

## SSNP303 - Element in plane constraint and traction - Perfect plasticity

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### Summary:

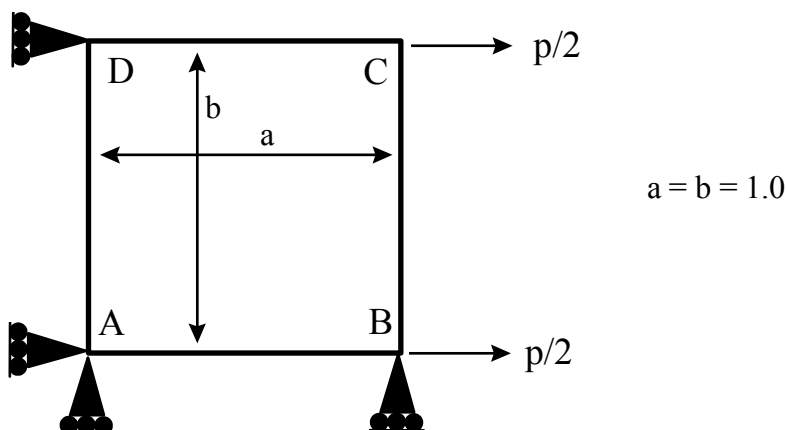
This test of nonlinear quasi-static mechanics 2D consist in charging three superimposed elements of plate (standard with element MECPOU4). The 4 nodes are common to the 3 elements. The elements have different properties (perfect plasticity) to obtain a suitable traction diagram. This test is drawn from guide NAFEMS.

The goal is to compare the various methods of the Newton-Raphson type making it possible to solve the system of nonlinear equations (NEWTON: ('ELASTIC' MATRIX) and NEWTON: ('TANGENT' MATRIX) with REAC\_INCR and REAC\_ITER). The selected convergence criteria correspond to 0.01% residual force (RESI\_GLOB\_RELA).

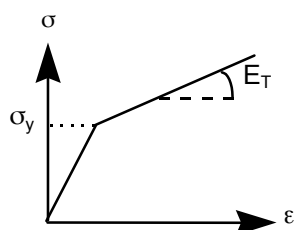
Modeling is carried out with elements plans of the type MECPOU4.

## 1 Problem of reference

### 1.1 Geometry



### 1.2 Material properties



Matériau 1	Matériau 2	Matériau 3
$E = 100000 \text{ MPa}$	$E = 60000 \text{ MPa}$	$E = 40000 \text{ MPa}$
$\nu = 0.25$	$\nu = 0.25$	$\nu = 0.25$
$\sigma_y = 3 \text{ MPa}$	$\sigma_y = 6 \text{ MPa}$	$\sigma_y = 8 \text{ MPa}$
$E_T = 0.0$	$E_T = 0.0$	$E_T = 0.0$

### 1.3 Boundary conditions and loadings

Not  $A$  :  $u_x = 0.$

$u_y = 0.$

Not  $B$  :  $u_y = 0.$

Not  $D$  :  $u_x = 0.$

Loading by a force  $P/2$  on the point  $B$  and  $C$ . The force  $P$  is increased in 6 stages in the following way:

Force $P$	3.00	6.00	9.00	12.95	15.00	16.93
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## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

The results considered as reference were got by the SAMCEF software by using 60 increments for each stage of loading. The characteristics of the various methods of resolution are the following ones:

- Wi Use of the elastic matrix.
- th
- B Use of the tangent matrix; this one is revalued only with the first iteration of each increment (REAC\_INCR = 1 and REAC\_ITER = 0). It is the method of Newton modified.
- C Use of the tangent matrix; this one is revalued with each iteration of each increment (REAC\_INCR = 1 and REAC\_ITER = 1). It is the method of Newton classical.

