

SSNV102 - Tensile test shearing with the model of TAHERI

Summary:

The problem is quasi-static nonlinear in mechanics of the structures.

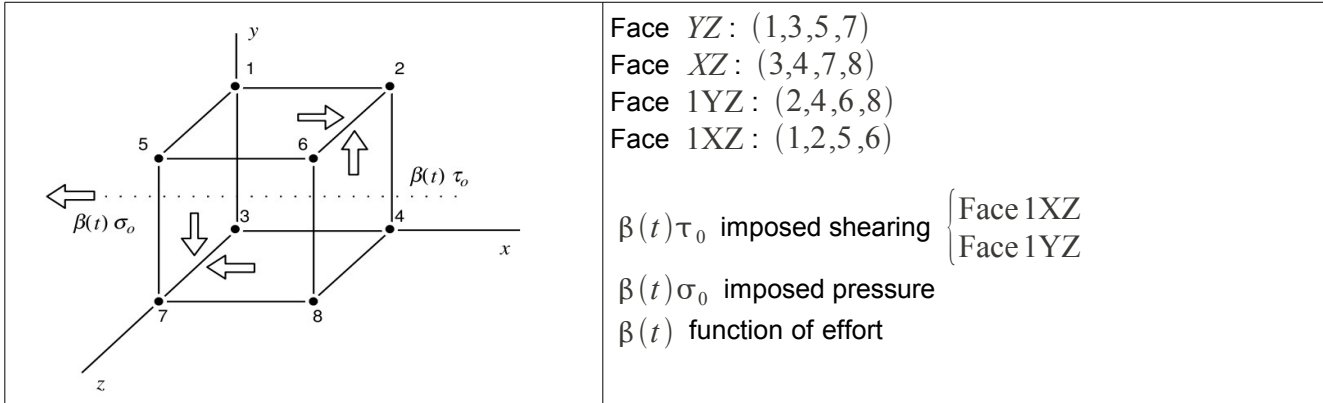
One analyzes the response of an element of volume to a loading in traction-shearing, carried out in such way that imposes a state of uniform stress-strain in the element.

There are 2 modelings: one in 3D voluminal and another in plane constraints 2D .

One validates by this test the digital integration of the elastoplastic model of behavior of Saïd Taheri.

1 Problem of reference

1.1 Geometry

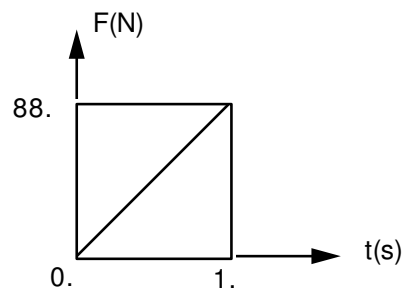


1.2 Material properties

isotropic elasticity	$E = 200\,000\text{ MPa}$	$\nu = 0,3$		
Taheri plasticity	$C_{inf} = 0.065\text{ MPa}$	$C_1 = -0.012\text{ Mpa}$	$s = 450$	$b = 30$
	$m = 0.1$	$a = 312$	$\alpha = 0.3$	$R_o = 72$

1.3 Boundary conditions and loadings

N04	$dx = dy = 0$	Face YZ :	$F_X = F_Y = -F(t)$
N08	$dx = dy = dz = 0$	Face XZ :	$F_X = -F(t)$
N02, N06	$dx = 0$	Face 1YZ :	$F_Y = F(t)$
		Face 1XZ :	$F_X = F(t)$



1.4 Initial conditions

Worthless constraints and deformations with $t = 0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

One integrates the following system numerically enters $t=0$ and $t=1$.

