:

$$u_r = C_1 r + \frac{D_1}{r} \quad u_\theta = 0 \quad u_z = 0 \qquad \text{éq 2.6}$$

and for solid 2 the following shape of the fields of displacement:

$$u_r = C_2 r \quad u_\theta = 0 \quad u_z = 0 \qquad \text{éq 2.7}$$

To determine the constants C_1, D_1, C_2, it remains us to impose the limiting conditions in pressure and displacement. For that, the deformations should initially be calculated then constraints associated with the field with displacement.

Déformations are the symmetrical part of the gradient of displacements. One obtains for solid 1:

$$\begin{aligned}\epsilon_{rr}&=C_1-\frac{D_1}{r^2}\\ \epsilon_{\theta\theta}&=C_1+\frac{D_1}{r^2}\\ \epsilon_{zz}&=\epsilon_{r\theta}=\epsilon_{rz}=\epsilon_{\theta z}=0\end{aligned} \qquad \text{éq 2.8}$$

And for solid 2:

$$\begin{aligned}\epsilon_{rr}&=C_1\\ \epsilon_{\theta\theta}&=C_1\\ \epsilon_{zz}&=\epsilon_{r\theta}=\epsilon_{rz}=\epsilon_{\theta z}=0\end{aligned} \qquad \text{éq 2.9}$$

By applying the law of Hooke:

$$\underline{\underline{\sigma}}=\lambda\, tr(\underline{\underline{\epsilon}})\underline{\underline{1}}+2\mu\underline{\underline{\epsilon}} \qquad \text{éq 2.10}$$

one obtains the following general form for the constraints for solid 1:

$$\begin{aligned}\sigma_{rr}&=\frac{E_1}{1+\nu_1}\left(\frac{C_1}{1-2\nu_1}-\frac{D_1}{r^2}\right)\\ \sigma_{\theta\theta}&=\frac{E_1}{1+\nu_1}\left(\frac{C_1}{1-2\nu_1}+\frac{D_1}{r^2}\right)\\ \sigma_{zz}&=\frac{2\nu_1 E_1 C_1}{(1+\nu_1)(1-2\nu_1)}\\ \sigma_{r\theta}&=\sigma_{rz}=\sigma_{\theta z}=\underline{0}\end{aligned} \qquad \text{éq 2.11}$$

and the following general form for the constraints for solid 2:

$$\begin{aligned}\sigma_{rr}&=\frac{E_2}{1+\nu_2}\left(\frac{C_2}{1-2\nu_2}\right)\\ \sigma_{\theta\theta}&=\frac{E_2}{1+\nu_2}\left(\frac{C_2}{1-2\nu_2}\right)\\ \sigma_{zz}&=\frac{2\nu_2 E_2 C_2}{(1+\nu_2)(1-2\nu_2)}\\ \sigma_{r\theta}&=\sigma_{rz}=\sigma_{\theta z}=\underline{0}\end{aligned} \qquad \text{éq 2.12}$$

One poses:

$$A_1=\frac{E_1}{(1+\nu_1)(1-2\nu_1)}C_1 \qquad B_1=\frac{E_1}{1+\nu_1}D_1 \qquad A_2=\frac{E_2}{(1+\nu_2)(1-2\nu_2)}C_2 \qquad \text{éq 2.13}$$

It any more but does not remain us to calculate the values of A_1, B_1, A_2. One will note λ_n the contact pressure between the two crowns such as:

$$\begin{aligned}\underline{\underline{\sigma}}_{1rr}(R_2).(-\underline{e}_r)&=\lambda_n\underline{e}_r\\ \underline{\underline{\sigma}}_{2rr}(R_2).\underline{e}_r&=-\lambda_n\underline{e}_r\end{aligned} \qquad \text{éq 2.14}$$

with the boundary conditions:

$$\underline{\sigma}_{1rr}(R_1).e_r=-p.e_r$$ éq 2.15

The condition of continuity on displacement with the interface between the two groups of contact gives moreover:

$$u_{r;1}(R2)=u_{r;2}(R2)$$ éq 2.16

We thus have 4 equations for 4 unknown factors A_1, B_1, A_2, λ_n .

