

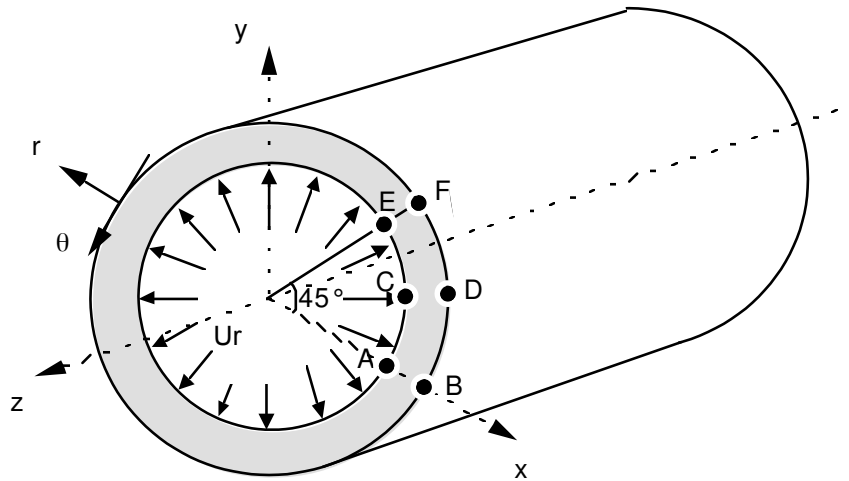
SSNV112 - Hollow roll into incompressible (great deformations)

Summary:

This test makes it possible to validate the quasi-incompressible elements in great deformations, in statics for a three-dimensional, axisymmetric or two-dimensional problem (plane deformations). One considers a hollow roll subjected to an internal radial displacement. The material has a Poisson's ratio equal to 0.4999 and one uses the quasi-incompressible elements `INCO_UPG/INCO_UPGB` with the deformations of `SIMO_MIEHE` and `GDEF_LOG` and elements `INCO_UP` with the deformations `GDEF_LOG`.

1 Problem of reference

1.1 Geometry



Rayon interne $a = 0.1 \text{ m}$
Rayon externe $b = 0.2 \text{ m}$

Coordinates of the points:

	A	B	E	F	C	D
x	0.1	0.2	$0.1 \times \cos(45)$	$0.2 \times \cos(45)$	$0.1 \times \cos(22.5)$	$0.2 \times \cos(22.5)$
y	0	0	$0.1 \times \sin(45)$	$0.1 \times \sin(45)$	$0.1 \times \sin(22.5)$	$0.1 \times \sin(22.5)$
z	0	0	0	0	0	0

1.2 Properties of material

$E = 2.10^5 \text{ MPa}$
 $\nu = 0.4999$

1.3 Boundary conditions and loadings

Radial displacement $U_0 = 6.10^{-5} \text{ m}$ (expansion)

2 Reference solution

2.1 Method of calculating

For the studied problem, displacement \mathbf{u} is radial and thus of the form $\mathbf{u}=[u,0,0]$.

One from of deduced the general form from the tensor of the deformations in great deformations:

$$\mathbf{b} = \mathbf{F} \mathbf{F}^T = \begin{bmatrix} (1+u')^2 & 0 & 0 \\ 0 & \left(1+\frac{u}{r}\right)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

as well as the form of the tensor of the constraints, which is written simply if one takes into account the fact that $J = \det \mathbf{F} = 1$ for an incompressible problem:

$\boldsymbol{\sigma} = -p \mathbf{I}_d + \mu \mathbf{b}^d$, that is to say:

$$\begin{cases} \sigma_{rr} = -p + \mu \left(\frac{2}{3}(1+u')^2 - \frac{1}{3} \left(1+\frac{u}{r}\right)^2 - \frac{1}{3} \right) \\ \sigma_{\theta\theta} = -p + \mu \left(-\frac{1}{3}(1+u')^2 + \frac{2}{3} \left(1+\frac{u}{r}\right)^2 - \frac{1}{3} \right) \\ \sigma_{zz} = -p + \mu \left(-\frac{1}{3}(1+u')^2 - \frac{1}{3} \left(1+\frac{u}{r}\right)^2 + \frac{2}{3} \right) \\ \sigma_{r\theta} = \sigma_{rz} = \sigma_{\theta z} = 0 \end{cases}$$

The writing of the equilibrium equations leads to the checking of only one equation:

$$\sigma'_{rr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

who allows to determine the pressure p knowing the field of radial displacement u :

$$p' = \mu \left(\frac{4}{3}(1+u')u'' - \frac{2}{3} \left(1+\frac{u}{r}\right) \left(\frac{u'}{r} - \frac{u}{r^2} \right) + \frac{(1+u')^2}{r} - \frac{\left(1+\frac{u}{r}\right)^2}{r} \right)$$

2.2 Particularization of the solution

The condition of incompressibility is written $\det \mathbf{F} = 1$ with $\mathbf{F} = \begin{bmatrix} 1+u' & 0 & 0 \\ 0 & 1+\frac{u}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Displacement

u thus check the following differential equation:

$$ru' + u + u'u = 0 \quad \text{éq 2.2-1}$$

The imposed loading is the following $u = U_0$ in $r = a$.

The solution in displacement is thus:

$$\begin{cases} u_r = -r + r \sqrt{r^2 + U_0(U_0 + 2a)} \\ u_\theta = u_z = 0 \end{cases}$$

The tensor of the deformations thus has as an expression:

$$\begin{cases} b_{rr} = \frac{r^2}{r^2 + U_0(U_0 + 2a)} \\ b_{\theta\theta} = \frac{r^2 + U_0(U_0 + 2a)}{r^2} \\ b_{zz} = 1 \\ b_{r\theta} = b_{z\theta} = b_{\theta z} = 0 \end{cases}$$

And the constraints are worth:

$$\begin{cases} \sigma_{rr} = -p + \mu \left(\frac{2}{3} \frac{r^2}{r^2 + U_0(U_0 + 2a)} - \frac{1}{3} \frac{r^2 + U_0(U_0 + 2a)}{r^2} - \frac{1}{3} \right) \\ \sigma_{\theta\theta} = -p + \mu \left(-\frac{1}{3} \frac{r^2}{r^2 + U_0(U_0 + 2a)} + \frac{2}{3} \frac{r^2 + U_0(U_0 + 2a)}{r^2} - \frac{1}{3} \right) \\ \sigma_{zz} = -p + \mu \left(-\frac{1}{3} \frac{r^2}{r^2 + U_0(U_0 + 2a)} - \frac{1}{3} \frac{r^2 + U_0(U_0 + 2a)}{r^2} + \frac{2}{3} \right) \\ \sigma_{r\theta} = \sigma_{z\theta} = \sigma_{\theta z} = 0 \end{cases}$$

with p obtained by integration of [éq 2.2-1] which is worth:

$$p = \mu \left(\frac{U_0(U_0 + 2a)}{6r^2} - \frac{2U_0(U_0 + 2a)}{3(U_0(U_0 + 2a) + r^2)} - \log(r) + \frac{1}{2} \log(U_0(U_0 + 2a) + r^2) \right) + C$$

where C is a constant

One obtains finally the following digital values:

in $r=0.1$:	in $r=0.2$:
$u_r = 6.10^{-5}$	$u_r = 3.006710^{-5}$
$\sigma_{rr} = -59.9955$	$\sigma_{rr} = 0.$
$\sigma_{\theta\theta} = 99.9566$	$\sigma_{\theta\theta} = 40.006$
$\sigma_{zz} = 19.9326$	$\sigma_{zz} = 20.$
$E_{rr} = 0,0005994604316761909$	
$E_{\theta\theta} = -0.0006001799999999502$	

The passage in the Cartesian system is done using the following relations:

$$\begin{aligned}\sigma_{xx} &= \sigma_{rr} \cos^2 \theta + \sigma_{\theta\theta} \sin^2 \theta - 2 \sigma_{r\theta} \sin \theta \cos \theta \\ \sigma_{\theta\theta} &= \sigma_{rr} \sin^2 \theta + \sigma_{\theta\theta} \cos^2 \theta + 2 \sigma_{r\theta} \sin \theta \cos \theta \\ \sigma_{zz} &= \sigma_{rr} \sin \theta \cos \theta - \sigma_{\theta\theta} \sin \theta \cos \theta - 2 \sigma_{r\theta} (\cos^2 \theta - \sin^2 \theta)\end{aligned}$$

2.3 Sizes and results of reference

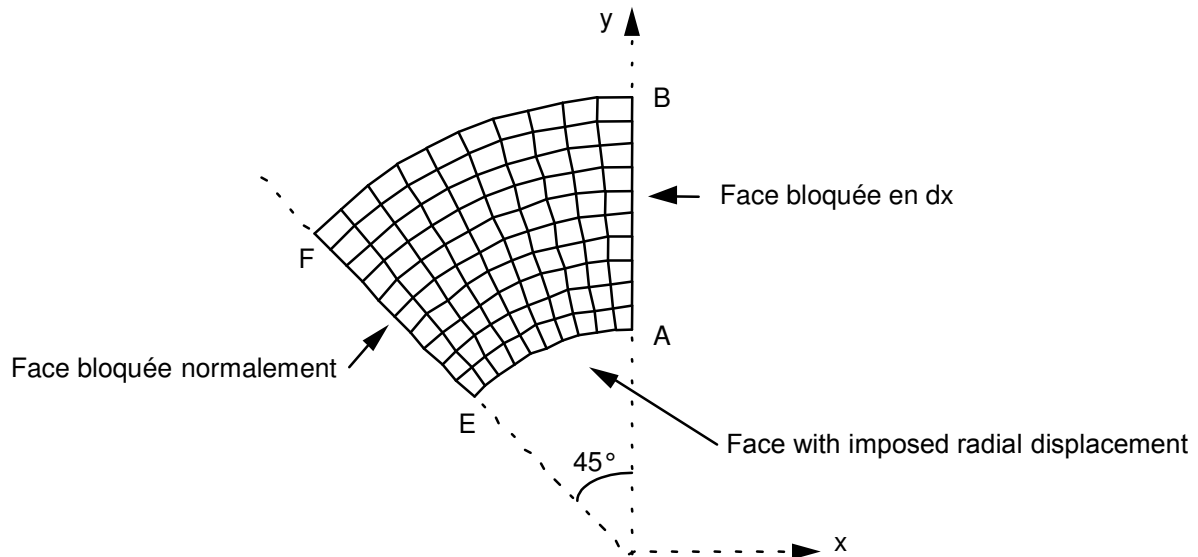
One compared to values of reference:

- displacements (u, v) at the points A and F ,
- constraints $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy})$ at the points A and F ,
- constraints of Von Mises and Tresca as well as the eigenvalues of the tensor of the constraints at the point A .