### 2.2 Results of reference

Results got with 60 increments for each stage of loading

Force	Element 1		Element 2		Element 3	
	$\sigma^{xx}$	$\sigma^{yy}$	$\sigma^{xx}$	$\sigma^{yy}$	$\sigma^{xx}$	$\sigma^{yy}$
3.00	1.500000D+00	6.938894D-18	9.000000D-01	-1.040834D-17	6.000000D-01	-6.938894D-18
6.00	3.000000D+00	4.861944D-13	1.800000D+00	-2.081668D-17	1.200000D+00	-3.469447D-18
9.00	3.147155D+00	3.199571D-01	3.511707D+00	-1.900098D-01	2.341138D+00	-1.279828D-01
12.95	3.252919D+00	5.950074D-01	5.814267D+00	-3.523377D-01	3.878832D+00	-2.380030D-01
15.00	3.213822D+00	4.873069D-01	6.017834D+00	3.174572D-02	5.768340D+00	-5.231355D-01
16.93	3.209297D+00	4.753345D-01	6.149462D+00	3.048490D-01	7.571241D+00	-7.863557D-01

### 2.3 Uncertainty on the solution

Uncertainty lower than 1 % .

### 2.4 Bibliographical references

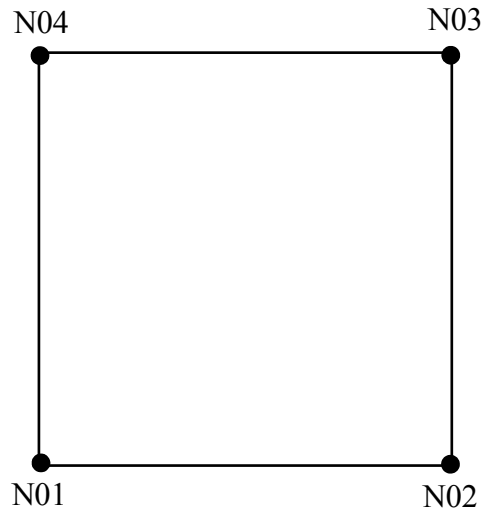
- 1) Fundamental tests for two and three dimensional, small strain, elastoplastic finite element analysis, 1987, NAFEMS

## 3 Modeling A

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### 3.1 Characteristics of modeling A

Use of the elements QUAD4



Modeling in plane constraints: C\_PLAN

The loading and the boundary conditions are modelled by:

- DDL\_IMPO: (Node N01 DX = 0, DY = 0)  
(Node N02 DY = 0)  
(Node N04 DX = 0)
- forces imposed on the nodes N02 and N03 .

### 3.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 3 MEC PQU4

## 3.3 Sizes tested and results

Identification	Moment	Reference	Aster	% difference
<i>SIXX</i> (mesh 1)	1	1,500000E+00	1,500000E+00	0
<i>SIYY</i> (mesh 1)	1	6,938894E-18	1,291047E-14	0
<i>SIXX</i> (mesh 2)	1	9,000000E-01	9,000000E-01	0
<i>SIYY</i> (mesh 2)	1	- 1,040834E-17	- 1,509903E-14	0
<i>SIXX</i> (mesh 3)	1	6,000000E-01	6,000000E-01	0
<i>SIYY</i> (mesh 3)	1	- 6,938894E-18	- 2,107830E-14	0
<i>SIXX</i> (mesh 1)	2	3,000000E+00	3,000000E+00	0
<i>SIYY</i> (mesh 1)	2	4,861944E-13	1,359551E-13	0
<i>SIXX</i> (mesh 2)	2	1,800000E+00	1,800000E+00	0
<i>SIYY</i> (mesh 2)	2	- 2,081668E-17	- 1,093097E-13	0
<i>SIXX</i> (mesh 3)	2	1,200000E+00	1,200000E+00	0
<i>SIYY</i> (mesh 3)	2	- 3,469447E-18	- 1,007126E-13	0
<i>SIXX</i> (mesh 1)	3	3,147155E+00	3,145788E+00	- 0.043
<i>SIYY</i> (mesh 1)	3	3,199571E-01	3,167040E-01	- 1.017
<i>SIXX</i> (mesh 2)	3	3,511707E+00	3,512527E+00	0.023
<i>SIYY</i> (mesh 2)	3	- 1,900098E-01	- 1,900224E-01	0.007
<i>SIXX</i> (mesh 3)	3	2,341138E+00	2,341685E+00	0.023
<i>SIYY</i> (mesh 3)	3	- 1,279828E-01	- 1,266816E-01	- 1.017
<i>SIXX</i> (mesh 1)	4	3,252919E+00	3,250728E+00	- 0.067
<i>SIYY</i> (mesh 1)	4	5,950074E-01	5,887464E-01	- 1.052
<i>SIXX</i> (mesh 2)	4	5,814267E+00	5,816159E+00	0.033
<i>SIYY</i> (mesh 2)	4	- 3,523377E-01	- 3,521695E-01	- 0.048
<i>SIXX</i> (mesh 3)	4	3,878832E+00	3,882391E+00	0.092
<i>SIYY</i> (mesh 3)	4	- 2,380030E-01	- 2,364650E-01	- 0.646
<i>SIXX</i> (mesh 1)	5	3,213822E+00	3,214367E+00	0.017
<i>SIYY</i> (mesh 1)	5	4,873069E-01	4,887527E-01	0.297
<i>SIXX</i> (mesh 2)	5	6,017834E+00	6,015985E+00	- 0.031
<i>SIYY</i> (mesh 2)	5	3,174572E-02	3,209818E-02	1.11
<i>SIXX</i> (mesh 3)	5	5,768340E+00	5,769681E+00	0.023
<i>SIYY</i> (mesh 3)	5	- 5,231355E-01	- 5,207413E-01	- 0.458
<i>SIXX</i> (mesh 1)	6	3,209297E+00	3,210194E+00	0.028
<i>SIYY</i> (mesh 1)	6	4,753345E-01	4,777143E-01	0.501
<i>SIXX</i> (mesh 2)	6	6,149462E+00	6,146791E+00	- 0.043
<i>SIYY</i> (mesh 2)	6	3,048490E-01	3,052330E-01	0.126
<i>SIXX</i> (mesh 3)	6	7,571241E+00	7,572486E+00	0.016
<i>SIYY</i> (mesh 3)	6	- 7,863557E-01	- 7,826545E-01	- 0.471