The system of the first 3 equations enables us to obtain:

$$A_1=\frac{-p R_1^2+\lambda_n R_2^2}{R_1^2-R_2^2};\, B_1=(-p+\lambda_n)\frac{R_1^2 R_2^2}{R_1^2-R_2^2}$$
$$A_2=-\lambda_n$$ éq 2.17

and the equation of continuity on displacement finally makes it possible to have the contact pressure:

$$\lambda_n=\frac{2 p R_1^2(1-\nu_1)}{R_1^2+R_2^2(1-2\nu_1)+\frac{E_1}{E_2}\frac{1+\nu_2}{1+\nu_1}(1-2\nu_2)(R_1^2-R_2^2)}$$ éq 2.18

2.1.2 Variable pressure

By using the assumption of invariance according to Z of the plane deformations, the solution of the problem takes the following shape:

$$u_r=u_r(r,\theta)$$
$$u_\theta=u_\theta(r,\theta)$$
$$u_z=0$$ éq 2.19

Subsequently, one will note the specific parameters to each solid by an index I, with i=1,2.

In the absence of forces of volume, one will use a form of the function of Airy proposed by Michel [1], in polar coordinates:

$$\begin{aligned}\chi(r,\theta)=&A_{01}r^2+A_{02}r^2\log(r)+A_{03}\log(r)+A_{04}\theta\\&+(A_{11}r^3+A_{12}r\log(r)+A_{13}r^{-1})\cos(\theta)+A_{14}r\theta\sin(\theta)\\&+(B_{11}r^3+B_{12}r\log(r)+B_{13}r^{-1})\sin(\theta)+B_{14}r\theta\cos(\theta)\\&+\sum_{n=2}^{\infty}(A_{n1}r^{n+2}+A_{n2}r^{-n+2}+A_{n3}r^n+A_{n4}r^{-n})\cos(n\theta)\\&+\sum_{n=2}^{\infty}(B_{n1}r^{n+2}+B_{n2}r^{-n+2}+B_{n3}r^n+B_{n4}r^{-n})\sin(n\theta)\end{aligned}$$ éq 2.20

The terms of the tensor of the nonworthless constraints of Cauchy are expressed according to the function of Airy as follows:

$$\sigma_{rr}=\frac{1}{r}\frac{\partial\chi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\chi}{\partial\theta^2}$$
$$\sigma_{\theta\theta}=\frac{\partial^2\chi}{\partial r^2}$$
$$\sigma_{r\theta}=\frac{-\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\chi}{\partial\theta}\right)$$
$$\sigma_{zz}=\nu(\sigma_{rr}+\sigma_{\theta\theta})$$

éq 2.21

Our pressure $p=\alpha_1.\cos(2\theta)$ varying in $\cos(2\theta)$, one will take only the part varying in $\cos(2\theta)$ in the function of Airy. The function of Airy will be written then:

$$\chi(r,\theta)=\left(Ar^2+Br^4+\frac{C}{r^2}+D\right)\cos(2\theta)$$

éq 2.22

From there, one can express the nonworthless constraints in the polar reference mark for solid 1:

$$\sigma_{rr}^1=\left(-2A_1-6\frac{C_1}{r^4}-4\frac{D_1}{r^2}\right)\cos(2\theta)$$
$$\sigma_{\theta\theta}^1=\left(2A_1+12B_1r^2+6\frac{C_1}{r^4}\right)\cos(2\theta)$$
$$\sigma_{r\theta}^1=2\left(A_1+3B_1r^2-3\frac{C_1}{r^4}-\frac{D_1}{r^2}\right)\sin(2\theta)$$
$$\sigma_{zz}^1=\nu(\sigma_{rr}+\sigma_{\theta\theta})$$

éq 2.23

And for solid 2:

$$\sigma_{rr}^2=(-2A_2)\cos(2\theta)$$
$$\sigma_{\theta\theta}^2=(2A_2+12B_2r^2)\cos(2\theta)$$
$$\sigma_{r\theta}^2=2(A_2+3B_2r^2)\sin(2\theta)$$
$$\sigma_{zz}^2=\nu(\sigma_{rr}+\sigma_{\theta\theta})$$

