SSNP503 – Contact in great slips with X-FEM for horizontal cracks

Summary:

The purpose of this test is to test and validate the features of the approach great slips with X-FEM in cases presenting of the horizontal cracks. One tests structures 2D in plane constraints and plane deformations for elements quadrangles and triangles as well as structures 3D for elements hexahedrons, pentahedrons and tetrahedrons. This approach, described in [R5.03.53], allows in particular the taking into account of the contact, with friction, on the level of the cracks introduced by method X-FEM, in the presence of great displacements but under the assumption of the small deformations. New algorithms, compared to the treatment of contact with X-FEM under the assumption of the small disturbances [R7.02.12], which is tested and validated by this case test relate to the geometrical reactualization of the lips of the cracks, master-slave pairing and the creation of the new hybrid elements of contact.

One considers a rectangular structure presenting two horizontal cracks crossing it completely, placed symmetrically compared to the median axis of the structure. The two cracks thus cut the rectangle in three blocks. The blocking of horizontal displacements is imposed on the four corners of the rectangle, of vertical displacements are imposed on the edges inferior and superior of the rectangle in order to tighten the block medium which is seen forcing a horizontal displacement to make it slip along the cracks. Following the request of compression thus created, contact pressures appear on the zones in contact, with an evolution of their values according to the advance of the block medium.

The validation is done by comparison of the values of contact pressure with the similar values obtained starting from a homologous test (even geometry, same boundary conditions etc), treated within the classical framework of the finite element method, with *Code_Aster*, where the cracks are in conformity with the grid.

1 Problem of reference

1.1 Geometry

The structure is a healthy rectangle into which two horizontal cracks are introduced. The cracks are placed symmetrically compared to the median axis of the structure, as shown on the Figure 1.1-a. Dimensions of the structure as well as the distance between the cracks are:

$H = 9\,m$;
$L = 4\,m$;
$b = 4\,m$;

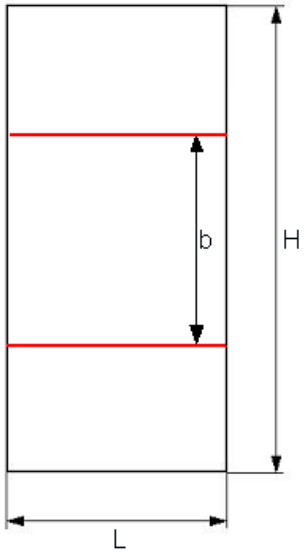

Figure 1.1-a: Geometry of the structure and positioning of the cracks.

1.2 Properties of material

Young modulus: $E = 100\,MPa$

Poisson's ratio: $\nu = 0.3$

1.3 Boundary conditions and loadings

The blocking of horizontal displacements is imposed on the four corners of the structure (Figure 1.3-a). On the edges inferior and superior of the rectangle, one imposes displacements along the axis Y who will close the cracks in order to generate contact pressure. The block medium is subjected to an important slip by applying a displacement controlled along the axis X on its left edge.

The digital values of imposed displacements are:

$Depl_X = 2.00\,m$
$Depl_Y = 1.0\mathrm{E}-3\,m$

Their application is done according to a function crawls classical, in 4 pas de charges.

The coefficient of friction of Coulomb is taken equal to $0,5$.

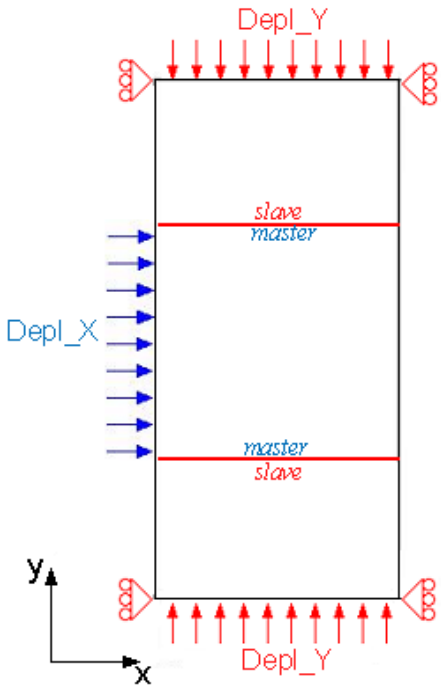

Figure 1.3-a: Illustration of the boundary conditions and the loadings.

2 Reference solution

The reference solutions for this CAS-test are provided by the results resulting from calculations Aster, for the same structure but with the cracks respecting the grid, carried out with the method continues contact already existing for the classical framework of the finite element method [R5.03.52].

The geometry (except for the introduction the cracks), the boundary conditions, the loadings as well as the parameters of contact are the same ones as those considered for this CAS-test modelled with X-FEM.

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling FEM, in plane deformations. The three blocks are with a grid conformément and the conditions of contact are imposed on the edges of these blocks. One declares the edges Masters on the block medium and the edges slaves on the blocks inferior and superior in order to conform to the figure 1.3-a.

3.2 Characteristics of the grid

The grid is regulated (Figure 3.2-a) and comprises 3 blocks made up of meshs of the type `QUAD4`. The blocks superior and inferior have each one 12 meshs, while the block medium has 20 of them.

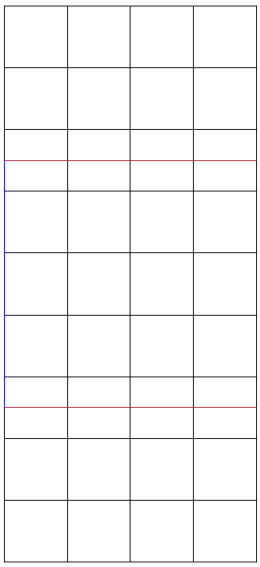

Figure 3.2-a: Grid of modeling A.

3.3 Sizes tested and results

One tests contact pressures on the upper lip of the lower block, this one being declared slave, at the end of each step of load considered. The site of the nodes which store the degrees of freedom of contact which one tests the values, is illustrated on the Figure 3.4-a.

Not	Identification	*Reference*
1	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-8.1059677E+03
	LAGS_C for N19	-6.4501080E+03
	LAGS_C for N18	-6.4542489E+03
	LAGS_C for N17	-4.1383235E+03
2	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-1.0392388E+04
	LAGS_C for N19	-1.3120234E+04
	LAGS_C for N18	-1.1868000E+04
	LAGS_C for N17	-8.7124511E+03
3	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-2.1971033E+04
	LAGS_C for N18	-1.8731887E+04
	LAGS_C for N17	-1.1800265E+04
4	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-1.9159216E+04
	LAGS_C for N18	-2.3596467E+04
	LAGS_C for N17	-1.7951778E+04

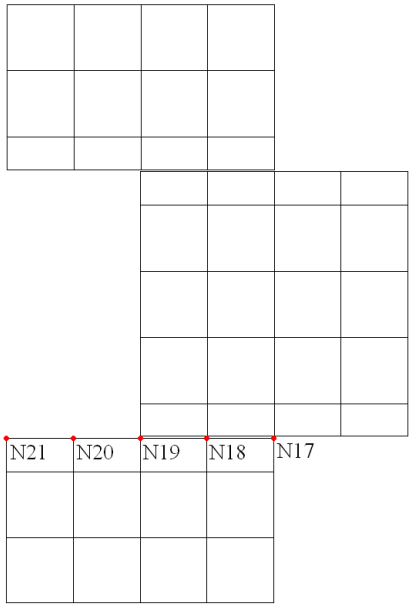

Figure 3.4-a: The site of the nodes which store the degrees of freedom of contact tested.