(Nonlinear ordinary Differential connection of 6 equations to 6 unknown factors solved using library NAG by a 'Backward difference method')

$$\dot{\varepsilon} - \dot{\varepsilon}_p = \dot{\beta} \frac{\sigma_o}{E} \quad \text{éq 2.1- 1}$$

$$\dot{\gamma} - \dot{\gamma}_p = \dot{\beta} \frac{\tau_o}{2\mu} \quad \text{éq 2.1- 2}$$

$$\dot{\varepsilon}_p - \dot{p} \frac{\partial F}{\partial \sigma} = 0 \quad \text{éq 2.1- 3}$$

$$\dot{\gamma}_p - \dot{p} \frac{\partial F}{\partial \tau} = 0 \quad \text{éq 2.1- 4}$$

$$\begin{aligned} & - \frac{3}{2} \frac{\partial F}{\partial \sigma} \left(Kx + C_s \frac{\partial F}{\partial \sigma} \right) - 2 \frac{\partial F}{\partial \tau} \left(Ky + C_s \frac{\partial F}{\partial \tau} \right) - HR - a D \alpha Z^{(\alpha-2)} \\ & \left(\varepsilon_p - \varepsilon_p^n \right) \frac{\partial F}{\partial \sigma} + \frac{4}{3} \left(\gamma_p - \gamma_p^n \right) \frac{\partial F}{\partial \tau} + \dot{p} + \frac{3}{2} \frac{\partial F}{\partial \sigma} \left(Qx + C \varepsilon_p^n \right) + 2 \frac{\partial F}{\partial \tau} \left(Qy + C \gamma_p^n \right) + JR \dot{\sigma}_p \\ & = - \frac{\partial F}{\partial \sigma} \dot{\beta} \sigma_o - 2 \frac{\partial F}{\partial \tau} \dot{\beta} \tau_o \end{aligned} \quad \text{éq 2.1- 5}$$

$$\begin{aligned} & 1 + JR + \frac{3C}{2U} \left(\frac{3}{2} C \left(\varepsilon_p - \sigma_p \varepsilon_p^n \right) \varepsilon_p^n + 2C \left(s \gamma_p - \sigma_p \gamma_p^n \right) \gamma_p^n \right) \dot{\sigma}_p \\ & - \left[HR + a D \alpha Z^{(\alpha-2)} \left(\left(\varepsilon_p - \varepsilon_p^n \right) \frac{\partial F}{\partial \sigma} + \frac{4}{3} \left(\gamma_p - \gamma_p^n \right) \frac{\partial F}{\partial \tau} \right) + KX \right. \\ & \left. + 3 \frac{CS}{2U} \left(\frac{3}{2} C \left(s \varepsilon_p - \sigma_p \varepsilon_p^n \right) \frac{\partial F}{\partial \sigma} + 2C \left(s \gamma_p - \sigma_p \gamma_p^n \right) \frac{\partial F}{\partial \tau} \right) \dot{p} \right] = 0 \end{aligned} \quad \text{éq 2.1- 6}$$

<p>avec</p> $u = b \left(1 - \frac{\sigma_p}{S} \right)$ $v = \frac{C_\infty - C}{C}$ $D = 1 - m e^{-up} \quad w = \frac{1 - D}{D}$ $C = C_\infty - C_1 e^{-up}$ $K = v u \quad Q = v \frac{bp}{s}$ $H = w u \quad J = w \frac{bp}{s}$	<p>et</p> $X = C \left(s \varepsilon_p - \sigma_p \varepsilon_p^n \right)$ $Y = C \left(s \gamma_p - \sigma_p \gamma_p^n \right)$ $U = \left[\frac{9}{4} X^2 + 3 Y^2 \right]^{1/2}$ $R = D \left(a Z^\alpha + r_0 \right)$ $Z = \left[\left(\varepsilon_p - \varepsilon_p^n \right)^2 + \frac{4}{3} \left(\gamma_p - \gamma_p^n \right)^2 \right]^{1/2}$
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with the initial conditions:

$$\beta(0) = \frac{R(0)}{(\sigma_o^2 + 3\tau_o^2)^{1/2}}$$

$$\varepsilon(0) = \beta(0) \frac{\sigma_o}{E}$$

$$\gamma(0) = \beta(0) \frac{\tau_o}{2\mu}$$

$$p(0) = \varepsilon_p(0) = \varepsilon_p^n = 0$$

$$R(0) = (1-m)r_o = \sigma_p(o)$$

$$\text{d'où } \sigma(t=1) = \begin{pmatrix} 88. & 88. & 0 \\ 88. & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.2 Results of reference