éq 2.24

and terms of the tensor of deformations by using the law of Hooke:

$$\underline{\underline{\varepsilon}}=\frac{1}{E}\left((1+\nu)\underline{\underline{\sigma}}-\nu\,tr(\underline{\underline{\sigma}})\underline{\underline{I}}\right)$$

éq 2.25

One will use the expression of the deformations to express displacements in the polar reference mark. One a:

$$\frac{\partial u_r}{\partial r}=\varepsilon_{rr}$$
$$\frac{\partial u_\theta}{\partial\theta}=r\,\varepsilon_{\theta\theta}-u_r$$
$$\frac{1}{2}\left(\frac{\partial u_\theta}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r}\right)=\varepsilon_{r\theta}$$

éq 2.26

By integrating these relations and by using symmetries of the problem, one can express displacements for solid 1:

$$u_r^1=\frac{1+\nu_1}{E_1}\left[\left(-2A_1r+2\frac{C_1}{r^3}+4\frac{D_1}{r}\right)-\nu_1\left(4B_1r^3+4\frac{D_1}{r}\right)\right]\cos(2\theta)$$
$$u_\theta^1=\frac{1+\nu_1}{E_1}\left[\left(2A_1r+6B_1r^3+2\frac{C_1}{r^3}-2\frac{D_1}{r}\right)-\nu_1\left(4B_1r^3-4\frac{D_1}{r}\right)\right]\sin(2\theta)$$

éq 2.27

And for solid 2:

$$u_r^2=\frac{1+\nu_2}{E_2}[(-2A_2 r)-\nu_2(4B_2 r^3)]\cos(2\theta)$$
$$u_\theta^2=\frac{1+\nu_2}{E_2}[(2A_2 r+6B_2 r^3)-\nu_1(4B_1 r^3)]\sin(2\theta)$$ éq

2.28

Now that one expressed all our fields according to the constants $A_1, B_1, C_1, D_1, A_2, B_2$, one must calculate these last according to the geometrical characteristics and of the loading.
One will note λ the contact pressure between the two solids.
The boundary conditions are:

$$\sigma_{rr}^1(R_1)=-\alpha_1\cos(2\theta)\text{: pression externe appliquée}$$
$$\sigma_{r\theta}^1(R_1)=0\text{: pression tangentielle nulle sur le bord extérieur du solide 1}$$
$$\sigma_{rr}^2(R_2)=-\lambda\text{: pression de contact appliquée par le solide 1 sur le solide 2}$$
$$\sigma_{rr}^1(R_2)=-\lambda\text{: pression de contact appliquée par le solide 2 sur le solide 1}$$
$$\sigma_{r\theta}^1(R_2)=0\text{: pas de frottement entre les deux solides}$$
$$\sigma_{r\theta}^2(R_2)=0\text{: pas de frottement entre les deux solides}$$

We thus have 6 equations for the 6 unknown factors: $A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2$.
While posing: $f_1=(\frac{R_2}{R_1})^2$
The system of 6 equations enables us to have:

$$A_1=\frac{\alpha_1(2f_1^2+f_1+1)-\lambda(f_1^3+f_1^2+2f_1)}{2(1-f_1)^3}; B_1=\frac{-1}{R_2^2}\frac{\alpha_1(3f_1^2+f_1)-\lambda(f_1^3+3f_1^2)}{6(1-f_1)^3};$$
$$C_1=R_2^4\frac{\alpha_1(f_1+3)-\lambda(3f_1+1)}{6(1-f_1)^3}; D_1=-R_2^2\frac{\alpha_1(f_1^2+f_1+2)-\lambda(2f_1^2+f_1+1)}{2(1-f_1)^3}$$ éq
$$A_2=\frac{\lambda}{2}; B_2=\frac{-\lambda}{6R_2^2}$$