3.4 Remarks

This test is used as reference to modeling B, it is thus normal to have residues close to zero ($10^{-10}\%$), which corresponds to the precision required in the operator IMPR_TABLE command file. It makes it possible to make sure that there is no evolution in the calculation algorithms for method FEM of contact in great slips, in which case it would then be necessary also to re-examine method X-FEM, based on this one.

4 Modeling B

4.1 Characteristics of modeling

It is about a modeling X-FEM, in plane deformations, with definition of contact on the interfaces generated by the cracks, themselves defined by functions of level (level set noted normal LN) directly in the command file using the operator DEFI_FISS_XFEM [U4.82.08].

The statute main slave/for a surface of contact X-FEM is given by the sign of the normal function of level LN : surface slave is negative side while surface Master is positive side.

The equations of the functions of levels for the two horizontal cracks are the following ones:

$$\mathrm{LN}1 = Y - 2.5 \qquad \text{éq 4.1-1}$$
$$\mathrm{LN}2 = -Y + 6.5 \qquad \text{éq 4.1-2}$$

The levels set tangent are not defined because one does not model basic of crack. The keyword should then be informed **TYPE_DISCONTINUITE** with the option **'INTERFACE'** in the operator **DEFI_FISS_XFEM**.

So that the approach great slips with X-FEM is activated, the keyword should be informed **REAC_GEOM** in the operator **DEFI_CONTACT** with the option '**AUTOMATIQUE'**. If not, for **REAC_GEOM=' SANS'** (option by default), the treatment HP is active.

For the activation of friction, it is necessary to inform the keyword **FRICTION** in the operator **DEFI_CONTACT** with the option **'COULOMB'** then to indicate the value of the coefficient of friction for each zone by informing the keyword **COULOMB** under the operand **ZONE**.

4.2 Characteristics of the grid

The grid is regulated (Figure 4.2-a) and comprises 36 meshs of the type QUAD4. Following the definition of the cracks, the 8 cut meshs are transformed into QUAD8 in order to store the degrees of freedom of contact rubbing with the nodes tops and mediums.

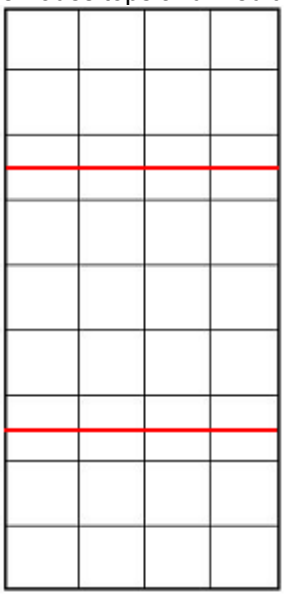

Figure 4.2-a: Grid of modeling B.

The choice amongst elements in the direction Y (on the vertical) was made by taking account of the restriction imposed by the modeling of multi-cracking with method X-FEM: the cracks must be separate of at least two healthy meshs.

4.3 Sizes tested and results

One tests the values of the contact pressure on the lip slave of the first crack, at the end of each step of load considered. On the cut elements, one can choose to arbitrarily record the values on the nodes above or below the crack (one chose with the top), because the degrees of freedom of contact are bound by relation of equality (it is not necessary to make an interpolation). The site of the nodes which store the degrees of freedom of contact which one tests the values, is illustrated on the Figure 4.4-a.

Not	Identification	Reference
1	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-8.1059677E+03
	LAGS_C for N34	-6.4501080E+03
	LAGS_C for N35	-6.4542489E+03
	LAGS_C for N10	-4.1383235E+03
2	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-1.0392388E+04
	LAGS_C for N34	-1.3120234E+04
	LAGS_C for N35	-1.1868000E+04
	LAGS_C for N10	-8.7124511E+03
3	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-2.1971033E+04
	LAGS_C for N35	-1.8731887E+04
	LAGS_C for N10	-1.1800265E+04
4	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-1.9159216E+04
	LAGS_C for N35	-2.3596467E+04
	LAGS_C for N10	-1.7951778E+04

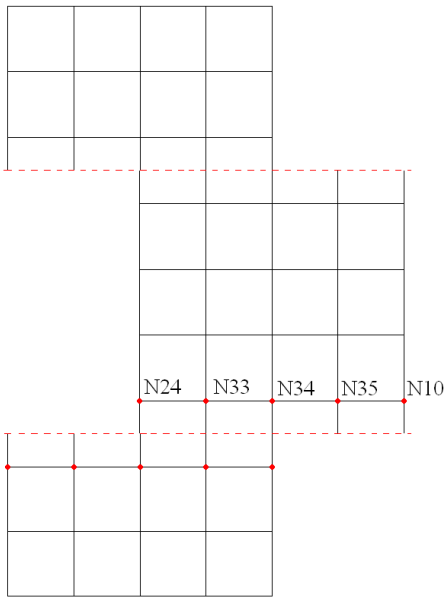

Figure 4.4-a: The site of the nodes which store the degrees of freedom of contact tested.

4.4 Remarks

In both cases (methods of contact X-FEM and classical FEM) there is the same number of degrees of freedom of displacements, the same number of degrees of freedom of contact, and the position of the nodes of the grid (except the interface which is not with a grid for X-FEM) are the same ones, one integrates the contact on the same elements and rigidity on the quasi-same elements (except for the cut elements). One thus solves the same digital problem in the 2 cases. It is thus normal to observe differences close to zero ($10^{-7}\%$).

5 Modeling C

5.1 Characteristics of modeling

They is the same characteristics of modeling as modeling A but in plane constraints.

5.2 Characteristics of the grid

They is the same characteristics of grid as modeling A.

5.3 Sizes tested and results

One tests contact pressures as for modeling A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 3.4-a.

Not	Identification	Reference
1	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-7.1041388E+03
	LAGS_C for N19	-5.5751916E+03
	LAGS_C for N18	-5.6055420E+03
	LAGS_C for N17	-3.6920556E+03
2	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-9.1709839E+03
	LAGS_C for N19	-1.1435732E+04
	LAGS_C for N18	-1.0391562E+04
	LAGS_C for N17	-7.7671507E+03
3	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-1.9485289E+04
	LAGS_C for N18	-1.6399583E+04
	LAGS_C for N17	-1.0605893E+04
4	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-1.7065781E+04
	LAGS_C for N18	-2.0872174E+04
	LAGS_C for N17	-1.6114706E+04

5.4 Remarks

This test is used as reference to modeling D.

6 Modeling D

6.1 Characteristics of modeling

They is the same characteristics of modeling as modeling B but in plane constraints.

6.2 Characteristics of the grid

They is the same characteristics of grid as modeling B.

6.3 Sizes tested and results

One tests the values of contact pressure as for modeling B.