3 Modeling A

3.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of elements

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.	
	GROUP_NO = ' FACINF'	DZ = 0.	faces <i>A E F D</i> ($z=0$ and
			$z=0.01$)
	GROUP_NO = ' FACEAB'	DX = 0.	face <i>AB</i>
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face <i>EF</i>
	GROUP_MA = ' FACEAE'	DNOR = -6.10^{-5}	face <i>AE</i>

3.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshes: 240 HEXA20

3.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	$1. 10^{-5}$
	v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.02
	σ_{zz}	ANALYTICAL	19.9326	0,035
	σ_{xy}	ANALYTICAL	0.	0,012
	VMIS	ANALYTICAL	138.5226	0.02
	TRESCA	ANALYTICAL	159.9521	0.02
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0,035
	PRIN_3	ANALYTICAL	99.9566	0.01
	VMIS_SG	ANALYTICAL	138.5226	0.02

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$-2.1217 10^{-5}$	$1. 10^{-3}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	20,003	0,005
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,005
	σ_{xy}	ANALYTICAL	20,003	0,005

For the deformations of Green-Lagrange:

Identification		Type of reference	Reference	Tolerance
A	E_{xx}	ANALYTICAL	0.000599576100401	2.E-4
	E_{yy}	ANALYTICAL	-0.00059885996551	2.2E-3

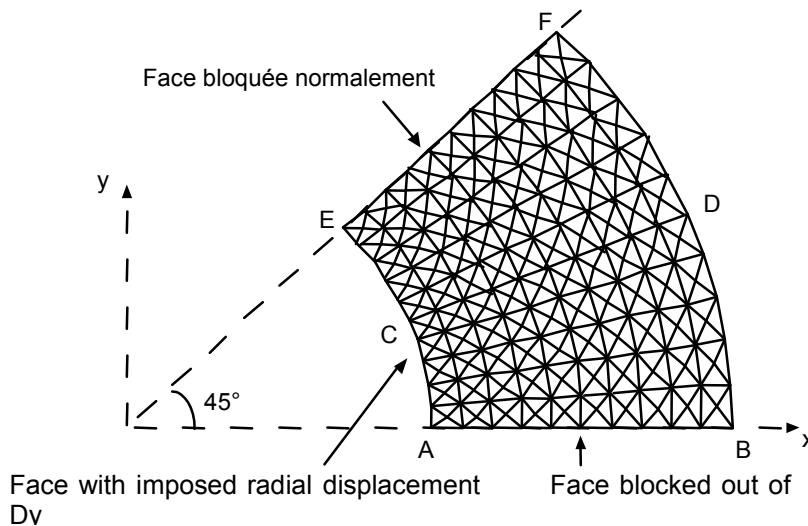
3.4 Remarks

One obtains very good performances since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.1% for displacements, lower than 3.5 % for the constraints and lower than 0.03% for the deformations.

4 Modeling B

4.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).

The grid was obtained with GMSH for a density of 0,01 .

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.	
	GROUP_NO = ' FACINF'	DZ = 0.	faces $A E F D$ ($z=0$ and
			$z=0.01$)
	GROUP_NO = ' FACEAB'	DY = 0.	face AB
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF
	GROUP_MA = ' FACEAE'	DNOR = -6.10^{-5}	face AE

4.2 Characteristics of the grid

Many nodes: 2064

Many meshes: 1121 TETRA10

4.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	$6. 10^{-5}$	$2. 10^{-3}$
	v	ANALYTICAL	0.	$1. 10^{-5}$
	σ_{xx}	ANALYTICAL	-59.9955	0,025
	σ_{yy}	ANALYTICAL	99.9566	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0.01
	TRESCA	ANALYTICAL	159.9521	0.01
	PRIN_1	ANALYTICAL	-59.9955	0,025
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0,015
	VMIS_SG	ANALYTICAL	138.5226	0.01

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-3}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	20,003	0,005
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,005
	σ_{xy}	ANALYTICAL	-20,003	0.01

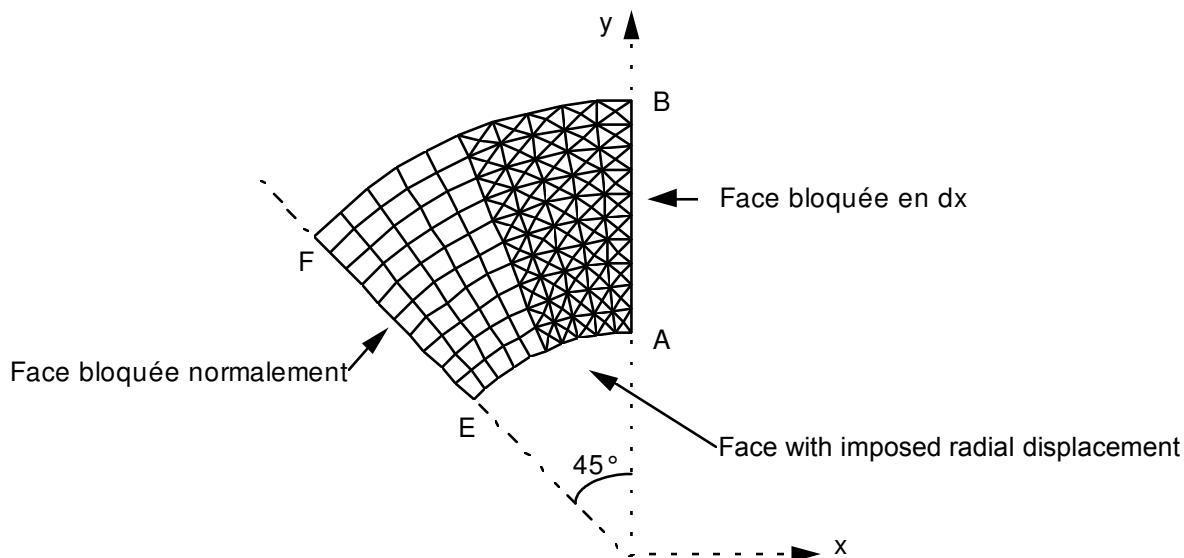
4.4 Remarks

The got results are completely correct since the constraints are obtained with a precision lower than 3 % even 1 % at the point F. the variation is a little more important here than for HEXA20, but can be explained by the fact that the loading is imposed here in a way a little less precise since displacement U at point A, is defined only with one precision of 0.158% against 0.077% (evening the factor 2, qu ' one finds on the constraints).

5 Modeling C

5.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UPG (DEFORMATION=' SIMO_MIEHE ') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' GRNM11 '	DX = 0.	side <i>AB</i>
FACE_IMPO =	GROUP_MA = ' GRMA12 '	DNOR = 0.	dimensioned <i>EF</i>
	= GROUP_MA = ' GRMA13 '	DNOR = -6. 10 ⁻⁵	face <i>AE</i>

Name of the nodes:

A = N2 , *B* = N361 , *C* = N121 , *D* = N584 , *E* = N155 , *F* = N503

5.2 Characteristics of the grid

Many nodes: 591

Many meshes: 200 TRIA6, 50 QUAD8.

5.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	$1. 10^{-5}$
	v	ANALYTICAL	$6. 10^{-5}$	$5. 10^{-3}$
	σ_{xx}	ANALYTICAL	99.9566	$5. 10^{-3}$
	σ_{yy}	ANALYTICAL	-59.9955	0.03
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0.02
	TRESCA	ANALYTICAL	159.9521	0.02
	PRIN_1	ANALYTICAL	-59.9955	0.03
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0.02
	VMIS_SG	ANALYTICAL	138.5226	0.02

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$-2.1217 10^{-5}$	$5. 10^{-3}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$5. 10^{-3}$
	σ_{xx}	ANALYTICAL	20,003	$5. 10^{-3}$
	σ_{yy}	ANALYTICAL	20,003	$5. 10^{-3}$
	σ_{zz}	ANALYTICAL	20,003	$5. 10^{-3}$
	σ_{xy}	ANALYTICAL	20,003	$5. 10^{-3}$

For the deformations of Green-Lagrange:

Identification		Type of reference	Reference	Tolerance
A	E_{xx}	ANALYTICAL	0.000601357472911	4 %
	E_{yy}	ANALYTICAL	-0.000604235469839	0.7 %

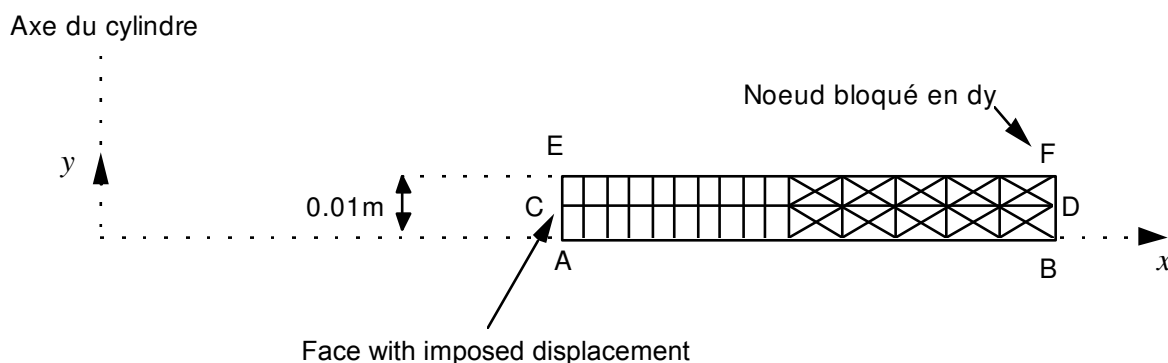
5.4 Remarks

As for the modelisation 3D, the got results are completely satisfactory.