2.29
One can express the contact pressure analytically
By using the continuity of radial displacement on the level of the interface of contact:

$$u_r^1(R_2)=u_r^2(R_2)$$ éq

2.30
one can express the contact pressure analytically:

$$\lambda=\frac{coef_1}{coef_2+coef_3}\alpha_1$$ éq

2.31

such as:

$$coef_1=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-12f_1^2-8f_1-12)+\nu_1(12f_1^2+8f_1+12)]$$
$$coef_2=\frac{1+\nu_1}{6E_1(1-f_1)^3}[(-3f_1^3-15f_1^2-9f_1-5)+\nu_1(2f_1^3+18f_1^2+6f_1+6)]$$ éq
$$coef_3=\frac{1+\nu_2}{6E_2}(-3+2\nu_2)$$

2.32

2.1.3 Values tested

One tests the contact pressure to the interface between the two solids, as well as displacements according to X and Y: u_x, u_y , in plane deformations.

The value of the pressure applied to the edge external of crown in $r=R_1$ express yourself in the form: $p(\theta)=10^7+10^5\cos(2\theta)(Pa)$, with $\theta=\arctan(\frac{Y}{X})$.
One will test the values the min and max values of displacements and the contact pressure. One will carry out for each modeling two calculations: the first calculation with contact and a second calculation without contact. One will test the values for each calculation. One will call “calcul_1” calculation with contact and “calcul_2” calculation without contact.

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN).
Young moduli $E_1 = E_2$ and Poisson's ratios $\nu_1 = \nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the external edge is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.

3.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:
- 240 meshs of the type SEG2;
- 3120 meshs of the type QUAD4.

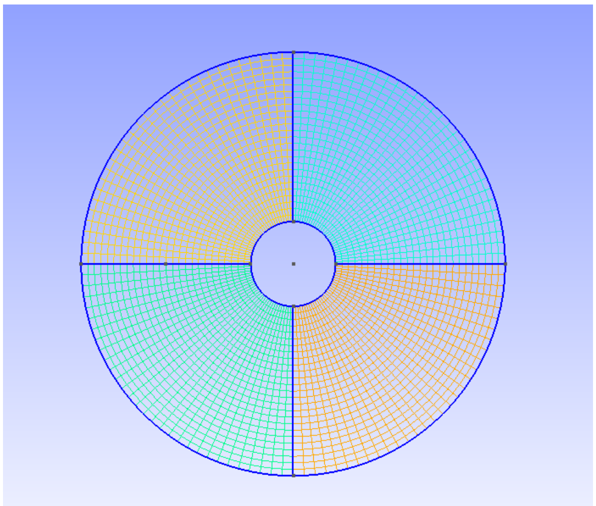

Figure 3.2-a: Grid of modeling A

3.3 Sizes tested and results

One tests the minimal value and the maximum value of displacements and the contact pressure on the level of the interface.

Identification	Type of reference	Value of reference	tolerance
DX max (calcul_1)	analytical	0.00441230599078	0.1%
DY max (calcul_1)	analytical	0.00422769400922	0.1%
LAGS_C max (calcul_1)	analytical	-9852073.73272	0.1%
DX min (calcul_1)	analytical	-0.00441230599078	0.1%
DY min (calcul_1)	analytical	-0.00422769400922	0.1%
LAGS_C min (calcul_1)	analytical	-10147926.2673	0.1%
DX max (calcul_2)	analytical	0.00441230599078	0.1%
DY max (calcul_2)	analytical	0.00422769400922	0.1%
DX min (calcul_2)	analytical	-0.00441230599078	0.1%
DY min (calcul_2)	analytical	-0.00422769400922	0.1%

4 Modeling B

4.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN).
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the external edge is worth $1.0E7+10E5.\cos(2\theta)\;(Pa)$, θ being the polar angle.

4.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:

- 240 meshs of the type SEG3;
- 3360 meshs of the type QUAD8.