Not	Identification	Reference
1	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-7.1041388E+03
	LAGS_C for N34	-5.5751916E+03
	LAGS_C for N35	-5.6055420E+03
	LAGS_C for N10	-3.6920556E+03
2	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-9.1709839E+03
	LAGS_C for N34	-1.1435732E+04
	LAGS_C for N35	-1.0391562E+04
	LAGS_C for N10	-7.7671507E+03
3	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-1.9485289E+04
	LAGS_C for N35	-1.6399583E+04
	LAGS_C for N10	-1.0605893E+04
4	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-1.7065781E+04
	LAGS_C for N35	-2.0872174E+04
	LAGS_C for N10	-1.6114706E+04

6.4 Remarks

Idem 4.4

7 Modeling E

7.1 Characteristics of modeling

They is the same characteristics of modeling as modeling A.

7.2 Characteristics of the grid

The grid is regulated (Figure 7.2-a) and comprises 3 blocks made up of meshs of the type TRI3. The blocks superior and inferior have each one 336 meshs, while the block medium has 608 of them.

The grid is finer than in the preceding tests, because one wants to avoid a loss of precision between the methods of contact FEM and X-FEM caused by the differences in grid. The grid is in addition built so as to respect the median symmetry of the model.

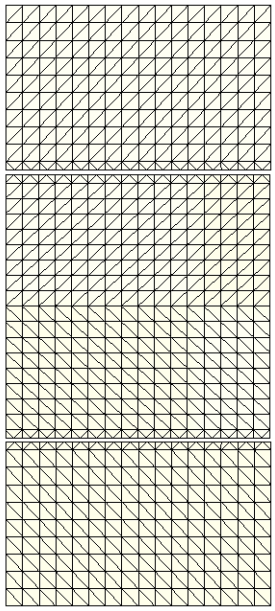

Figure 7.2-a: Grid of modeling E.

7.3 Sizes tested and results

One tests the values of the contact pressure as for modeling A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 7.4-a.

Not	Identification	Reference
1	LAGS_C for N1254	0.0000000E+00
	LAGS_C for N1262	-7.1942608E+03
	LAGS_C for N1270	-6.5408344E+03
	LAGS_C for N1278	-6.1360067E+03
	LAGS_C for N1286	-2.4914329E+03
2	LAGS_C for N1254	0.0000000E+00
	LAGS_C for N1262	-2.4822528E+04
	LAGS_C for N1270	-1.2993429E+04
	LAGS_C for N1278	-1.2178296E+04
	LAGS_C for N1286	-3.4642908E+03
	LAGS_C for N1254	0.0000000E+00

3	LAGS_C for N1262	0.0000000E+00
	LAGS_C for N1270	-2.0219781E+04
	LAGS_C for N1278	-1.8168929E+04
	LAGS_C for N1286	-4.8271686E+03
4	LAGS_C for N1254	0.0000000E+00
	LAGS_C for N1262	0.0000000E+00
	LAGS_C for N1270	-4.3387251E+04
	LAGS_C for N1278	-2.4197889E+04
	LAGS_C for N1286	-7.2647040E+03

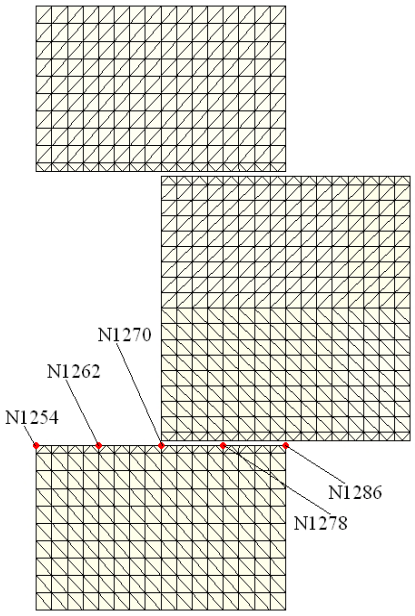

Figure 7.4-a: The site of the nodes which store the degrees of freedom of contact tested.

7.4 Remarks

This test is used as reference to modeling F.

8 Modeling F

8.1 Characteristics of modeling

They is the same characteristics of modeling as modeling B.

8.2 Characteristics of the grid

The grid is regulated (Figure 8.2-a) and comprises 1152 meshs of the type TRI3. Following the definition of the cracks, the 64 cut meshs store with their nodes the degrees of freedom of contact friction. The grid is built as for modeling E, so as to respect the median symmetry of the model.

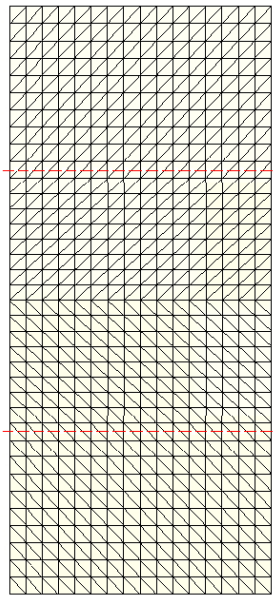

Figure 8.2-a: Grid of modeling F.

8.3 Sizes tested and results

One tests the values of contact pressure as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 8.4-a.

One graphically compares on the figure 8.4-b the results of approaches FEM and X-FEM.

Not	Identification	Reference
1	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	-7.1942608E+03
	LAGS_C for N230	-6.5408344E+03
	LAGS_C for N306	-6.1360067E+03
	LAGS_C for N677	-2.4914329E+03
2	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	-2.4822528E+04
	LAGS_C for N230	-1.2993429E+04
	LAGS_C for N306	-1.2178296E+04
	LAGS_C for N677	-3.4642908E+03
3	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	0.0000000E+00
	LAGS_C for N230	-2.0219781E+04

	LAGS_C for N306	-1.8168929E+04
	LAGS_C for N677	-4.8271686E+03
4	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	0.0000000E+00
	LAGS_C for N230	-4.3387251E+04
	LAGS_C for N306	-2.4197889E+04
	LAGS_C for N677	-7.2647040E+03

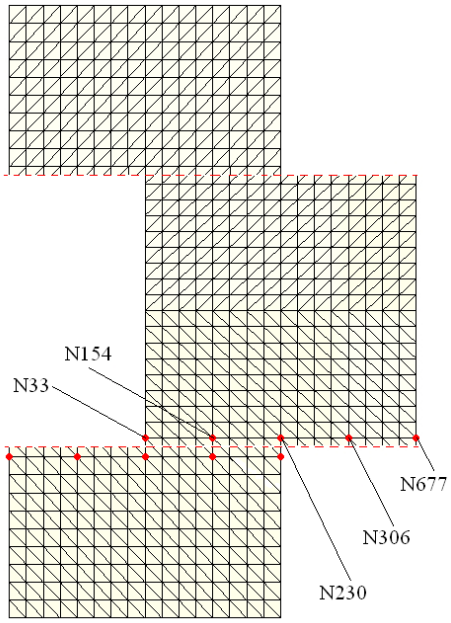

Figure 8.4-a: The site of the nodes which store the degrees of freedom of contact tested.

Figure 8.4-b: SSNP503-e and F: Comparison of the reactions of contact (4 pas de loads), for methods FEM (in blue) and XFEM (in red).

8.4 Remarks

These results validate in 2D the management of the integration of the contact with slipping friction when a linear relation between the degrees of freedom of contact friction is imposed on the way contacting/not contacting by the algorithm of stabilization of the LBB (see [R5 03 53] and [D9 05 06]).