6 Modeling D

6.1 Characteristics of modeling

Grid with elements AXIS_INCO_UPG (DEFORMATION=' SIMO_MIEHE ') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DY = 0.	side <i>EF</i>
	GROUP_NO = ' FACINF '	DY = 0.	side <i>AB</i>
FACE_IMPO =	GROUP_MA = ' FACEAE '	DX = $6 \cdot 10^{-5}$	face <i>AE</i>

6.2 Characteristics of the grid

Many nodes: 175.

Many meshes and types: 20 QUAD8, 40 TRIA6.

6.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
v	ANALYTICAL	0.	$1. 10^{-5}$
σ_{xx}	ANALYTICAL	-59.9955	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	19.9326	$5. 10^{-3}$
σ_{zz}	ANALYTICAL	99.9566	$5. 10^{-3}$
σ_{xy}	ANALYTICAL	0.	$1. 10^{-5}$
VMIS	ANALYTICAL	138.5226	$5. 10^{-3}$
TRESCA	ANALYTICAL	159.9521	$5. 10^{-3}$
PRIN_1	ANALYTICAL	-59.9955	$5. 10^{-3}$
PRIN_2	ANALYTICAL	19.9326	$5. 10^{-3}$
PRIN_3	ANALYTICAL	99.9566	$5. 10^{-3}$
VMIS_SG	ANALYTICAL	138.5226	$5. 10^{-3}$

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$3. 10^{-5}$	$1. 10^{-3}$
v	ANALYTICAL	0.	$1. 10^{-5}$
σ_{xx}	ANALYTICAL	0.	0.03
σ_{yy}	ANALYTICAL	20.0	$5. 10^{-3}$
σ_{zz}	ANALYTICAL	40,006	$5. 10^{-3}$
σ_{xy}	ANALYTICAL	0.	$5. 10^{-3}$

For the deformations of Green-Lagrange:

Identification	Type of reference	Reference	Tolerance
A E_{xx}	ANALYTICAL	-0.000598704996172	2nd-3

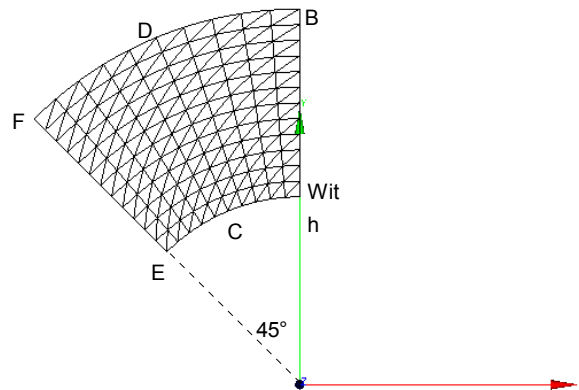
6.4 Remarks

The precision obtained is very good since all the constraints are obtained with a precision lower than 0.5% .

7 Modeling E

7.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DZ = 0.	
	GROUP_NO = ' FACINF '	DZ = 0.	faces <i>A E F D</i> ($z=0$ and
$z=0.01$)			
	GROUP_NO = ' FACEAB '	DX = 0.	face <i>AB</i>
FACE_IMPO =	GROUP_MA = ' FACEEF '	DNOR = 0.	face <i>EF</i>
	GROUP_MA = ' FACEAE '	DNOR = -6.10^{-5}	face <i>AE</i>

7.2 Characteristics of the grid

Many nodes: 1861
Many meshes: 480 PENTA15

7.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

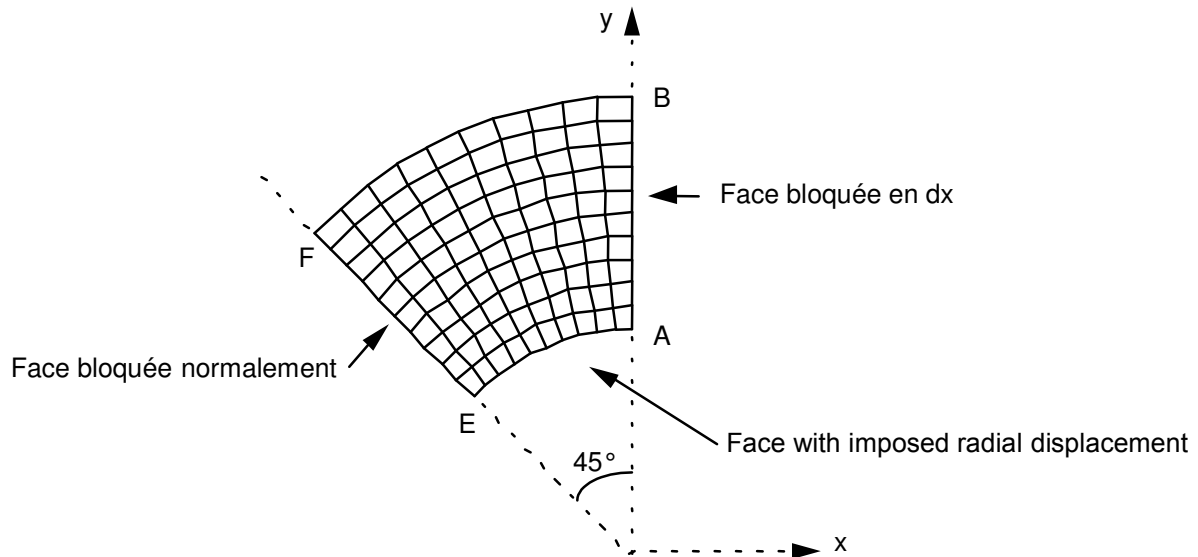
Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	$1. 10^{-5}$
	v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	99.9566	0.03
	σ_{yy}	ANALYTICAL	-59.9955	0.09
	σ_{zz}	ANALYTICAL	19.9326	0.08
	σ_{xy}	ANALYTICAL	0.	0,005
	VMIS	ANALYTICAL	138.5226	0.05
	TRESCA	ANALYTICAL	159.9521	0.05
	PRIN_1	ANALYTICAL	-59.9955	0.09
	PRIN_2	ANALYTICAL	19.9326	0.08
	PRIN_3	ANALYTICAL	99.9566	0.03
	VMIS_SG	ANALYTICAL	138.5226	0.05

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$-2.1217 10^{-5}$	$3. 10^{-3}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$3. 10^{-3}$
	σ_{xx}	ANALYTICAL	20,003	0.04
	σ_{yy}	ANALYTICAL	20,003	0.17
	σ_{zz}	ANALYTICAL	20,003	0.07
	σ_{xy}	ANALYTICAL	20,003	0,008

8 Modeling F

8.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of elements

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.	
	GROUP_NO = ' FACINF'	DZ = 0.	faces $AEFD$ ($z=0$ and
			$z=0.01$)
	GROUP_NO = ' FACEAB'	DX = 0.	face AB
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF
	GROUP_MA = ' FACEAE'	DNOR = -6.10^{-5}	face AE

8.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshes: 240 HEXA20

8.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	$1. 10^{-3}$
	v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.03
	σ_{zz}	ANALYTICAL	19.9326	0.05
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,001
	TRESCA	ANALYTICAL	159.9521	0,001
	PRIN_1	ANALYTICAL	-59.9955	0.0025
	PRIN_2	ANALYTICAL	19.9326	0,005
	PRIN_3	ANALYTICAL	99.9566	0.0005
	VMIS_SG	ANALYTICAL	138.5226	0,001

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$-2.1217 10^{-5}$	0,005
	v	ANALYTICAL	$2.1217 10^{-5}$	0,005
	σ_{xx}	ANALYTICAL	20,003	0,002
	σ_{yy}	ANALYTICAL	20,003	0,002
	σ_{zz}	ANALYTICAL	20,003	0.0025
	σ_{xy}	ANALYTICAL	20,003	0.0015

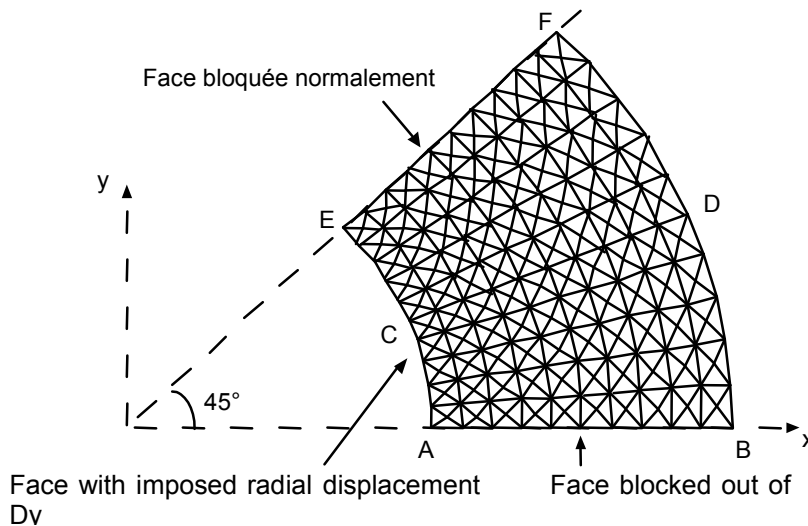
8.4 Remarks

One obtains very good performances since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.5% for displacements and lower than 5% for the constraints.

9 Modeling G

9.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).

The grid was obtained with GMSH for a density of 0,01 .

