SSNV103 - Tensile test shearing model of Rousselier

Summary:

It is about a nonlinear quasi-static problem in mechanics of the structures.

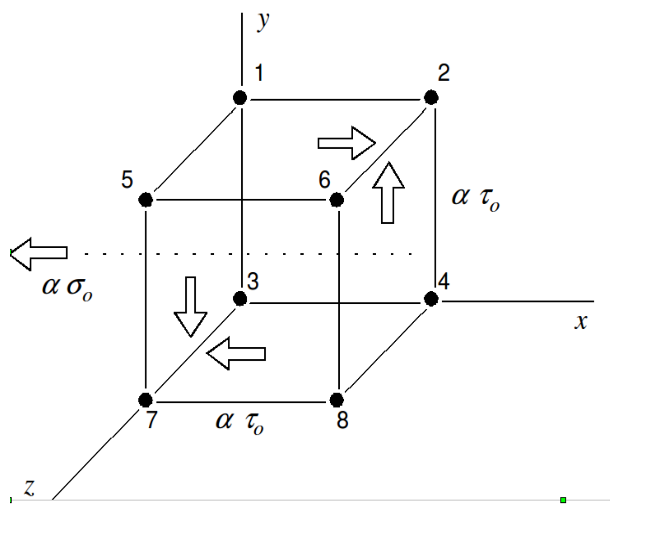
One analyzes the response of an element of volume to a loading in traction-shearing, carried out in such way that imposes a state of uniform stress-strain.

The case test understands 1 modeling: in $3D$.

It validates the digital integration of the elastoplastic model of behavior with damage of G. Rousselier.

1 Problem of reference

1.1 Geometry



Face YZ: $(1,3,5,7)$
Face XZ: $(3,4,7,8)$
Face $1YZ$: $(2,4,6,8)$
Face $1XZ$: $(1,2,5,6)$

$\alpha\tau_0$ imposed shearing $\begin{cases}\text{Face } 1XZ \\ \text{Face } 1YZ\end{cases}$

$\alpha\sigma_0$ imposed pressure Face YZ

$\alpha(t)$ function of effort

1.2 Material properties

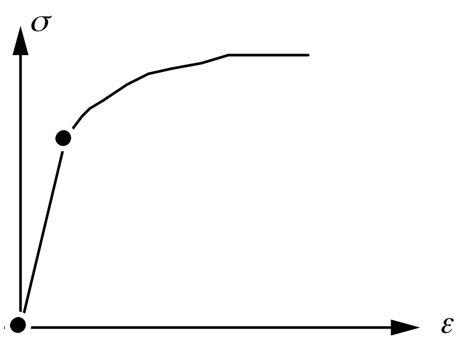
isotropic elasticity: $E=206\,400.\text{MPa}$ $\nu=0.3$

plasticity:
(coefficients of the model of Rousselier)
$D=2.$
$f_0=5.10^{-4}$
$\sigma_1=490.\text{MPa}$

The rational traction diagram entered point by point with:

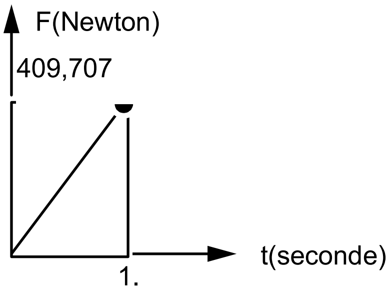
$$R(p)=r_i+(r_0-r_i)e^{-bp}$$

with p : cumulated plastic deformation
and $r_i=1500\ MPa$
$r_0=520.\text{MPa}$
$b=2.4$



1.3 Boundary conditions and loadings

$N04$	$dx=dy=0$	Face YZ :	$FX=FY=-F(t)$
$N08$	$dx=dy=dz=0$	Face XZ :	$FX=-F(t)$
$N02$, $N06$	$dx=0$	Face $1YZ$:	$FY=F(t)$
		Face $1XZ$:	$FX=F(t)$



1.4 Initial conditions

Worthless constraints and deformations with $t=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The model $3\mathrm{D}$ in speed is written:

$$\begin{cases} \dot\sigma - \dot\rho\,\Lambda\,\varepsilon_e - \rho\,\Lambda\,\dot\varepsilon_e & = 0 \quad (\Lambda \text{ tenseur élasticité isotrope linéaire}) \\ \dot\beta - \dot p\, D \exp\left(\dfrac{\sigma_H}{\sigma_1 \rho}\right) & = 0 \\ \dot\varepsilon - \dot\varepsilon_e - \rho\,\dot p \dfrac{\partial f}{\partial \sigma} & = 0 \\ \dot f & = 0 \end{cases}$$

what, in the case of a loading of imposed traction-shearing $\left(\sigma(t)=\alpha(t)\begin{bmatrix}\sigma_0 & \tau_0 & 0\\ \tau_0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right)$ conduit to integrate a system of 6 ordinary differential equations in $y=(\varepsilon, \gamma, \varepsilon_e, \gamma_e, \beta, p)$ form $A(y,t)\dot y = G(y,t)$.