So that results of the two approaches (methods of contact X-FEM and Classical FEM) corresponds, one imposes that the positions of the nodes of the grid as well as the meshs triangles (except the interface which is not with a grid for X-FEM) are the same ones, that the integration of the contact is made on the same segments, and that the integration of rigidity is made on the same triangles (except for the cut elements).

However by thus making, the interfaces of method of contact FEM have almost twice more degrees of freedom (contact and displacement) that those of method of contact X-FEM. One thus does not solve exactly the same digital problem in the 2 cases.

Taking account of that, one can conclude that one observes very good performances (relative differences lower than 0.5%) if one does not take account of the effects edges (relative differences of about 5% on the last node).

9 Modeling G

9.1 Characteristics of modeling

They is the same characteristics of modeling as modeling A, but in 3D. One thus extrudes the rectangle considered in the depth of the plan a unit length. One blocks the movements of rigid body in the new direction considered.

9.2 Characteristics of the grid

The grid is regulated (Figure9.2-a) and comprises 3 blocks made up of meshs of the type HEXA8. The blocks superior and inferior have each one 12 meshs, while the block medium has 20 of them.

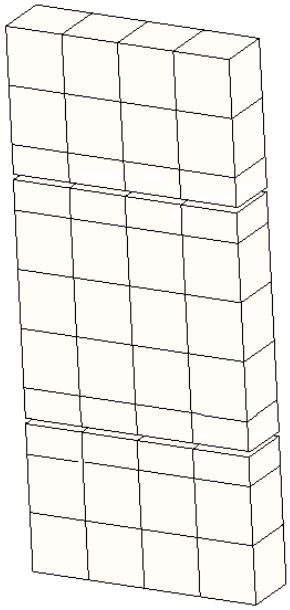

Figure 9.2-a: Grid of modeling G.

9.3 Sizes tested and results

One tests contact pressures as for modeling A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 9.4-a.

Not	Identification	Reference
1	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	-7.4394584E+03
	LAGS_C for N121	-5.5601371E+03
	LAGS_C for N123	-5.7894274E+03
	LAGS_C for N108	-3.8723295E+03
2	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	-9.9465544E+03
	LAGS_C for N121	-1.1457450E+04
	LAGS_C for N123	-1.0644747E+04
	LAGS_C for N108	-8.1152553E+03
3	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	0.0000000E+00
	LAGS_C for N121	-2.0437614E+04

	LAGS_C for N123	-1.6583379E+04
	LAGS_C for N108	-1.1024368E+04
4	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	0.0000000E+00
	LAGS_C for N121	-1.8510255E+04
	LAGS_C for N123	-2.1051214E+04
	LAGS_C for N108	-1.6537311E+04

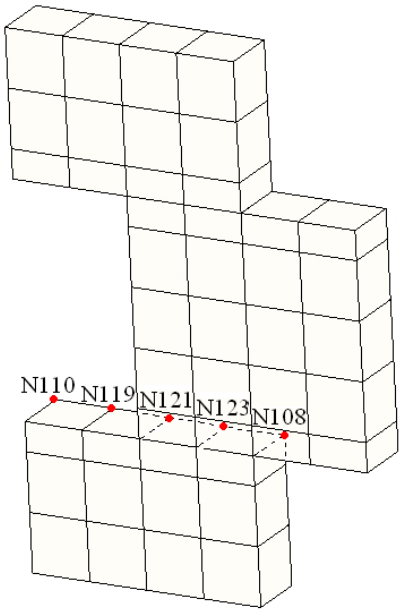

Figure 9.4-a: The site of the nodes which store the degrees of freedom of contact tested.

9.4 Remarks

This test is used as reference to modeling H.

10 Modeling H

10.1 Characteristics of modeling

They is the same characteristics of modeling as modeling B but in 3D. One thus extrudes the rectangle considered in the depth of the plan a unit length. One blocks the movements of rigid body in the new direction considered.

10.2 Characteristics of the grid

The grid is regulated (Figure 10.2-a) and comprises 36 meshs of the type HEXA8. Following the definition of the cracks, the 8 cut meshs store with their nodes the degrees of freedom of contact friction.

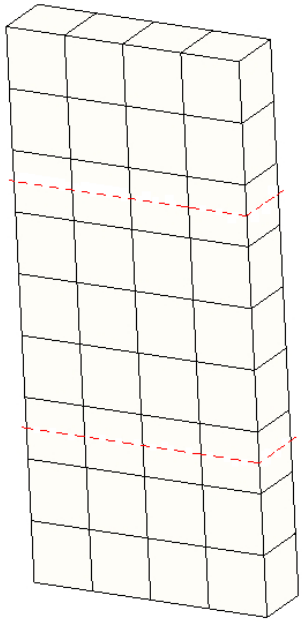

Figure 10.2-a: Grid of modeling H.

10.3 Sizes tested and results

One tests the values of contact pressure as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 10.4-a.

Not	Identification	Reference
1	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	-7.4394584E+03
	LAGS_C for N15	-5.5601371E+03
	LAGS_C for N16	-5.7894274E+03
	LAGS_C for N85	-5.8084941D+03
2	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	-9.9465544E+03
	LAGS_C for N15	-1.1457450E+04
	LAGS_C for N16	-1.0644747E+04
	LAGS_C for N85	-1.2172882D+04
	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	0.0000000E+00

3	LAGS_C for N15	-2.0437614E+04
	LAGS_C for N16	-1.6583379E+04
	LAGS_C for N85	-1.6536552D+04
4	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	0.0000000E+00
	LAGS_C for N15	-1.8510255E+04
	LAGS_C for N16	-2.1051214E+04
	LAGS_C for N85	-2.4805965D+04

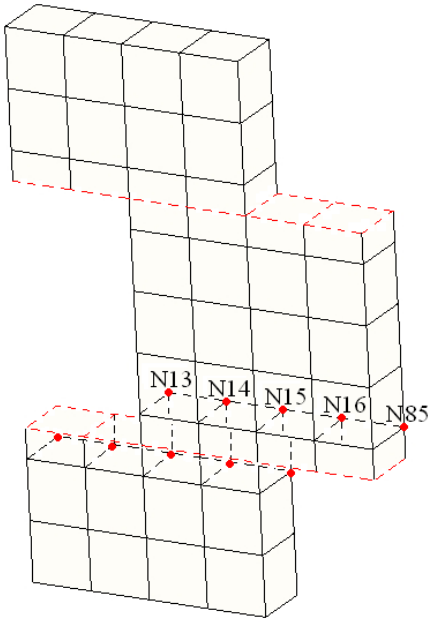

Figure 10.4-a: The site of the nodes which store the degrees of freedom of contact tested.

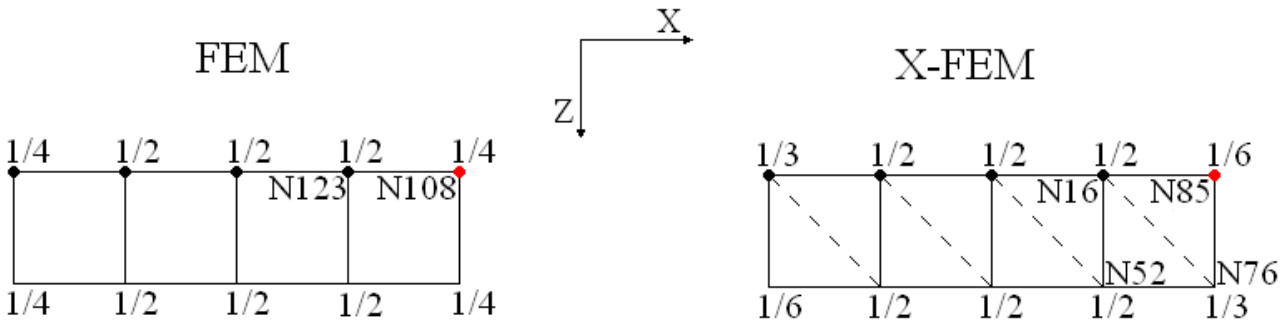

Figure 10.4-b: Surfaces (in m^2) on which the multipliers of contact of each node are divided into considering nodal integration (on the quadrangles for FEM, of the triangles for X-FEM).