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.	faces $A E F D$ ($z=0$ and $z=0.01$)
	GROUP_NO = ' FACINF'	DZ = 0.	
FACE_IMPO =	GROUP_NO = ' FACEAB'	DY = 0.	face AB
	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF
	GROUP_MA = ' FACEAE'	DNOR = -6.10^{-5}	face AE

9.2 Characteristics of the grid

Many nodes: 2064

Many meshes: 1121 TETRA10

9.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	$6. 10^{-5}$	$2. 10^{-3}$
	v	ANALYTICAL	0.	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	-59.9955	0.02
	σ_{yy}	ANALYTICAL	99.9566	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,002
	TRESCA	ANALYTICAL	159.9521	0,002
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0,015
	VMIS_SG	ANALYTICAL	138.5226	0,002

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-4}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-4}$
	σ_{xx}	ANALYTICAL	20,003	0,003
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,002
	σ_{xy}	ANALYTICAL	-20,003	0.01

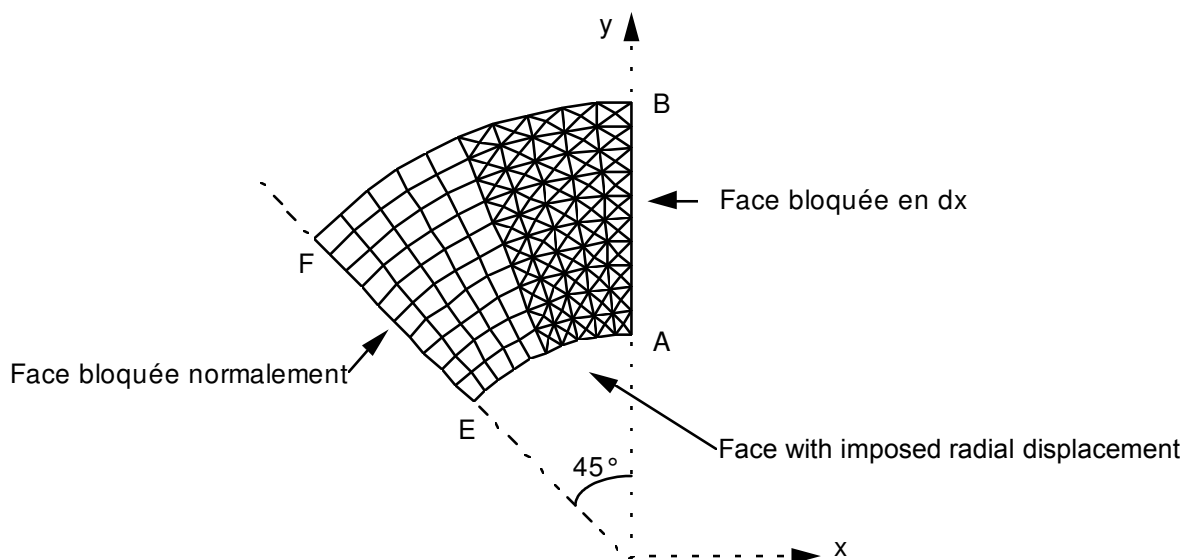
9.4 Remarks

The got results are completely correct since the constraints are obtained with a precision lower than 3 % even 1 % at the point F. the variation is a little more important here than for HEXA20, but can be explained by the fact that the loading is imposed here in a way a little less precise since displacement U at point A, is defined only with one precision of 0.158% against 0.077% (evening the factor 2, qu 'one finds on the constraints).

10 Modeling H

10.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' GRNM11 '	DX = 0.	side <i>AB</i>
FACE_IMPO =	GROUP_MA = ' GRMA12 '	DNOR = 0.	dimensioned <i>EF</i>
	= GROUP_MA = ' GRMA13 '	DNOR = -6. 10 ⁻⁵	face <i>AE</i>

Name of the nodes:

A = N2 , *B* = N361 , *C* = N121 , *D* = N584 , *E* = N155 , *F* = N503

10.2 Characteristics of the grid

Many nodes: 591

Many meshes: 200 TRIA6, 50 QUAD8.

10.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-5}$
v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-4}$
σ_{xx}	ANALYTICAL	99.9566	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	-59.9955	0.02
σ_{zz}	ANALYTICAL	19.9326	0.02
σ_{xy}	ANALYTICAL	0.	0.02
VMIS	ANALYTICAL	138.5226	0,006
TRESCA	ANALYTICAL	159.9521	0,006
PRIN_1	ANALYTICAL	-59.9955	0.02
PRIN_2	ANALYTICAL	19.9326	0.02
PRIN_3	ANALYTICAL	99.9566	0,003
VMIS_SG	ANALYTICAL	138.5226	0,006

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$3. 10^{-4}$
v	ANALYTICAL	$2.1217 10^{-5}$	$3. 10^{-4}$
σ_{xx}	ANALYTICAL	20,003	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	20,003	$2.5 10^{-3}$
σ_{zz}	ANALYTICAL	20,003	$5. 10^{-4}$
σ_{xy}	ANALYTICAL	20,003	$2. 10^{-3}$

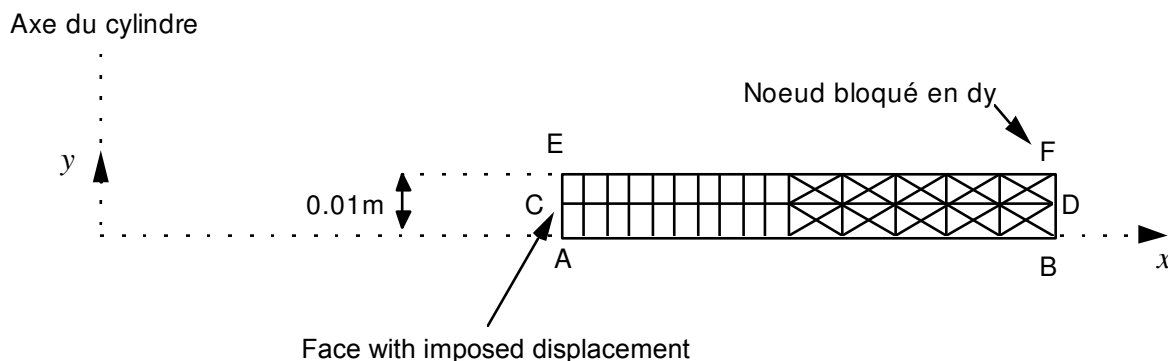
10.4 Remarks

As for modeling 3D, the got results are completely satisfactory.

11 Modeling I

11.1 Characteristics of modeling

Grid with elements `AXIS_INCO_UPG` (`DEFORMATION=' GDEF_LOG '`) incompressible of type `TRIA6` and `QUAD8`



Limiting conditions:

<code>DDL_IMPO =</code>	<code>GROUP_NO = ' FACSUP '</code>	<code>DY = 0.</code>	side <i>EF</i>
	<code>GROUP_NO = ' FACINF '</code>	<code>DY = 0.</code>	side <i>AB</i>
<code>FACE_IMPO =</code>	<code>GROUP_MA = ' FACEAE '</code>	<code>DX = 6. 10⁻⁵</code>	face <i>AE</i>

11.2 Characteristics of the grid

Many nodes: 175.

Many meshes and types: 20 `QUAD8`, 40 `TRIA6`.

11.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
v	ANALYTICAL	0.	$1. 10^{-5}$
σ_{xx}	ANALYTICAL	-59.9955	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	19.9326	$5. 10^{-3}$
σ_{zz}	ANALYTICAL	99.9566	$5. 10^{-3}$
σ_{xy}	ANALYTICAL	0.	$1. 10^{-5}$
VMIS	ANALYTICAL	138.5226	$2. 10^{-3}$
TRESCA	ANALYTICAL	159.9521	$2. 10^{-3}$
PRIN_1	ANALYTICAL	-59.9955	$3. 10^{-3}$
PRIN_2	ANALYTICAL	19.9326	$5. 10^{-3}$
PRIN_3	ANALYTICAL	99.9566	$5. 10^{-4}$
VMIS_SG	ANALYTICAL	138.5226	$2. 10^{-3}$

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$3. 10^{-5}$	$5. 10^{-4}$
v	ANALYTICAL	0.	$1. 10^{-5}$
σ_{xx}	ANALYTICAL	0.	0.03
σ_{yy}	ANALYTICAL	20.0	$3. 10^{-3}$
σ_{zz}	ANALYTICAL	40,006	$3. 10^{-3}$
σ_{xy}	ANALYTICAL	0.	$5. 10^{-3}$

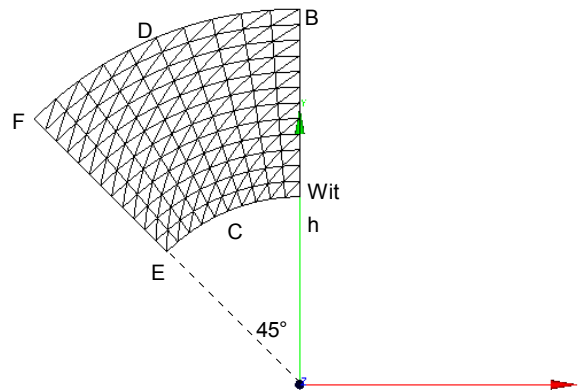
11.4 Remarks

The precision obtained is very good since all the constraints are obtained with a precision lower than 0.5% .

12 Modeling J

12.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DZ = 0.	
	GROUP_NO = ' FACINF '	DZ = 0.	faces <i>A E F D</i> ($z=0$ and
$z=0.01$)			
	GROUP_NO = ' FACEAB '	DX = 0.	face <i>AB</i>
FACE_IMPO =	GROUP_MA = ' FACEEF '	DNOR = 0.	face <i>EF</i>
	GROUP_MA = ' FACEAE '	DNOR = -6.10^{-5}	face <i>AE</i>