$$(S)\quad \begin{cases} \dot\alpha\,\sigma_o + \rho^2\, F_o\, e^{\beta}\, E\,\varepsilon_e\,\dot\beta - \rho\, E\,\dot\varepsilon_e = 0 \\ \dot\alpha\,\tau_o + \rho^2\, F_o\, e^{\beta}\, 2\mu\,\gamma_e\,\dot\beta - 2\rho\,\mu\,\dot\gamma_e = 0 \\ \dot\varepsilon - \dfrac{\sigma_o}{\sigma_{eq\ o}}\rho\,\dot p - \dot\varepsilon_e = 0 \\ \dot\gamma - \dfrac{3\,\tau_o}{2\,\sigma_{eq\ o}}\rho\,\dot p - \dot\gamma_e = 0 \\ \dot\beta - \dot p\, D\, \exp\left(\dfrac{\sigma_H}{\sigma_1\,\rho}\right) = 0 \\ \dot\alpha\,\sigma_o\left(\dfrac{\sigma_o}{\rho\,\sigma_{eq\ o}} + \dfrac{1}{3}D\,F_o\,e^{\beta}\exp\left(\dfrac{\sigma_H}{\sigma_1\,\rho}\right)\right) + 3\,\dot\alpha\,\dfrac{\tau_o^2}{\rho\,\sigma_{eq\ o}} - \dfrac{\partial R}{\partial p}\dot p \\ \quad + \left(\sigma_{eq\ o}\, F_o\, e^{\beta} + D\,\sigma_1\,\rho\, F_o\, e^{\beta}\exp\left(\dfrac{\sigma_H}{\sigma_1\,\rho}\right)\left(1-\rho\, F_o\, e^{\beta}\left(1-\dfrac{\sigma_H}{\sigma_1\,\rho}\right)\right)\right)\dot\beta = 0 \end{cases}$$

with $t=0$:

$$f=0,\ \rho(0)=1,\ \beta(0)=0$$

from where:

$$\alpha(0)\,\sigma_{eq0} - R(0) + D\sigma_1 F_0 \exp\left(\frac{\alpha(0)\sigma_0}{3\,\sigma_1}\right)$$

who is solved by a method of NEWTON for $\alpha(0)$:

$$\begin{cases} \varepsilon(0) & = & \frac{1}{E}\alpha(0)\sigma_0 & = & \varepsilon_e(0) \\ \gamma(0) & = & \frac{1}{2\mu}\alpha(0)\tau_0 & = & \gamma_e(0) \\ p(0) & = & 0 & & \end{cases}$$

2.2 Results of reference

One imposes $\alpha(t)=\alpha(0)+t$ with $\sigma_0=\tau_0=150\,MPa$.

One obtains $\alpha(0)=1.73138$ and $\alpha(1)=2.73138$.

The system (S) is then solved numerically by a 'Backward difference formulated' using scientific library NAG on CRAY. Result of reference = $(\varepsilon,\gamma,\beta,\rho)$ with the nodes with $t=1$.

2.3 Uncertainty on the solution

Uncertainty related to library NAG.

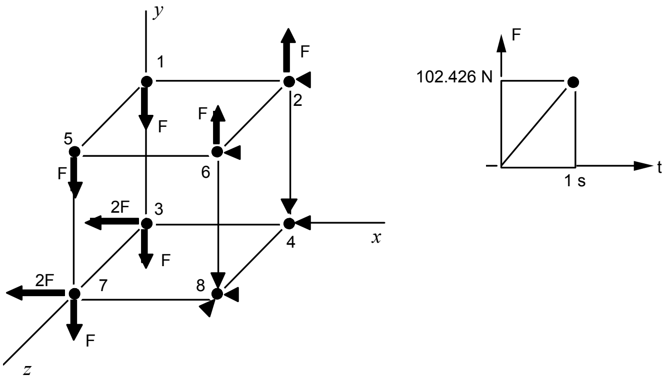
2.4 Bibliographical references

1) User's manual library NAG on CRAY.

3 Modeling A

3.1 Characteristics of modeling

Modeling 3D



3.2 Characteristics of the grid

The grid contains 1 element of the type HEXA8.

3.3 Sizes tested and results of modeling A

Identification	Reference	Test	Tolerance
ε on $NO1$ with $t=1s$	0.07830	ANALYTICAL	0,11%
γ on $NO1$ with $t=1s$	0.11700	ANALYTICAL	0,20%
p on $NO1$ with $t=1s$	0.15260	ANALYTICAL	0,10%
σ_{11} on $NO1$ with $t=1s$	409,70700	ANALYTICAL	0,05%

One also tests the structural parameters of data results:

Identification	Reference	Test	Tolerance
INST for NUME_ORDRE=8	1	ANALYTICAL	0%
ITER_GLOB for NUME_ORDRE=8	4	NON_REGRESSION	0%

3.4 Remarks

One could expect a better correlation, but it should be stressed that library NAG uses the function $R(p)$ in algebraic form, whereas *Code_Aster* uses in the form of a point by point given curve.

Moreover, it seems that the integration of the rate of the function threshold poses problems with NAG, whatever the precision required in addition (the value of the threshold f being appreciably different from 0 at the end of the integration). However, one can note the constancy of this correlation throughout integration $(t \in [0,1])$.

4 Summary of the results

Values of *Code_Aster* are in concord with the values of reference.