## 3.4 Remarks

To obtain a correct precision, it is necessary to force a significant number of increments for each stage of loading (60 increments).

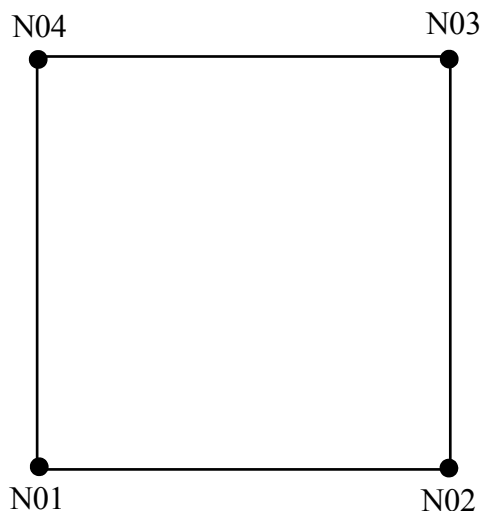
The iteration count is equal to 411.

## 4 Modeling B

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### 4.1 Characteristics of modeling B

Use of the elements QUAD4



### 4.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 3 MECPQU4

Modeling in plane constraints: C\_PLAN

The loading and the boundary conditions are modelled by:

- DDL\_IMPO: (Node NO1 DX = 0, DY = 0)  
(Node NO2 DY = 0)  
(Node NO4 DX = 0)
- forces imposed on the nodes NO2 and NO3 .