10.4 Remarks

Idem 4.5 except that one does not integrate the contributions of contact on the same elements (see figure 10.4.b): in FEM the meshs of contact are quadrangles, in X-FEM the facets are triangles. That causes a dissymmetry for approach X-FEM between the nodes $N85$ (which contributes on a surface of $1/6\,m^2$) and $N76$ (which contributes on a surface of $1/3\,m^2$). The surfaces clarified on the figure 10.4-b act numerically like weights for the multipliers, one thus has logically $\frac{1}{6}\lambda_{N85}=\frac{1}{3}\lambda_{N76}$ that is to say $\lambda_{N85}=2\,\lambda_{N76}$.

So now one compares the N108 nodes of approach FEM and N85 of approach X-FEM, one can write $\frac{1}{6}\lambda_{N85}^{XFEM}=\frac{1}{4}\lambda_{N108}^{FEM}$ that is to say $\lambda_{N85}^{XFEM}=\frac{3}{2}\lambda_{N108}^{FEM}$.

The value LAGS_C of $N108$ coming from modeling G must thus be multiplied by 1.5 to be compared with LAGS_C of $N85$ modeling H. When one makes thus, one observes many differences close to zero ($10^{-3}\%$) on all the nodes tested.

11 Modeling I

11.1 Characteristics of modeling

They is the same characteristics of modeling as modeling G.

11.2 Characteristics of the grid

The grid is regulated (Figure 11.2-a) and comprises 3 blocks made up of meshs of the type PENTA6. The blocks superior and inferior have each one 24 meshs, while the block medium has 40 of them.

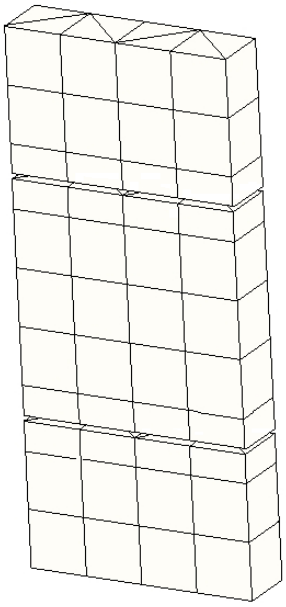

Figure 11.2-a: Grid of modeling I.

11.3 Sizes tested and results

One tests contact pressures as for modeling A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 11.4-a.

Not	Identification	Reference
1	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	-9.2184408E+03
	LAGS_C for N70	-5.3136225E+03
	LAGS_C for N73	-6.3318943E+03
	LAGS_C for N50	-3.8408176E+03
2	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	-1.5003217E+04
	LAGS_C for N70	-8.0031951E+03
	LAGS_C for N73	-1.4765678E+04
	LAGS_C for N50	-6.7122964E+03
3	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	0.0000000E+00
	LAGS_C for N70	-1.7575098E+04
	LAGS_C for N73	-1.6339667E+04
	LAGS_C for N50	-1.2605826E+04

4	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	0.0000000E+00
	LAGS_C for N70	-1.7152487E+04
	LAGS_C for N73	-2.1471849E+04
	LAGS_C for N50	-1.7685794E+04

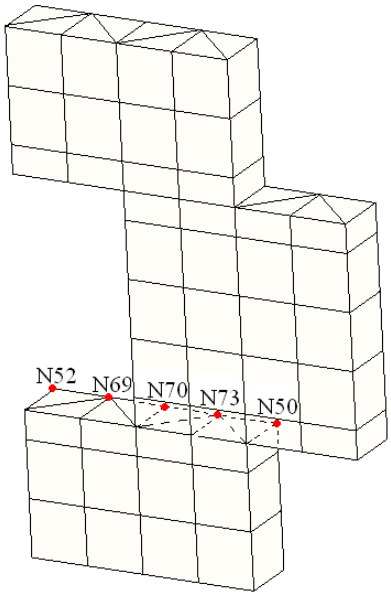

Figure 11.4-a: The site of the nodes which store the degrees of freedom of contact tested.

11.4 Remarks

This test is used as reference to modeling J.

12 Modeling J

12.1 Characteristics of modeling

They is the same characteristics of modeling as modeling H.

12.2 Characteristics of the grid

The grid is regulated (Figure 12.2-a) and comprises 72 meshs of the type PENTA6. Following the definition of the cracks, the 16 cut meshs store with their nodes the degrees of freedom of contact friction.

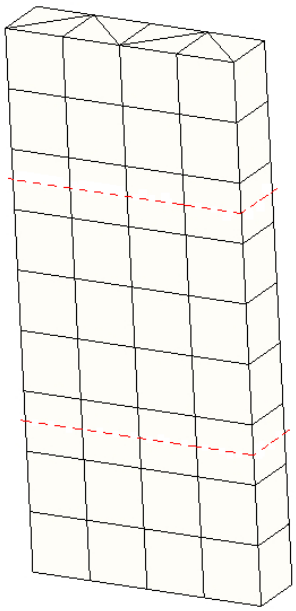

Figure 12.2-a: Grid of modeling J.

12.3 Sizes tested and results

One tests the values of contact pressure as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 12.4-a.

Not	Identification	Reference
1	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	-9.2184408E+03
	LAGS_C for N22	-5.3136225E+03
	LAGS_C for N23	-6.3318943E+03
	LAGS_C for N85	-3.8408176E+03
2	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	-1.5003217E+04
	LAGS_C for N22	-8.0031951E+03
	LAGS_C for N23	-1.4765678E+04
	LAGS_C for N85	-6.7122964E+03
3	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N22	-1.7575098E+04
	LAGS_C for N23	-1.6339667E+04

	LAGS_C for N85	-1.2605826E+04
4	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N22	-1.7152487E+04
	LAGS_C for N23	-2.1471849E+04
	LAGS_C for N85	-1.7685794E+04

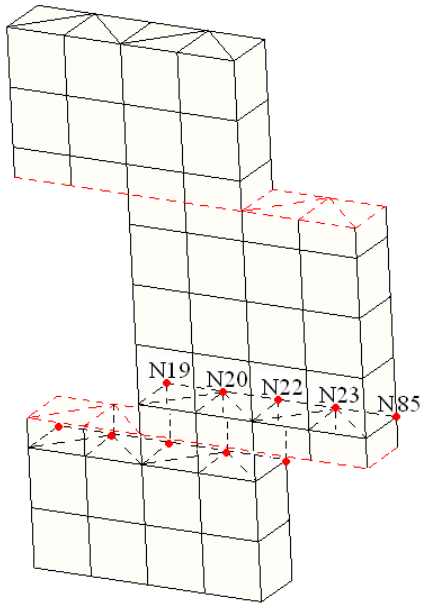

Figure 12.4-a: The site of the nodes which store the degrees of freedom of contact tested.