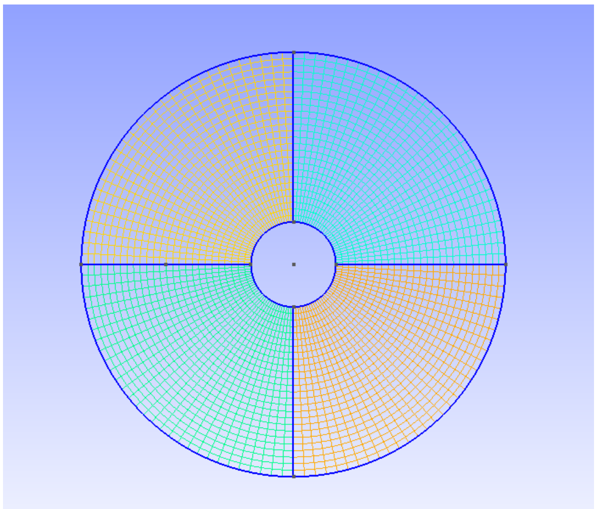

Figure 4.2-a: Grid of modeling B

4.3 Sizes tested and results

One tests the minimal value and the maximum value of displacements and the contact pressure on the level of the interface.

Identification	Type of reference	Value of reference	tolerance
DX max (calcul_1)	analytical	0.00441230599078	0.1%
DY max (calcul_1)	analytical	0.00422769400922	0.1%
LAGS_C max (calcul_1)	analytical	-9852073.73272	0.1%
DX min (calcul_1)	analytical	-0.00441230599078	0.1%
DY min (calcul_1)	analytical	-0.00422769400922	0.1%
LAGS_C min (calcul_1)	analytical	-10147926.2673	0.1%
DX max (calcul_2)	analytical	0.00441230599078	0.1%
DY max (calcul_2)	analytical	0.00422769400922	0.1%
DX min (calcul_2)	analytical	-0.00441230599078	0.1%
DY min (calcul_2)	analytical	-0.00422769400922	0.1%

5 Modeling C

5.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN).
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the external edge is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.

5.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:

- 232 meshs of the type SEG2;
- 2900 meshs of the type QUAD4.

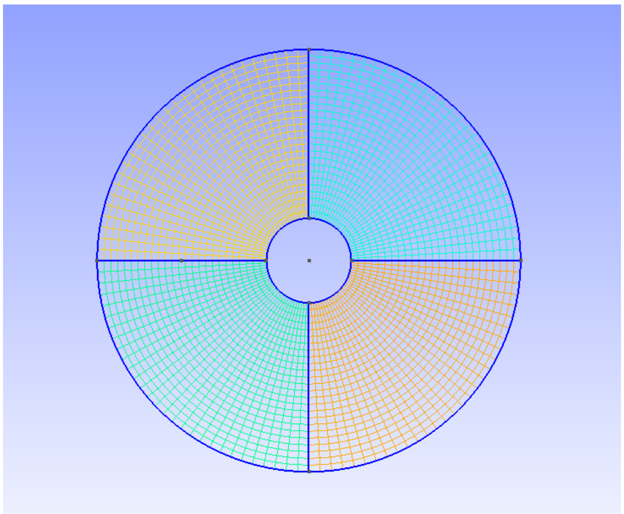

Figure 5.2-a: Grid of modeling C

5.3 Sizes tested and results

One tests the minimal value and the maximum value of displacements and the contact pressure on the level of the interface.

Identification	Type of reference	Value of reference	tolerance
DX max (calcul_1)	analytical	0.00441230599078	0.1%
DY max (calcul_1)	analytical	0.00422769400922	0.1%
LAGS_C max (calcul_1)	analytical	-9852073.73272	0.1%
DX min (calcul_1)	analytical	-0.00441230599078	0.1%
DY min (calcul_1)	analytical	-0.00422769400922	0.1%
LAGS_C min (calcul_1)	analytical	-10147926.2673	0.1%
DX max (calcul_2)	analytical	0.00441230599078	0.1%
DY max (calcul_2)	analytical	0.00422769400922	0.1%
DX min (calcul_2)	analytical	-0.00441230599078	0.1%
DY min (calcul_2)	analytical	-0.00422769400922	0.1%

6 Modeling D

6.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN).
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0E9\,Pa$ and 0.2. The pressure applied to the external edge is worth $1.0E7+10E5.\cos(2\theta)\ (Pa)$, θ being the polar angle.