12.2 Characteristics of the grid

Many nodes: 1861
Many meshes: 480 PENTA15

12.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

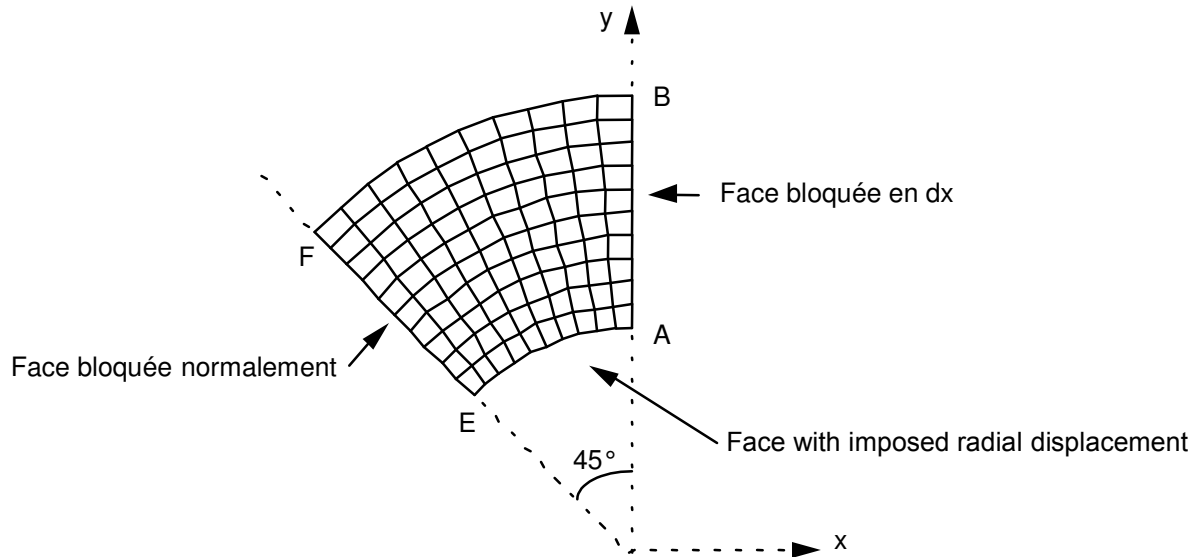
Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-5}$
v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
σ_{xx}	ANALYTICAL	99.9566	0.0002
σ_{yy}	ANALYTICAL	-59.9955	0,003
σ_{zz}	ANALYTICAL	19.9326	0,007
σ_{xy}	ANALYTICAL	0.	0.0008
VMIS	ANALYTICAL	138.5226	0.05
TRESCA	ANALYTICAL	159.9521	0.05
PRIN_1	ANALYTICAL	-59.9955	0.09
PRIN_2	ANALYTICAL	19.9326	0.08
PRIN_3	ANALYTICAL	99.9566	0.03
VMIS_SG	ANALYTICAL	138.5226	0.05

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$4. 10^{-3}$
v	ANALYTICAL	$2.1217 10^{-5}$	$4. 10^{-3}$
σ_{xx}	ANALYTICAL	20,003	0,007
σ_{yy}	ANALYTICAL	20,003	0.0006
σ_{zz}	ANALYTICAL	20,003	$3. 10^{-5}$
σ_{xy}	ANALYTICAL	20,003	0.0004

13 Modeling K

13.1 Characteristics of modeling

Grid with elements 3D_INCO_UP (DEFORMATION=' GDEF_LOG ') incompressible of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of elements

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DZ = 0.	faces $AEFD$ ($z=0$ and $z=0.01$)
	GROUP_NO = ' FACINF '	DZ = 0.	
FACE_IMPO =	GROUP_NO = ' FACEAB '	DX = 0.	face AB
	GROUP_MA = ' FACEEF '	DNOR = 0.	face EF
	GROUP_MA = ' FACEAE '	DNOR = -6.10^{-5}	face AE

13.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshes: 240 HEXA20

13.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	$1. 10^{-3}$
	v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-4}$
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.03
	σ_{zz}	ANALYTICAL	19.9326	0.05
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,001
	TRESCA	ANALYTICAL	159.9521	0,001
	PRIN_1	ANALYTICAL	-59.9955	0.0025
	PRIN_2	ANALYTICAL	19.9326	0,005
	PRIN_3	ANALYTICAL	99.9566	0.0005
	VMIS_SG	ANALYTICAL	138.5226	0,001

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$-2.1217 10^{-5}$	0,005
	v	ANALYTICAL	$2.1217 10^{-5}$	0,005
	σ_{xx}	ANALYTICAL	20,003	0,002
	σ_{yy}	ANALYTICAL	20,003	0,002
	σ_{zz}	ANALYTICAL	20,003	0.0025
	σ_{xy}	ANALYTICAL	20,003	0.0015

For the deformations of Green-Lagrange:

Identification		Type of reference	Reference	Tolerance
A	E_{xx}	ANALYTICAL	0.000599576100401	2nd4
	E_{yy}	ANALYTICAL	-0.00059885996551	2.2E3

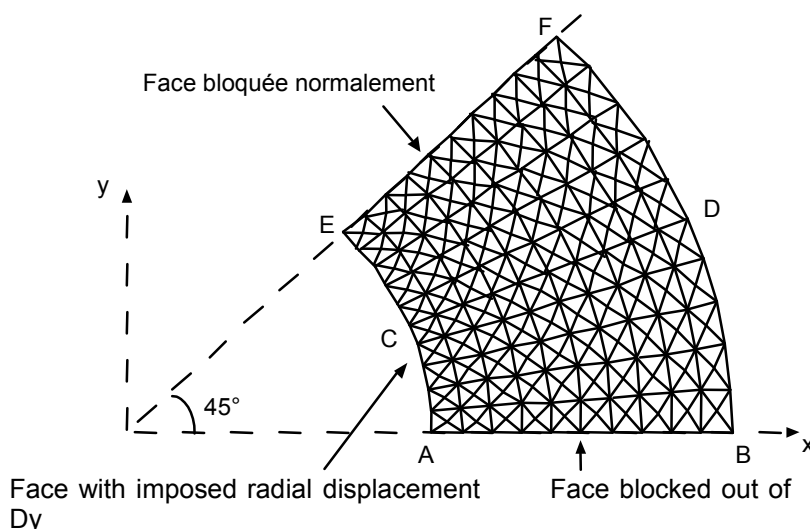
13.4 Remarks

One obtains very good performances since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.5% for displacements and lower than 5 % for the constraints.

14 Modeling L

14.1 Characteristics of modeling

Grid with elements 3D_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).

The grid was obtained with GMSH for a density of 0,01 .

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DZ = 0.	faces $A E F D$ ($z=0$ and $z=0.01$)
	GROUP_NO = ' FACINF '	DZ = 0.	
	GROUP_NO = ' FACEAB '	DY = 0.	face AB
FACE_IMPO =	GROUP_MA = ' FACEEF '	DNOR = 0.	face EF
	GROUP_MA = ' FACEAE '	DNOR = -6.10^{-5}	face AE

14.2 Characteristics of the grid

Many nodes: 2064

Many meshes: 1121 TETRA10

14.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6.10^{-5}	2.10^{-4}
	v	ANALYTICAL	0.	1.10^{-3}
	σ_{xx}	ANALYTICAL	-59.9955	0.02
	σ_{yy}	ANALYTICAL	99.9566	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,002
	TRESCA	ANALYTICAL	159.9521	0,002
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0,015
	VMIS_SG	ANALYTICAL	138.5226	0,002

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$2.1217 \cdot 10^{-5}$	1.10^{-4}
	v	ANALYTICAL	$2.1217 \cdot 10^{-5}$	1.10^{-4}
	σ_{xx}	ANALYTICAL	20,003	0,003
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,002
	σ_{xy}	ANALYTICAL	-20,003	0.01

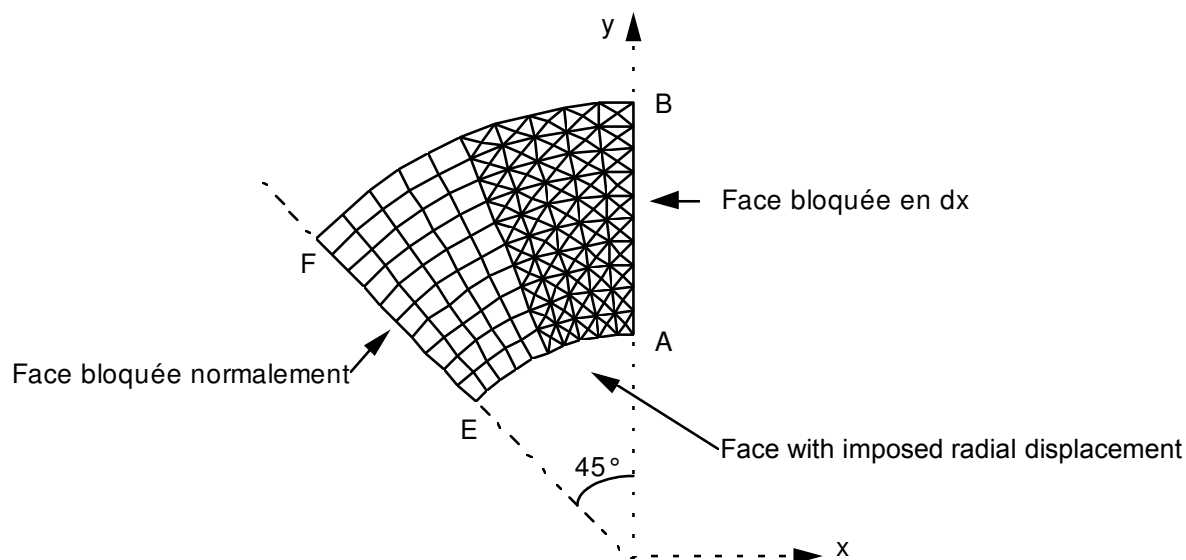
14.4 Remarks

The got results are completely correct since the constraints are obtained with a precision lower than 3 % even 1 % at the point F. the variation is a little more important here than for HEXA20, but can be explained by the fact that the loading is imposed here in a way a little less precise since displacement U at point A, is defined only with one precision of 0.158% against 0.077% (evening the factor 2, qu ' one finds on the constraints).

15 Modeling M

15.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' GRNM11 '	DX = 0.	side <i>AB</i>
FACE_IMPO =	GROUP_MA = ' GRMA12 '	DNOR = 0.	dimensioned <i>EF</i>
	= GROUP_MA = ' GRMA13 '	DNOR = -6. 10 ⁻⁵	face <i>AE</i>

Name of the nodes:

A = N2 , *B* = N361 , *C* = N121 , *D* = N584 , *E* = N155 , *F* = N503

15.2 Characteristics of the grid

Many nodes: 591

Many meshes: 200 TRIA6, 50 QUAD8.