12.4 Remarks

Idem 4.5. One observes differences close to zero ($10^{-3}\%$).

13 Modeling K

13.1 Characteristics of modeling

They is the same characteristics of modeling as modeling G.

13.2 Characteristics of the grid

The grid is regulated (Figure 13.2-a) and comprises 3 blocks made up of meshs of the type TETRA4. The blocks superior and inferior have each one 1098 meshs, while the block medium has 1824 of them.

The grid is finer than in the preceding tests, because one wants to limit a loss of precision between the methods of contact FEM and X-FEM caused by the differences in grid. The grid is in addition built so as to respect the median symmetry of the model.

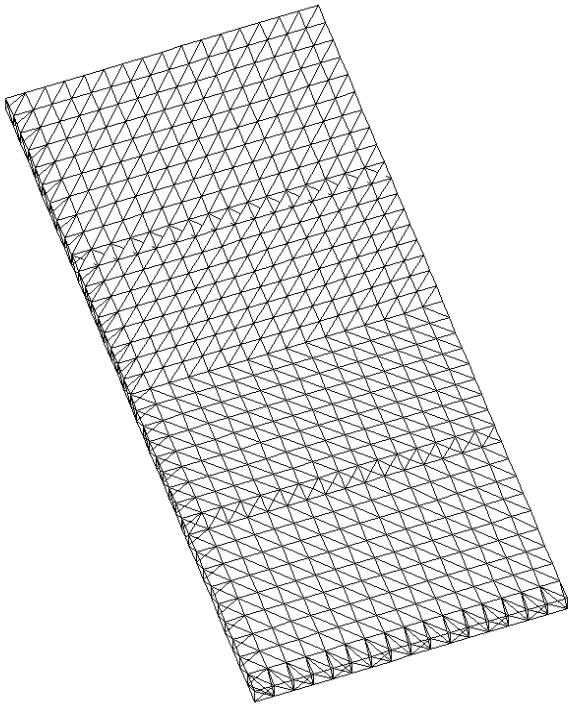

Figure 13.2-a: Grid of modeling K.

13.3 Sizes tested and results

One tests contact pressures as for modeling A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 13.4-a.

Not	Identification	Reference
1	LAGS_C for N3779	0,00E+000
	LAGS_C for N3802	-7,24E+003
	LAGS_C for N3826	-6,43E+003
	LAGS_C for N3850	-5,50E+003
	LAGS_C for N3874	-9,82E+002
2	LAGS_C for N3779	0,00E+000
	LAGS_C for N3802	-2,64E+004
	LAGS_C for N3826	-1,28E+004

	LAGS_C for N3850	-1,11E+004
	LAGS_C for N3874	-1,01E+003
3	LAGS_C for N3779	0,00E+000
	LAGS_C for N3802	0,00E+000
	LAGS_C for N3826	-1,93E+004
	LAGS_C for N3850	-1,68E+004
	LAGS_C for N3874	-1,57E+003
4	LAGS_C for N590	0,00E+000
	LAGS_C for N598	0,00E+000
	LAGS_C for N606	-4,43E+004
	LAGS_C for N614	-2,25E+004
	LAGS_C for N622	-3,17E+003

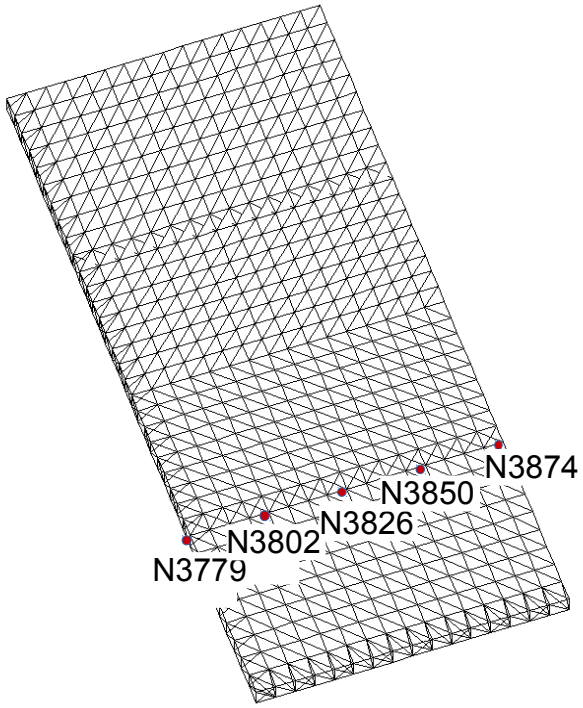

Figure 13.4-a: The site of the nodes which store the degrees of freedom of contact tested.

13.4 Remarks

This test is used as reference to modeling L.

14 Modeling L

14.1 Characteristics of modeling

They is the same characteristics of modeling as modeling H.

14.2 Characteristics of the grid

The grid is regulated (Figure 14.2-a) and comprises 3456 meshs of the type TETRA4. Following the definition of the cracks, the 192 cut meshs store with their nodes the degrees of freedom of contact friction. The grid is built as for modeling K, so as to respect the median symmetry of the model.

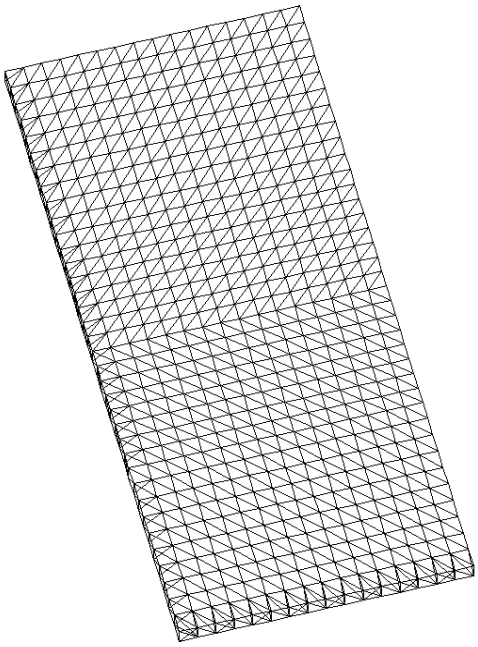

Figure 14.2-a: Grid of modeling L.

14.3 Sizes tested and results

One tests the values of contact pressure as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 14.4-a.
One graphically compares on the figure 14.4-b the results of approaches FEM and X-FEM.