6.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:
- 232 meshs of the type SEG3;
- 2900 meshs of the type QUAD8.

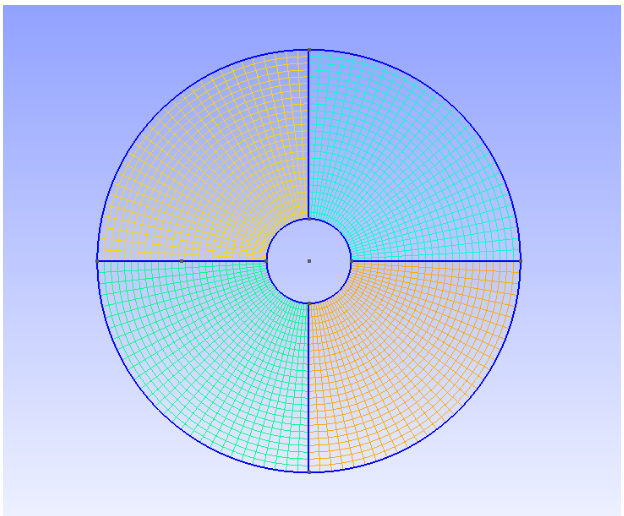

Figure 6.2-a: Grid of modeling D

6.3 Sizes tested and results

One tests the minimal value and the maximum value of displacements and the contact pressure on the level of the interface.

Identification	Type of reference	Value of reference	tolerance
DX max (calcul_1)	analytical	0.00441230599078	0.1%
DY max (calcul_1)	analytical	0.00422769400922	0.1%
LAGS_C max (calcul_1)	analytical	-9852073.73272	0.1%
DX min (calcul_1)	analytical	-0.00441230599078	0.1%
DY min (calcul_1)	analytical	-0.00422769400922	0.1%
LAGS_C min (calcul_1)	analytical	-10147926.2673	0.1%
DX max (calcul_2)	analytical	0.00441230599078	0.1%
DY max (calcul_2)	analytical	0.00422769400922	0.1%
DX min (calcul_2)	analytical	-0.00441230599078	0.1%
DY min (calcul_2)	analytical	-0.00422769400922	0.1%

7 Modeling E

7.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN).
Young moduli $E_1=E_2$ and Poisson's ratios $\nu_1=\nu_2$ are respectively $1.0\mathrm{E}9\,Pa$ and 0.2. The pressure applied to the external edge is that equivalent to the application of a pressure of $1.0\mathrm{E}7+10\mathrm{E}5.\cos(2\theta)$ (Pa), on a disc, θ being the polar angle.

7.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:

- 120 meshs of the type SEG2;
- 900 meshs of the type QUAD4.

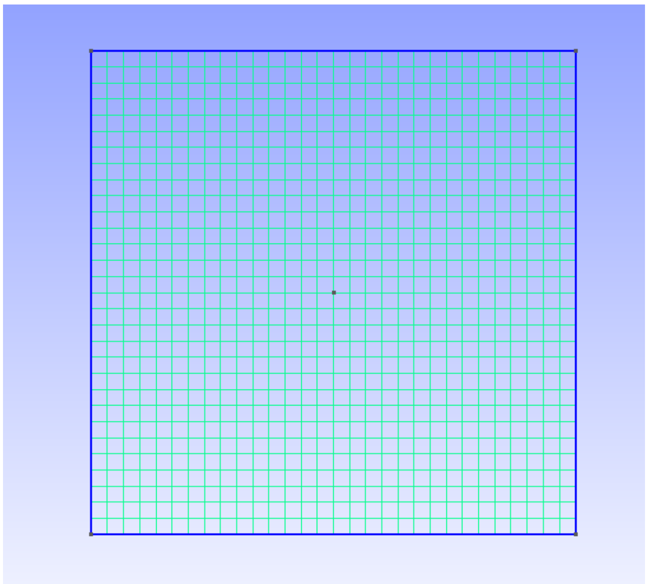

Figure 7.2-a: Grid of modeling E

7.3 Sizes tested and results

One tests the minimal value and the maximum value of displacements and the contact pressure on the level of the interface.