Values of $\varepsilon, \gamma, \varepsilon_p, \gamma_p, P$ and σ_p with the nodes with $t=1$ s .

2.3 Uncertainty on the solution

Uncertainty of library NAG.

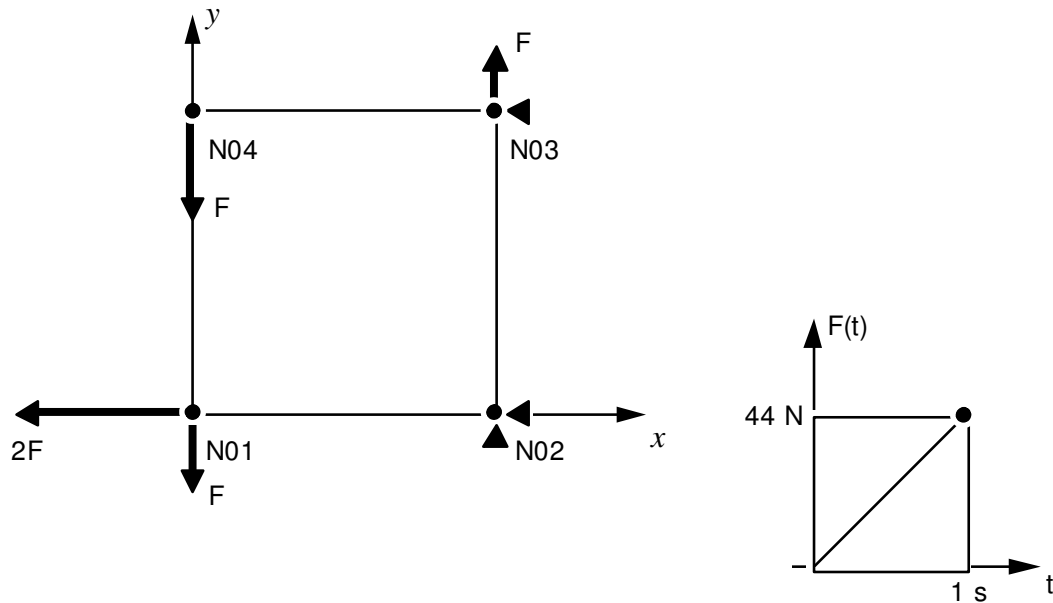
2.4 Bibliographical references

- 1) User's manual library NAG on CRAY.
- 2) S. ANDRIEUX - P. SCHOENBERGER - S. TAHERI: With three dimensional cyclic constitutive law for metals with has semi-discrete memory variable - HI - 71/8147 (1992)
- 3) P. GEYER - J.M. PROIX - P. SCHOENBERGER - S. TAHERI: Modeling of the phenomena of progressive deformation - Collection of the internal notes of DER 93NB00153

3 Modeling A

3.1 Characteristics of modeling

Modeling in plane constraints 2D , C_PLAN



3.2 Characteristics of the grid

Square quadrangle with 4 nodes in plane constraints with:

- *largeur* = 1 mm ,
- *épaisseur* = 1 mm .