15.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-5}$
v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-4}$
σ_{xx}	ANALYTICAL	99.9566	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	-59.9955	0.02
σ_{zz}	ANALYTICAL	19.9326	0.02
σ_{xy}	ANALYTICAL	0.	0.02
VMIS	ANALYTICAL	138.5226	0,006
TRESCA	ANALYTICAL	159.9521	0,006
PRIN_1	ANALYTICAL	-59.9955	0.02
PRIN_2	ANALYTICAL	19.9326	0.02
PRIN_3	ANALYTICAL	99.9566	0,003
VMIS_SG	ANALYTICAL	138.5226	0,006

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$3. 10^{-4}$
v	ANALYTICAL	$2.1217 10^{-5}$	$3. 10^{-4}$
σ_{xx}	ANALYTICAL	20,003	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	20,003	$2.5 10^{-3}$
σ_{zz}	ANALYTICAL	20,003	$5. 10^{-4}$
σ_{xy}	ANALYTICAL	20,003	$2. 10^{-3}$

For the deformations of Green-Lagrange:

Identification	Type of reference	Reference	Tolerance
A E_{xx}	ANALYTICAL	0.000601356410318	4E3
E_{yy}	ANALYTICAL	-0.000604235469839	7.0E3

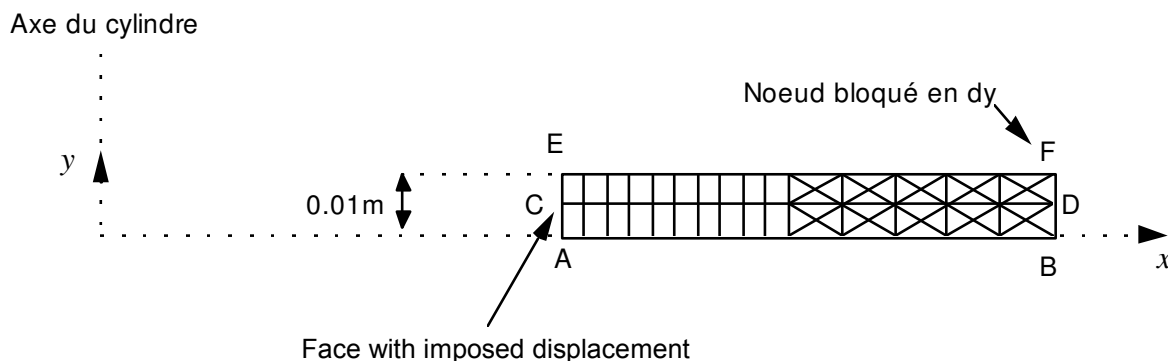
15.4 Remarks

As for modeling 3D, the got results are completely satisfactory.

16 Modeling NR

16.1 Characteristics of modeling

Grid with elements `AXIS_INCO_UP` (`DEFORMATION=' GDEF_LOG'`) incompressible of type `TRIA6` and `QUAD8`



Limiting conditions:

<code>DDL_IMPO</code>	<code>=</code>	<code>GROUP_NO</code>	<code>=</code>	<code>' FACSUP'</code>	<code>DY = 0.</code>	<code>side EF</code>
		<code>GROUP_NO</code>	<code>=</code>	<code>' FACINF'</code>	<code>DY = 0.</code>	<code>side AB</code>
<code>FACE_IMPO</code>	<code>=</code>	<code>GROUP_MA</code>	<code>=</code>	<code>' FACEAE'</code>	<code>DX = 6. 10⁻⁵</code>	<code>face AE</code>

16.2 Characteristics of the grid

Many nodes: 175.

Many meshes and types: 20 `QUAD8`, 40 `TRIA6`.

16.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	v	ANALYTICAL	0.	$1. 10^{-5}$
	σ_{xx}	ANALYTICAL	-59.9955	$5. 10^{-3}$
	σ_{yy}	ANALYTICAL	19.9326	$5. 10^{-3}$
	σ_{zz}	ANALYTICAL	99.9566	$5. 10^{-3}$
	σ_{xy}	ANALYTICAL	0.	$1. 10^{-5}$
	VMIS	ANALYTICAL	138.5226	$2. 10^{-3}$
	TRESCA	ANALYTICAL	159.9521	$2. 10^{-3}$
	PRIN_1	ANALYTICAL	-59.9955	$3. 10^{-3}$
	PRIN_2	ANALYTICAL	19.9326	$5. 10^{-3}$
	PRIN_3	ANALYTICAL	99.9566	$5. 10^{-4}$
	VMIS_SG	ANALYTICAL	138.5226	$2. 10^{-3}$

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	$3. 10^{-5}$	$5. 10^{-4}$
	v	ANALYTICAL	0.	$1. 10^{-5}$
	σ_{xx}	ANALYTICAL	0.	0.03
	σ_{yy}	ANALYTICAL	20.0	$3. 10^{-3}$
	σ_{zz}	ANALYTICAL	40,006	$3. 10^{-3}$
	σ_{xy}	ANALYTICAL	0.	$5. 10^{-3}$

For the deformations of Green-Lagrange:

Identification		Type of reference	Reference	Tolerance
A	E_{xx}	ANALYTICAL	-0.000598704996172	2E3

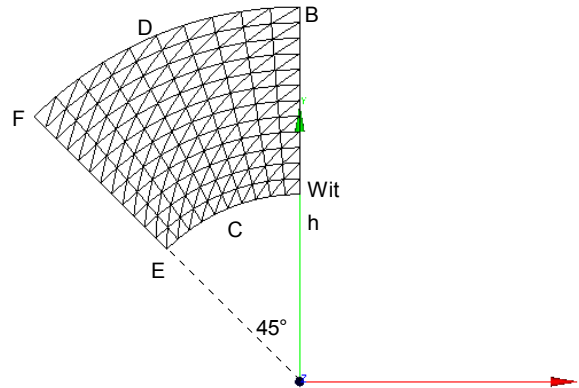
16.4 Remarks

The precision obtained is very good since all the constraints are obtained with a precision lower than 0.5% .

17 Modeling O

17.1 Characteristics of modeling

Grid with elements 3D_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DZ = 0.	
	GROUP_NO = ' FACINF '	DZ = 0.	faces <i>A E F D</i> ($z=0$ and
			$z=0.01$)
	GROUP_NO = ' FACEAB '	DX = 0.	face <i>AB</i>
FACE_IMPO =	GROUP_MA = ' FACEEF '	DNOR = 0.	face <i>EF</i>
	GROUP_MA = ' FACEAE '	DNOR = -6.10^{-5}	face <i>AE</i>

17.2 Characteristics of the grid

Many nodes: 1861
Many meshes: 480 PENTA15

17.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

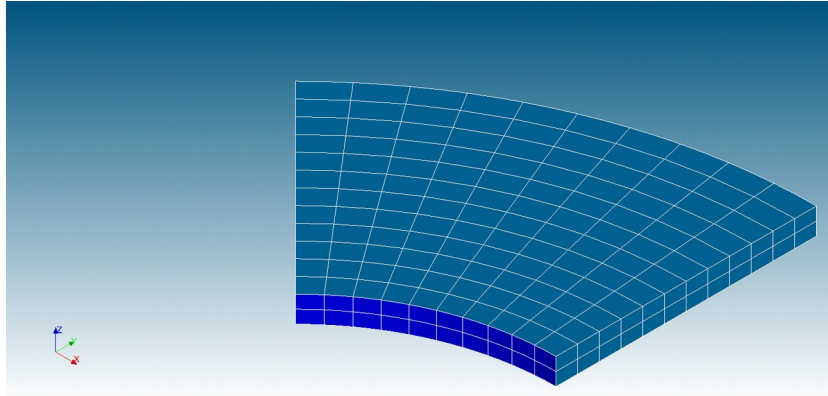
Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-5}$
v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
σ_{xx}	ANALYTICAL	99.9566	0.0002
σ_{yy}	ANALYTICAL	-59.9955	0,003
σ_{zz}	ANALYTICAL	19.9326	0,007
σ_{xy}	ANALYTICAL	0.	0.0008
VMIS	ANALYTICAL	138.5226	0,001
TRESCA	ANALYTICAL	159.9521	0,001
PRIN_1	ANALYTICAL	-59.9955	0,003
PRIN_2	ANALYTICAL	19.9326	0,008
PRIN_3	ANALYTICAL	99.9566	0.0002
VMIS_SG	ANALYTICAL	138.5226	0,001

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$4. 10^{-3}$
v	ANALYTICAL	$2.1217 10^{-5}$	$4. 10^{-3}$
σ_{xx}	ANALYTICAL	20,003	0,007
σ_{yy}	ANALYTICAL	20,003	0.0006
σ_{zz}	ANALYTICAL	20,003	$3. 10^{-5}$
σ_{xy}	ANALYTICAL	20,003	0.0004

18 Modeling P

18.1 Characteristics of modeling

Grid with elements 3D_INCO_UPGB (DEFORMATION='SIME_MIEHE') incompressible of type HEXA20 only.



18.2 Characteristics of the grid

Many nodes: 1501
Many meshes: 240 PENTA15

18.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU and EPSG_NOEU Stested at the point A only.

Identification	Type of reference	Reference	Tolerance	
A	u	ANALYTICAL	0.	$1. 10^{-5}$
	v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.035
	σ_{xy}	ANALYTICAL	0.	1.2
	E_{xx}	ANALYTICAL	0.00059946	0.0002
	E_{yy}	ANALYTICAL	-0.00060018	0.0025
	VMIS	ANALYTICAL	138.5226	0.02
	TRESCA	ANALYTICAL	159.9521	0.02
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.035
	PRIN_3	ANALYTICAL	99.9566	0.01
	VMIS_SG	ANALYTICAL	138.5226	0.02
Identification	Type of reference	Reference	Tolerance	
F	u	ANALYTICAL	$-2.1217 10^{-5}$	$1. 10^{-3}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	20,003	0.005
	σ_{yy}	ANALYTICAL	20,003	0.005

Code Aster

Version
default

Titre : SSNV112 - Cylindre creux en incompressible (grande[...]
Responsable : MICHEL-PONNELLE Sylvie

Date : 12/12/2017 Page : 37/54
Clé : V6.04.112 Révision :
a47c4afa52dc

σ_{zz}	ANALYTICAL	20,003	0.005
σ_{xy}	ANALYTICAL	20,003	0.005

Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification		Type of reference	Place
ERME_ELEM	NUEST	NON_REGRESSION	M1 mesh
ERME_ELNO	ERREST	NON_REGRESSION	Node A

19 Modeling Q

19.1 Characteristics of modeling

Grid with elements 3D_INCO_UPGB (DEFORMATION='SIME_MIEHE') incompressible of type TETRA4 only.