## 4.3 Sizes tested and results

Identification	Moment	Reference	Aster	% difference
<i>SIXX</i> (mesh 1)	1	1,500000E+00	1,500000E+00	0,000
<i>SIYY</i> (mesh 1)	1	6,938894E-18	1,351398E-14	0,000
<i>SIXX</i> (mesh 2)	1	9,000000E-01	9,000000E-01	0,000
<i>SIYY</i> (mesh 2)	1	- 1,040834E-17	- 2,142673E-14	0,000
<i>SIXX</i> (mesh 3)	1	6,000000E-01	6,000000E-01	0,000
<i>SIYY</i> (mesh 3)	1	- 6,938894E-18	- 2,924612E-14	0,000
<i>SIXX</i> (mesh 1)	2	3,000000E+00	3,000000E+00	0,000
<i>SIYY</i> (mesh 1)	2	4,861944E-13	9,617834E-14	0,000
<i>SIXX</i> (mesh 2)	2	1,800000E+00	1,800000E+00	0,000
<i>SIYY</i> (mesh 2)	2	- 2,081668E-17	- 8,301280E-14	0,000
<i>SIXX</i> (mesh 3)	2	1,200000E+00	1,200000E+00	0,000
<i>SIYY</i> (mesh 3)	2	- 3,469447E-18	- 9,360717E-14	0,000
<i>SIXX</i> (mesh 1)	3	3,147155E+00	3,145788E+00	- 0,043
<i>SIYY</i> (mesh 1)	3	3,199571E-01	3,167038E-01	- 1,017
<i>SIXX</i> (mesh 2)	3	3,511707E+00	3,512527E+00	0,023
<i>SIYY</i> (mesh 2)	3	- 1,900098E-01	- 1,900225E-01	0,007
<i>SIXX</i> (mesh 3)	3	2,341138E+00	2,341685E+00	0,023
<i>SIYY</i> (mesh 3)	3	- 1,279828E-01	- 1,266817E-01	- 1,017
<i>SIXX</i> (mesh 1)	4	3,252919E+00	3,250686E+00	- 0,069
<i>SIYY</i> (mesh 1)	4	5,950074E-01	5,886265E-01	- 1,072
<i>SIXX</i> (mesh 2)	4	5,814267E+00	5,816233E+00	0,034
<i>SIYY</i> (mesh 2)	4	- 3,523377E-01	- 3,520332E-01	- 0,086
<i>SIXX</i> (mesh 3)	4	3,878832E+00	3,883081E+00	0,110
<i>SIYY</i> (mesh 3)	4	- 2,380030E-01	- 2,365921E-01	- 0,593
<i>SIXX</i> (mesh 1)	5	3,213822E+00	3,214343E+00	0,016
<i>SIYY</i> (mesh 1)	5	4,873069E-01	4,886891E-01	0,284
<i>SIXX</i> (mesh 2)	5	6,017834E+00	6,015971E+00	- 0,031
<i>SIYY</i> (mesh 2)	5	3,174572E-02	3,207056E-02	1,023
<i>SIXX</i> (mesh 3)	5	5,768340E+00	5,769686E+00	0,023
<i>SIYY</i> (mesh 3)	5	- 5,231355E-01	- 5,207599E-01	- 0,454
<i>SIXX</i> (mesh 1)	6	3,209297E+00	3,210140E+00	0,026
<i>SIYY</i> (mesh 1)	6	4,753345E-01	4,775720E-01	0,471
<i>SIXX</i> (mesh 2)	6	6,149462E+00	6,146788E+00	- 0,043
<i>SIYY</i> (mesh 2)	6	3,048490E-01	3,052260E-01	0,124
<i>SIXX</i> (mesh 3)	6	7,571241E+00	7,573072E+00	0,024
<i>SIYY</i> (mesh 3)	6	- 7,863557E-01	- 7,827984E-01	- 0,452



## 4.4 Remarks

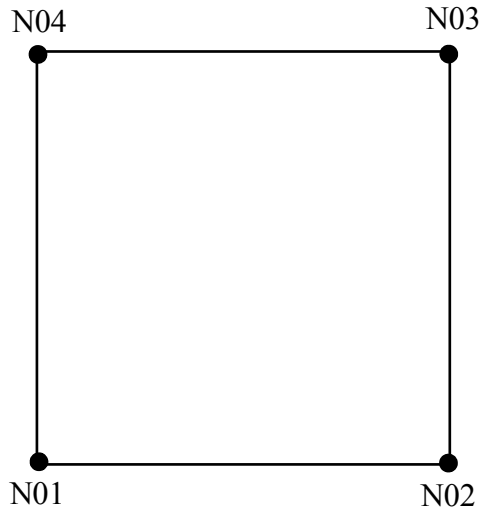
Idem modeling A concerning the number of increments on the precision of the results.  
The iteration count is equal to 361.

## 5 Modeling C

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### 5.1 Characteristics of modeling C

Use of the elements QUAD4



### 5.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 3 MECPCQU4

Modeling in plane constraints: C\_PLAN

The loading and the boundary conditions are modelled by:

- DDL\_IMPO: (Node NO1 DX = 0, DY = 0)  
(Node NO2 DY = 0)  
(Node NO4 DX = 0)
- forces imposed on the nodes NO2 and NO3 .