Not	Identification	Reference
1	LAGS_C for NM33	0,00E+000
	LAGS_C for NM394	-7,24E+003
	LAGS_C for NM842	-6,43E+003
	LAGS_C for NM1290	-5,50E+003
2	LAGS_C for NM33	0,00E+000
	LAGS_C for NM394	-2,64E+004
	LAGS_C for NM842	-1,28E+004
	LAGS_C for NM1290	-1,11E+004
3	LAGS_C for NM33	0,00E+000
	LAGS_C for NM394	0,00E+000
	LAGS_C for NM842	-1,93E+004
	LAGS_C for NM1290	-1,68E+004

4	LAGS_C for NM33	0,00E+000
	LAGS_C for NM394	0,00E+000
	LAGS_C for NM842	-4,43E+004
	LAGS_C for NM1290	-2,25E+004

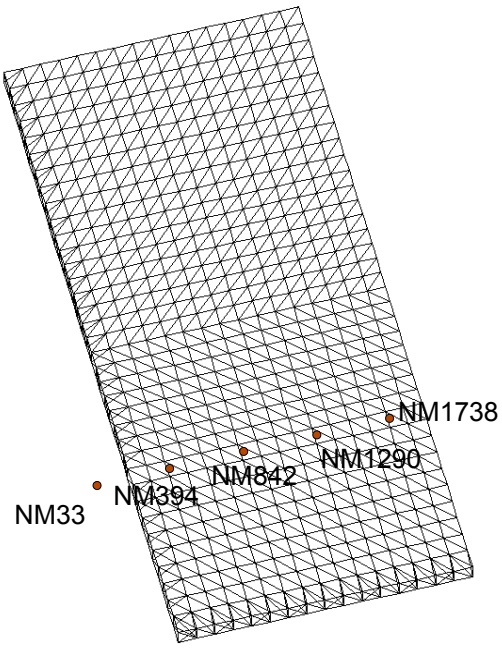

Figure 14.4-a: The site of the nodes which store the degrees of freedom of contact tested.

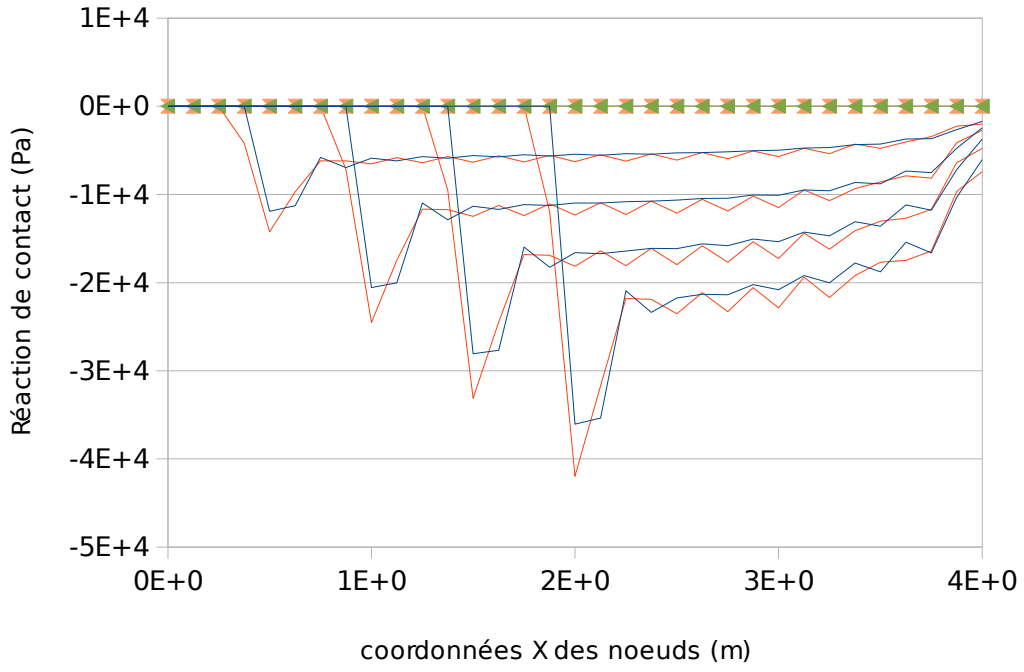

Figure 14.4-b: SSNP503-k and L: Comparison of the reactions of contact (4 pas de loads), for methods FEM (in blue) and XFEM (in red).

14.4 Remarks

These results validate in 3D the management of the integration of the contact with slipping friction when a linear relation between the degrees of freedom of contact friction is imposed on the way contacting/not contacting by the algorithm of stabilization of the LBB (see [R5 03 53] and [D9 05 06]).

So that results of the two approaches (methods of contact X-FEM and Classical FEM) corresponds, one imposes that the positions of the nodes of the grid as well as the tetrahedral meshs (except the interface which is not with a grid for X-FEM) are the same ones, that the integration of the contact is made on the same triangles, and that the integration of rigidity is made on the same tetrahedrons (except for the cut elements).
However by thus making, the interfaces of method of contact FEM have almost twice more degrees of freedom (contact and displacement) that those of method of contact X-FEM. One thus does not solve exactly the same digital problem in the 2 cases.

Taking account of that, one observes correct results which make it possible to validate the method (relative differences lower than 5%) if one does not take account of the effects edges (differences relative of about 20% on the points at the ends).

Nevertheless, these results are not as good as in 2D (comparison of modelings E and F). One can justify it by the fact that the reduction of the algorithm of the LBB is more severe on tetrahedrons than on triangles.

15 Modeling M

15.1 Characteristics of modeling

It is about a modeling X-FEM in small slips (one does not use the keyword ITER_GEOM_MAXI in the operator CONTACT).

It has the same characteristics of modeling as modeling B, but the two cracks have a point (see figure 15.1-a). One thus introduces into the modeling of the cracks the level sets tangents (noted LT). The points are positioned in $\mathrm{LT}=0$.

In order to test all the possible types of pairing is the late elements $H-H$, $HCT-H$, $H-HCT$, $HCT-HCT$ and CT (see part 3 of [R5.03.53]), one inverts for the second fissures the statutes Master/slave by changing the sign of $LN2$.

The first point (defined by $LT1$) is located inside the element, whereas the second (defined by $LT2$) is located on the edge of the element.

Equations 4.1-1 and 4.1-2 defining the level set initially are replaced by the following ones:

$$LN1=Y-2{,}5 \qquad \text{éq 15.1-1}$$
$$LT1=X-2{,}5 \qquad \text{éq 15.1-2}$$
$$LN2=Y-6{,}5 \qquad \text{éq 15.1-3}$$
$$LT2=X-3 \qquad \text{éq 15.1-4}$$

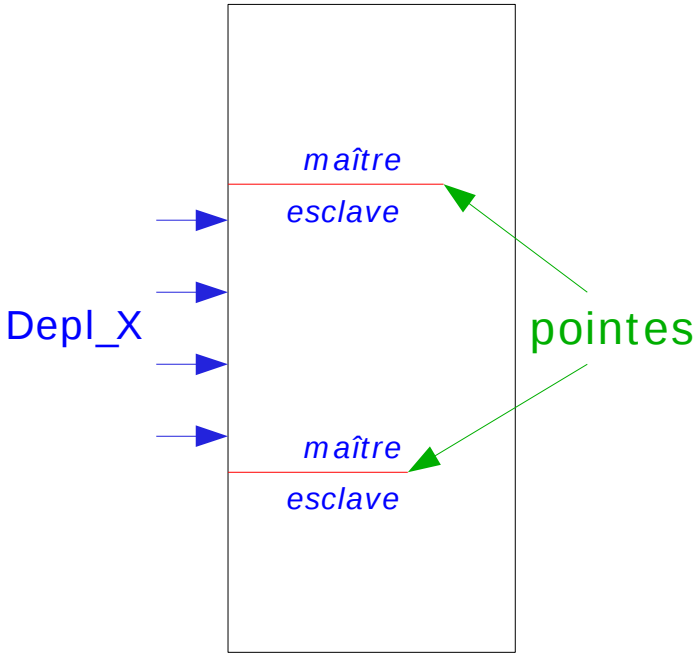

Figure 15.1-a: The site of the cracks for modeling Mr.