19.2 Characteristics of the grid

Many nodes: 336
Many meshes: 1126 TETRA4

19.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU and EPSG_NOEU Stested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-5}$
v	ANALYTICAL	$6. 10^{-5}$	$2. 10^{-3}$
σ_{xx}	ANALYTICAL	99.9566	0.025
σ_{yy}	ANALYTICAL	-59.9955	0.02
σ_{zz}	ANALYTICAL	19.9326	0.03
σ_{xy}	ANALYTICAL	0.	3.0
VMIS	ANALYTICAL	138.5226	0.01
TRESCA	ANALYTICAL	159.9521	0.01
PRIN_1	ANALYTICAL	-59.9955	0.025
PRIN_2	ANALYTICAL	19.9326	0.03
PRIN_3	ANALYTICAL	99.9566	0.015
VMIS_SG	ANALYTICAL	138.5226	0.01

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$1. 10^{-3}$
v	ANALYTICAL	$2.1217 10^{-5}$	$1. 10^{-3}$
σ_{xx}	ANALYTICAL	20,003	0.005
σ_{yy}	ANALYTICAL	20,003	0.005
σ_{zz}	ANALYTICAL	20,003	0.005
σ_{xy}	ANALYTICAL	20,003	0.01

Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification	Type of reference	Place
ERME_ELEM NUEST	NON_REGRESSION	M1 mesh
ERME_ELNO ERREST	NON_REGRESSION	Node A

20 Modeling R

20.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UPGB (DEFORMATION='SIME_MIEHE') incompressible of type TRIA6 and QUAD8.

20.2 Characteristics of the grid

Many nodes: 591
Many meshes: 200 TRIA6 and 50 QUAD8

20.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU and EPSG_NOEU Stested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-5}$
v	ANALYTICAL	$6. 10^{-5}$	$5. 10^{-3}$
σ_{xx}	ANALYTICAL	99.9566	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	-59.9955	0.03
σ_{zz}	ANALYTICAL	19.9326	0.03
σ_{xy}	ANALYTICAL	0.	3.0
E_{xx}	ANALYTICAL	0.00059946	0.013
E_{yy}	ANALYTICAL	-0.00060018	0.01
VMIS	ANALYTICAL	138.5226	0.02
TRESCA	ANALYTICAL	159.9521	0.02
PRIN_1	ANALYTICAL	-59.9955	0.03
PRIN_2	ANALYTICAL	19.9326	0.03
PRIN_3	ANALYTICAL	99.9566	0.02
VMIS_SG	ANALYTICAL	138.5226	0.02

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$5. 10^{-3}$
v	ANALYTICAL	$2.1217 10^{-5}$	$5. 10^{-3}$
σ_{xx}	ANALYTICAL	20,003	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	20,003	$5. 10^{-3}$
σ_{zz}	ANALYTICAL	20,003	$5. 10^{-3}$
σ_{xy}	ANALYTICAL	20,003	$5. 10^{-3}$

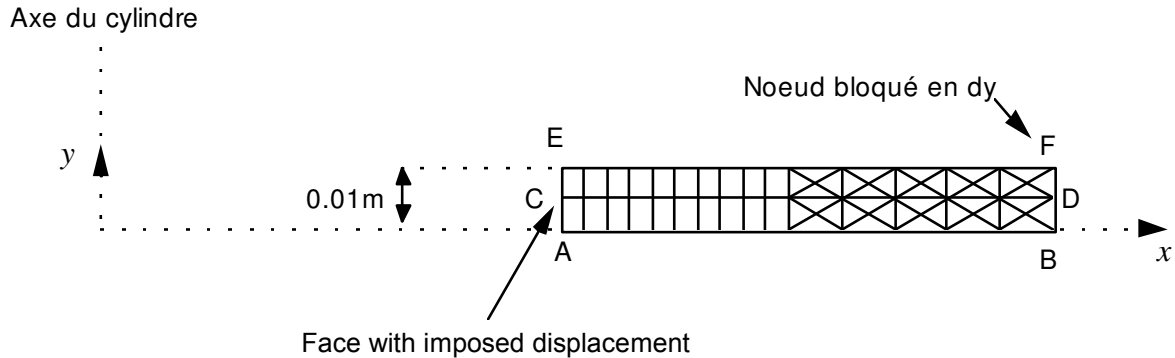
Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification	Type of reference	Place
ERME_ELEM NUEST	NON REGRESSION	M1 mesh
ERME_ELNO ERREST	NON REGRESSION	Node A

21 Modeling S

21.1 Characteristics of modeling

Grid with elements AXIS_INCO_UPGB (DEFORMATION=' SIMO_MIEHE ') incompressible of type TRIA6 and QUAD8



Limiting conditions:

```
DDL_IMPO = GROUP_NO = ' FACSUP '      DY = 0.      side EF
           GROUP_NO = ' FACINF '      DY = 0.      side AB
FACE_IMPO = GROUP_MA = ' FACEAE '     DX = 6. 10-5 face AE
```

21.2 Characteristics of the grid

Many nodes: 175.
Many meshes and types: 20 QUAD8, 40 TRIA6.

21.3 Sizes tested and results

Displacements and the constraints are evaluated at the points *A* and *F*. Components of the field SIEQ_NOEU are tested at the point *A* only.

Identification		Type of reference	Reference	Tolerance
<i>A</i>	<i>u</i>	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	<i>v</i>	ANALYTICAL	0.	$1. 10^{-5}$
	σ_{xx}	ANALYTICAL	-59.9955	$5. 10^{-3}$
	σ_{yy}	ANALYTICAL	19.9326	$5. 10^{-3}$
	σ_{zz}	ANALYTICAL	99.9566	$5. 10^{-3}$
	σ_{xy}	ANALYTICAL	0.	$1. 10^{-5}$
	VMIS	ANALYTICAL	138.5226	$5. 10^{-3}$
	TRESCA	ANALYTICAL	159.9521	$5. 10^{-3}$
	PRIN_1	ANALYTICAL	-59.9955	$5. 10^{-3}$
	PRIN_2	ANALYTICAL	19.9326	$5. 10^{-3}$
	PRIN_3	ANALYTICAL	99.9566	$5. 10^{-3}$
	VMIS_SG	ANALYTICAL	138.5226	$5. 10^{-3}$

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$3. 10^{-5}$	$1. 10^{-3}$
v	ANALYTICAL	0.	$1. 10^{-5}$
σ_{xx}	ANALYTICAL	0.	0.03
σ_{yy}	ANALYTICAL	20.0	$5. 10^{-3}$
σ_{zz}	ANALYTICAL	40,006	$5. 10^{-3}$
σ_{xy}	ANALYTICAL	0.	$5. 10^{-3}$

Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification	Type of reference	Place
ERME_ELEM NUEST	NON REGRESSION	M1 mesh
ERME_ELNO ERREST	NON REGRESSION	Node A

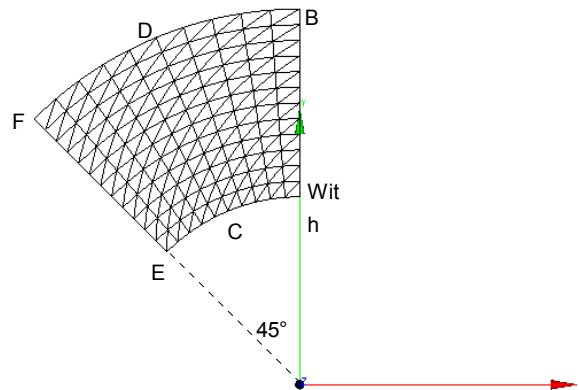
For the deformations of Green-Lagrange:

Identification	Type of reference	Reference	Tolerance
A E_{xx}	ANALYTICAL	-0.000598704996172	2nd-3

22 Modeling T

22.1 Characteristics of modeling

Grid with elements 3D_INCO_UPGB (DEFORMATION=' SIMO_MIEHE') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO = GROUP_NO = ' FACSUP' DZ = 0.
GROUP_NO = ' FACINF' DZ = 0. faces *AEFD* ($z=0$ and $z=0.01$)

GROUP_NO = ' FACEAB' DX = 0. face *AB*
FACE_IMPO = GROUP_MA = ' FACEEF' DNOR = 0. face *EF*
GROUP_MA = ' FACEAE' DNOR = -6.10^{-5} face *AE*

22.2 Characteristics of the grid

Many nodes: 1861
Many meshes: 480 PENTA15

22.3 Sizes tested and results

Displacements and the constraints are evaluated at the points *A* and *F*. Components of the field SIEQ_NOEU are tested at the point *A* only.

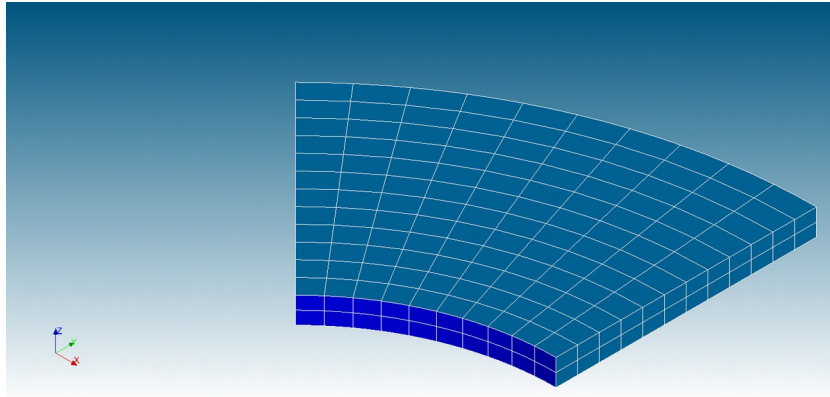
Identification	Type of reference	Reference	Tolerance
<i>A</i> u	ANALYTICAL	0.	1.10^{-5}
v	ANALYTICAL	6.10^{-5}	1.10^{-5}
σ_{xx}	ANALYTICAL	99.9566	0.03
σ_{yy}	ANALYTICAL	-59.9955	0.09
σ_{zz}	ANALYTICAL	19.9326	0.08
σ_{xy}	ANALYTICAL	0.	0.5
VMIS	ANALYTICAL	138.5226	0.05
TRESCA	ANALYTICAL	159.9521	0.05
PRIN_1	ANALYTICAL	-59.9955	0.09
PRIN_2	ANALYTICAL	19.9326	0.08
PRIN_3	ANALYTICAL	99.9566	0.03
VMIS_SG	ANALYTICAL	138.5226	0.05

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 \cdot 10^{-5}$	$3 \cdot 10^{-3}$
v	ANALYTICAL	$2.1217 \cdot 10^{-5}$	$3 \cdot 10^{-3}$
σ_{xx}	ANALYTICAL	20,003	0.04
σ_{yy}	ANALYTICAL	20,003	0.17
σ_{zz}	ANALYTICAL	20,003	0.07
σ_{xy}	ANALYTICAL	20,003	0,008

23 Modeling U

23.1 Characteristics of modeling

Grid with elements 3D_INCO_UPGB (DEFORMATION='GDEF_LOG ') incompressible of type HEXA20 only.