Identification	Type of reference	Value of reference	tolerance
DX max (calcul_1)	analytical	0.00441230599078	0.1%
DY max (calcul_1)	analytical	0.00422769400922	0.1%
DX min (calcul_1)	analytical	-0.00441230599078	0.1%
DY min (calcul_1)	analytical	-0.00422769400922	0.1%
LAGS_C min (calcul_1)	analytical	-10147926.2673	0.1%
DX max (calcul_2)	analytical	0.00441230599078	0.1%
DY max (calcul_2)	analytical	0.00422769400922	0.1%
DX min (calcul_2)	analytical	-0.00441230599078	0.1%
DY min (calcul_2)	analytical	-0.00422769400922	0.1%

8 Modeling F

8.1 Characteristics of modeling

It is about a modeling in plane deformations (D_PLAN).
Young moduli $E_1 = E_2$ and Poisson's ratios $\nu_1 = \nu_2$ are respectively $1.0\mathrm{E}9\,Pa$ and 0.2. The pressure applied to the external edge is that equivalent to the application of a pressure of $1.0\mathrm{E}7 + 10\mathrm{E}5.\cos(2\theta)\ (Pa)$, on a disc, θ being the polar angle.

8.2 Characteristics of the grid

The grid (Figure 3.2-1) comprises:

- 120 meshs of the type SEG3;
- 900 meshs of the type QUAD8.

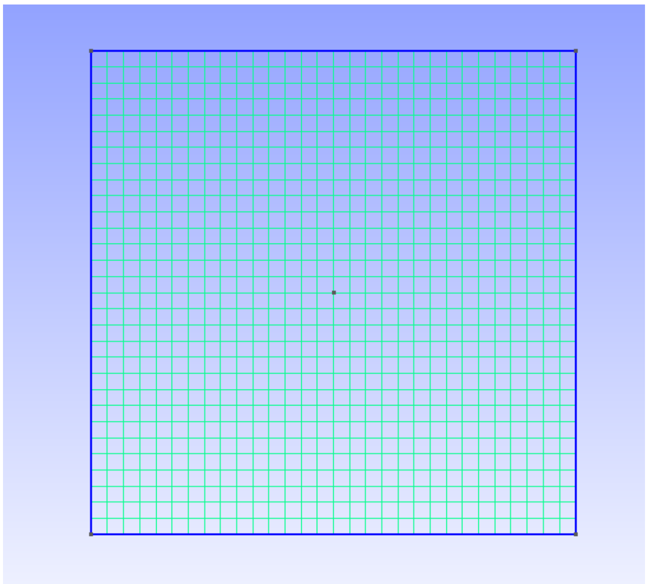

Figure 8.2-a: Grid of modeling F

8.3 Sizes tested and results

One tests the minimal value and the maximum value of displacements and the contact pressure on the level of the interface.

Identification	Type of reference	Value of reference	tolerance
DX max (calcul_1)	analytical	0.00441230599078	0.1%
DY max (calcul_1)	analytical	0.00422769400922	0.1%
LAGS_C max (calcul_1)	analytical	-9852073.73272	0.1%
DX min (calcul_1)	analytical	-0.00441230599078	0.1%
DY min (calcul_1)	analytical	-0.00422769400922	0.1%
LAGS_C min (calcul_1)	analytical	-10147926.2673	0.1%
DX max (calcul_2)	analytical	0.00441230599078	0.1%
DY max (calcul_2)	analytical	0.00422769400922	0.1%
DX min (calcul_2)	analytical	-0.00441230599078	0.1%
DY min (calcul_2)	analytical	-0.00422769400922	0.1%

9 Conclusion

This case test makes it possible to validate the contact formulation continues combined with X-FEM in the presence of curved surfaces of contact with linear and quadratic grids, in 2D. It in particular could make it possible to establish the orders of convergence for the method of contact continues combined to X-FEM and to check a convergence in energy of 1 and displacement of 2 for the linear grids, 2 in energy and 3 in displacement for the quadratic grids, for sufficiently rich diagrams of integration, when the interface of contact is in conformity with the grid and that the grid is radiating or not in conformity.