Lastly, the introduction of the funds of crack does not make it possible any more to impose a too large displacement according to X; one imposes now:
$Depl_X=0.20\,m$

15.2 Characteristics of the grid

Idem modeling B

15.3 Sizes tested and results

One tests contact pressures as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 4.4-a.

Not	Identification	*Reference*
1	LAGS_C for N24	-238385.373622
	LAGS_C for N33	-67269.0287535
	LAGS_C for N34	-44200.938843
2	LAGS_C for N24	-476772.622857
	LAGS_C for N33	-134537.599488
	LAGS_C for N34	-88401.7804325
3	LAGS_C for N24	-715159.872099
	LAGS_C for N33	-201806.170221
	LAGS_C for N34	-132602.622022
4	LAGS_C for N24	-953547.121341
	LAGS_C for N33	-269074.740954
	LAGS_C for N34	-176803.463611

15.4 Remarks

This test is used as test of nonregression for the contact with point in 2D small slip.

16 Modeling NR

16.1 Characteristics of modeling

They is the same characteristics of modeling as modeling M, but in great slips.

16.2 Characteristics of the grid

Idem modeling B

16.3 Sizes tested and results

One tests contact pressures as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 4.4-a.

Not	Identification	*Reference*
1	LAGS_C for N24	-235733.727207
	LAGS_C for N33	-67363.6213534
	LAGS_C for N34	-54390.634508
2	LAGS_C for N24	-465195.565576
	LAGS_C for N33	-133858.368841
	LAGS_C for N34	-110332.5788
3	LAGS_C for N24	-688183.601691
	LAGS_C for N33	-199310.458365
	LAGS_C for N34	-167912.167359
4	LAGS_C for N24	-904516.410316
	LAGS_C for N33	-263543.323143
	LAGS_C for N34	-227227.354139

16.4 Remarks

The only difference with modeling M is that for the great slips (modeling NR) a phase of D-pairing is introduced. This explains the differences in results between the two methods, since into our case one introduces a considerable slip compared to the size of the structure. It is also noticed that the difference increases linearly compared to the imposed slip. Moreover by not imposing any slip on these two CAS-tests, the difference becomes worthless.

As this test does not have an analytical solution, it is difficult to determine if the improvement of the model introduced by the great slip has a significant impact on the precision Dbe results (field of displacements, stress field) in the vicinity of the point of crack. Nevertheless, this test makes it possible to cover the functionality “great slip” in the elements at a peak of crack. It is thus about one test in not pure regression.

17 Modeling O

17.1 Characteristics of modeling

They is the same characteristics of modeling as modeling M, but in 3D. One thus extrudes the rectangle considered in the depth of the plan a unit length. One blocks the movements of rigid body in the new direction considered.

17.2 Characteristics of the grid

Idem modeling H.

17.3 Sizes tested and results

One tests contact pressures as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 10.4-a.

Not	Identification	*Reference*
1	LAGS_C for NR13	-1.6373123667042E+05
	LAGS_C for NR14	-44357.544505499
	LAGS_C for NR15	-35056.97086267
2	LAGS_C for NR13	-3.2746337860021E+05
	LAGS_C for NR14	-88714.735602067
	LAGS_C for NR15	-70113.84839032
3	LAGS_C for NR13	-4.9119551982913E+05
	LAGS_C for NR14	-1.3307192699716E+05
	LAGS_C for NR15	-105170.72552738
4	LAGS_C for NR13	-6.5492051932078E+05
	LAGS_C for NR14	-1.7743185103176E+05
	LAGS_C for NR15	-140228.33441009

17.4 Remarks

This test is used as test of nonregression for the contact with bottom of crack in 3D small slip.

18 Modeling P

18.1 Characteristics of modeling

They is the same characteristics of modeling as modeling O, but in great slips.

18.2 Characteristics of the grid

Idem modeling H.

18.3 Sizes tested and results

One tests contact pressures as for modeling B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 10.4-a.

Not	Identification	*Reference*
1	LAGS_C for NR13	-1.6238227696502E+05
	LAGS_C for NR14	-44471.272360813,
	LAGS_C for NR15	-47290.249552795
2	LAGS_C for NR13	-3.2167144997233E+05
	LAGS_C for NR14	-88579.687943577
	LAGS_C for NR15	-94685.282753714
3	LAGS_C for NR13	-4.7773723286992E+05
	LAGS_C for NR14	-1.3221454002148E+05
	LAGS_C for NR15	-142192.34718408
4	LAGS_C for NR13	-6.3045892009397E+05
	LAGS_C for NR14	-1.7526305992804E+05
	LAGS_C for NR15	-189824.71053565

18.4 Remarks

Idem modeling NR.

19 Modeling Q

19.1 Characteristics of modeling

They is the same characteristics of modeling as modeling F but by using the penalized formulation.

19.2 Characteristics of the grid

It is the same grid as that of modeling F.

19.3 Sizes tested and results

One tests the values of contact pressure with the same values of reference and tolerances that modeling F. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 8.4-a. It is checked that one gets results identical to the Lagrangian method.

Not	Identification	Reference	tolerance
1	LAGS_C for N33	0.0000000E+00	1,00E-010
	LAGS_C for N154	-7.1942608E+03	5,00%
	LAGS_C for N230	-6.5408344E+03	5,00%
	LAGS_C for N306	-6.1360067E+03	5,00%
	LAGS_C for N677	-2.4914329E+03	5,00%
2	LAGS_C for N33	0.0000000E+00	1,00E-010
	LAGS_C for N154	-2.4822528E+04	5,00%
	LAGS_C for N230	-1.2993429E+04	5,00%
	LAGS_C for N306	-1.2178296E+04	5,00%
	LAGS_C for N677	-3.4642908E+03	5,00%
3	LAGS_C for N33	0.0000000E+00	1.00E-10
	LAGS_C for N154	0.0000000E+00	1.00E-10
	LAGS_C for N230	-2.0219781E+04	5,00%
	LAGS_C for N306	-1.8168929E+04	5,00%
	LAGS_C for N677	-4.8271686E+03	6,50%
4	LAGS_C for N33	0.0000000E+00	0.00E+00
	LAGS_C for N154	0.0000000E+00	0.00E+00
	LAGS_C for N230	-4.3387251E+04	5,00%
	LAGS_C for N306	-2.4197889E+04	5,00%
	LAGS_C for N677	-7.2647040E+03	6,50%

20 Summary of the results

The goals of this test are achieved:

- It is a question of showing the feasibility of the taking into account of the contact and friction on the lips of the horizontal cracks with the approach great slips X-FEM.
- One also shows in 2D and 3D the effectiveness of the algorithms implemented to improve the results when the linear relations introduced on the degrees of freedom of contact by algorithm LBB enter in conflict with a contacting change of status/not contacting (modeling F in 2D and modeling L in 3D).
- The approach was validated with the taking into account of frictions in 2D (plane strains and plane stresses, elements QUAD4 and TRI3) and in 3D (elements HEXA8, PENTA6 and TETRA4)

- It is possible to define interfaces (modelings B, D, F, H, J and L) but also of the cracks not cutting entirely the structure (modelings NR and P).