## 5.3 Sizes tested and results

Identification	Moment	Reference	Aster	% difference
<i>SIXX</i> (mesh 1)	1	1,500000E+00	1,500000E+00	0
<i>SIYY</i> (mesh 1)	1	6,938894E-18	1,351398E-14	0
<i>SIXX</i> (mesh 2)	1	9,000000E-01	9,000000E-01	0
<i>SIYY</i> (mesh 2)	1	- 1,040834E-17	- 2,142673E-14	0
<i>SIXX</i> (mesh 3)	1	6,000000E-01	6,000000E-01	0
<i>SIYY</i> (mesh 3)	1	- 6,938894E-18	- 2,924612E-14	0
<i>SIXX</i> (mesh 1)	2	3,000000E+00	3,000000E+00	0
<i>SIYY</i> (mesh 1)	2	4,861944E-13	9,617834E-14	0
<i>SIXX</i> (mesh 2)	2	1,800000E+00	1,800000E+00	0
<i>SIYY</i> (mesh 2)	2	- 2,081668E-17	- 8,301280E-14	0
<i>SIXX</i> (mesh 3)	2	1,200000E+00	1,200000E+00	0
<i>SIYY</i> (mesh 3)	2	- 3,469447E-18	- 9,360717E-14	0
<i>SIXX</i> (mesh 1)	3	3,147155E+00	3,145788E+00	- 0.043
<i>SIYY</i> (mesh 1)	3	3,199571E-01	3,167040E-01	- 1.017
<i>SIXX</i> (mesh 2)	3	3,511707E+00	3,512527E+00	0.023
<i>SIYY</i> (mesh 2)	3	- 1,900098E-01	- 1,900224E-01	0.007
<i>SIXX</i> (mesh 3)	3	2,341138E+00	2,341685E+00	0.023
<i>SIYY</i> (mesh 3)	3	- 1,279828E-01	- 1,266816E-01	- 1.017
<i>SIXX</i> (mesh 1)	4	3,252919E+00	3,250686E+00	- 0.069
<i>SIYY</i> (mesh 1)	4	5,950074E-01	5,886268E-01	- 1.072
<i>SIXX</i> (mesh 2)	4	5,814267E+00	5,816233E+00	0.034
<i>SIYY</i> (mesh 2)	4	- 3,523377E-01	- 3,520328E-01	- 0.087
<i>SIXX</i> (mesh 3)	4	3,878832E+00	3,883081E+00	0.11
<i>SIYY</i> (mesh 3)	4	- 2,380030E-01	- 2,365919E-01	- 0.593
<i>SIXX</i> (mesh 1)	5	3,213822E+00	3,214343E+00	0.016
<i>SIYY</i> (mesh 1)	5	4,873069E-01	4,886892E-01	0.284
<i>SIXX</i> (mesh 2)	5	6,017834E+00	6,015971E+00	- 0.031
<i>SIYY</i> (mesh 2)	5	3,174572E-02	3,207066E-02	1.024
<i>SIXX</i> (mesh 3)	5	5,768340E+00	5,769686E+00	0.023
<i>SIYY</i> (mesh 3)	5	- 5,231355E-01	- 5,207598E-01	- 0.454
<i>SIXX</i> (mesh 1)	6	3,209297E+00	3,210140E+00	0.026
<i>SIYY</i> (mesh 1)	6	4,753345E-01	4,775721E-01	0.471
<i>SIXX</i> (mesh 2)	6	6,149462E+00	6,146788E+00	- 0.043
<i>SIYY</i> (mesh 2)	6	3,048490E-01	3,052262E-01	0.124
<i>SIXX</i> (mesh 3)	6	7,571241E+00	7,573072E+00	0.024
<i>SIYY</i> (mesh 3)	6	- 7,863557E-01	- 7,827982E-01	- 0.452

## 5.4 Remarks

Idem modeling A concerning the number of increments on the precision of the results.

The iteration count is equal to 360.

## 6 Summary of the results

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A significant number of increments is necessary to obtain a correct precision: for information, ten increments per stage of loading are not sufficient.