3.3 Sizes tested and results

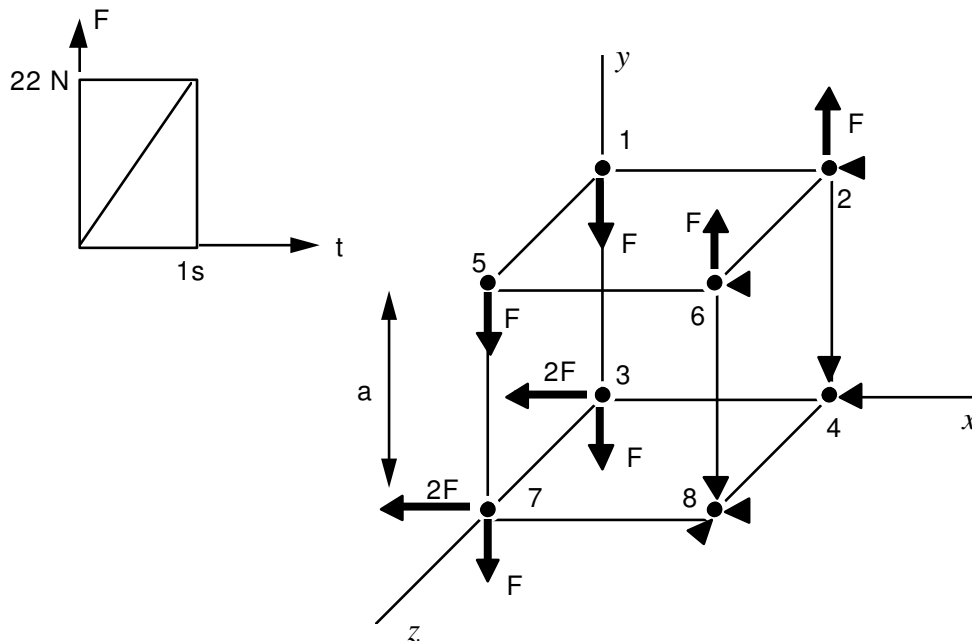
Identification	Reference	Test	Tolerance
ε on NO4 with $t=1s$	0.01721	ANALYTICAL	2,5%
γ on NO4 with $t=1s$	0.02573	ANALYTICAL	2,5%
ε_p on MA1 , not of Gauss 4 with $t=1s$	0.01678	ANALYTICAL	2,5%
γ_p on MA1 , not of Gauss 4 with $t=1s$	0.02515	ANALYTICAL	2,5%
ε_p on NO4 with $t=1s$	0.01678	ANALYTICAL	2,5%
γ_p on NO4 with $t=1s$	0.02515	ANALYTICAL	2,5%
σ_p on NO1 with $t=1s$	176.0	ANALYTICAL	0,10%

4 Modeling B

4.1 Characteristics of modeling

Modeling 3D :

Cubic elementary with a grid using a hexahedron with 8 nodes.



4.2 Characteristics of the grid

1 mesh HEXA8, width side $a = 1 \text{ mm}$.

4.3 Sizes tested and results

4.3.1 Values tested

Identification	Reference	Test	Tolerance
ε on NO4 with $t = 1\text{s}$	0.01721	ANALYTICAL	2,5%
γ on NO4 with $t = 1\text{s}$	0.02573	ANALYTICAL	2,5%
ε_p on MA1, not of Gauss 1 with $t = 1\text{s}$	0.01678	ANALYTICAL	2,5%
γ_p on MA1, not of Gauss 1 with $t = 1\text{s}$	0.02515	ANALYTICAL	2,5%
ε_p on NO4 with $t = 1\text{s}$	0.01678	ANALYTICAL	2,5%
γ_p on NO4 with $t = 1\text{s}$	0.02515	ANALYTICAL	2,5%
σ_p on NO1 with $t = 1\text{s}$	176.0	ANALYTICAL	0,10%

4.3.2 Remarks

The reduction in the tolerance on total convergence in displacement does not bring significant profit in precision.

The number of increments of load (6) led to a satisfactory precision of the result.

5 Summary of the results

Good precision at the time of the comparison with NAG in spite of some difficulties of convergence with this mathematical library.