23.2 Characteristics of the grid

Many nodes: 1501
Many meshes: 480 HEXA20

23.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU and EPSG_NOEU Stested at the point A only.

Identification	Type of reference	Reference	Tolerance	
A	u	ANALYTICAL	0.	$1. 10^{-3}$
	v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
	σ_{xx}	ANALYTICAL	99.9566	0.001
	σ_{yy}	ANALYTICAL	-59.9955	0.003
	σ_{zz}	ANALYTICAL	19.9326	0.005
	σ_{xy}	ANALYTICAL	0.	0.03
	$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.003
	$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.001
	$E(\log)_{zz}$	ANALYTICAL	0	$1. 10^{-6}$
	$E(\log)_{xy}$	ANALYTICAL	0	$1. 10^{-6}$
VMIS	ANALYTICAL	138.5226	0.001	
TRESCA	ANALYTICAL	159.9521	0.001	
PRIN_1	ANALYTICAL	-59.9955	0.0025	
PRIN_2	ANALYTICAL	19.9326	0.005	
PRIN_3	ANALYTICAL	99.9566	0.0005	
VMIS_SG	ANALYTICAL	138.5226	0.001	
F	u	ANALYTICAL	$-2.1217 10^{-5}$	$5. 10^{-5}$
	v	ANALYTICAL	$2.1217 10^{-5}$	$5. 10^{-5}$

σ_{xx}	ANALYTICAL	20,003	0,002
σ_{yy}	ANALYTICAL	20,003	0.002
σ_{zz}	ANALYTICAL	20,003	0.0025
σ_{xy}	ANALYTICAL	20,003	0.0015
$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.002
$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.002
$E(\log)_{zz}$	ANALYTICAL	0	1. 10 ⁻⁶
$E(\log)_{xy}$	ANALYTICAL	0.00015002	0.015

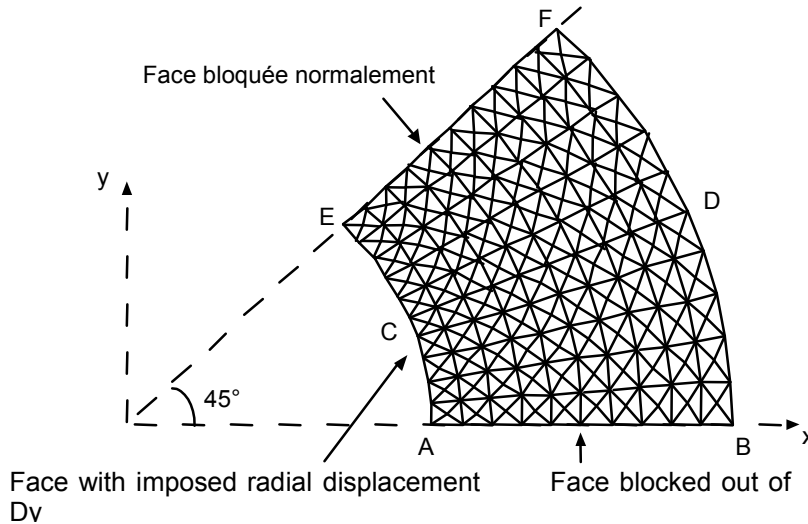
Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification		Type of reference	Place
ERME_ELEM	NUEST	NON_REGRESSION	M1 mesh
ERME_ELNO	ERREST	NON_REGRESSION	Node A

24 Modeling V

24.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).

The grid was obtained with GMSH for a density of 0,01 .

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP '	DZ = 0.	faces $AEFD$ ($z=0$ and $z=0.01$)
	GROUP_NO = ' FACINF '	DZ = 0.	
FACE_IMPO =	GROUP_NO = ' FACEAB '	DY = 0.	face AB
	GROUP_MA = ' FACEEF '	DNOR = 0.	face EF
	GROUP_MA = ' FACEAE '	DNOR = -6.10^{-5}	face AE

24.2 Characteristics of the grid

Many nodes: 2078

Many meshes: 1126 TETRA10

24.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU are tested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	6.10^{-5}	1.10^{-3}
v	ANALYTICAL	0.	2.10^{-3}
σ_{xx}	ANALYTICAL	-59.9955	0.02
σ_{yy}	ANALYTICAL	99.9566	0.03
σ_{zz}	ANALYTICAL	19.9326	0.03
σ_{xy}	ANALYTICAL	0.	3
$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.008
$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.01
$E(\log)_{zz}$	ANALYTICAL	0	1.10^{-6}
$E(\log)_{xy}$	ANALYTICAL	0	1.10^{-6}
VMIS	ANALYTICAL	138.5226	0.005
TRESCA	ANALYTICAL	159.9521	0.005
PRIN_1	ANALYTICAL	-59.9955	0.03
PRIN_2	ANALYTICAL	19.9326	0.03
PRIN_3	ANALYTICAL	99.9566	0,015
VMIS_SG	ANALYTICAL	138.5226	0.005

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$2.1217 \cdot 10^{-5}$	1.10^{-3}
v	ANALYTICAL	$2.1217 \cdot 10^{-5}$	1.10^{-3}
σ_{xx}	ANALYTICAL	20,003	0,003
σ_{yy}	ANALYTICAL	20,003	0,005
σ_{zz}	ANALYTICAL	20,003	0,002
σ_{xy}	ANALYTICAL	-20,003	0.01
$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.002
$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.002
$E(\log)_{zz}$	ANALYTICAL	0	1.10^{-6}
$E(\log)_{xy}$	ANALYTICAL	0.00015002	6.10^{-3}

Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification	Type of reference	Place
ERME_ELEM NUEST	NON REGRESSION	M1 mesh
ERME_ELNO ERREST	NON REGRESSION	Node A

25 Modeling W

25.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UPGB (DEFORMATION='GDEF_LOG') incompressible of type TRIA6 and QUAD8.

25.2 Characteristics of the grid

Many nodes: 591
Many meshes: 200 TRIA6 and 50 QUAD8

25.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F . Components of the field SIEQ_NOEU and EPSG_NOEU Stested at the point A only.

Identification	Type of reference	Reference	Tolerance
A u	ANALYTICAL	0.	$1. 10^{-3}$
v	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
σ_{xx}	ANALYTICAL	99.9566	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	-59.9955	0.03
σ_{zz}	ANALYTICAL	19.9326	0.03
σ_{xy}	ANALYTICAL	0.	3.0
$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.012
$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.012
$E(\log)_{zz}$	ANALYTICAL	0	$1. 10^{-6}$
$E(\log)_{xy}$	ANALYTICAL	0	$1. 10^{-6}$
VMIS	ANALYTICAL	138.5226	0.02
TRESCA	ANALYTICAL	159.9521	0.02
PRIN_1	ANALYTICAL	-59.9955	0.03
PRIN_2	ANALYTICAL	19.9326	0.03
PRIN_3	ANALYTICAL	99.9566	0,004
VMIS_SG	ANALYTICAL	138.5226	0.02

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 10^{-5}$	$3. 10^{-4}$
v	ANALYTICAL	$2.1217 10^{-5}$	$3. 10^{-4}$
σ_{xx}	ANALYTICAL	20,003	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	20,003	$2.5 10^{-3}$
σ_{zz}	ANALYTICAL	20,003	$5. 10^{-4}$
σ_{xy}	ANALYTICAL	20,003	$2. 10^{-3}$
$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.002
$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.002
$E(\log)_{zz}$	ANALYTICAL	0	$1. 10^{-6}$
$E(\log)_{xy}$	ANALYTICAL	0.00015002	0.0015

Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

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Titre : SSNV112 - Cylindre creux en incompressible (grande[...]
Responsable : MICHEL-PONNELLE Sylvie

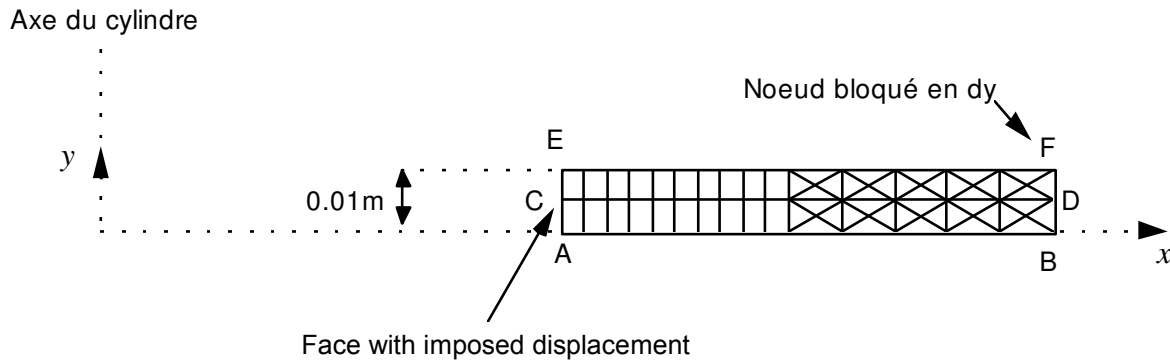
Date : 12/12/2017 Page : 49/54
Clé : V6.04.112 Révision :
a47c4afa52dc

Identification			Type of reference	Place
ERME	ELEM	NUEST	NON REGRESSION	M1 mesh
ERME	ELNO	ERREST	NON REGRESSION	Node A

26 Modeling X

26.1 Characteristics of modeling

Grid with elements AXIS_INCO_UPGB (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

```
DDL_IMPO = GROUP_NO = ' FACSUP'      DY = 0.          side EF
          GROUP_NO = ' FACINF'      DY = 0.          side AB
FACE_IMPO = GROUP_MA = ' FACEAE'    DX = 6. 10-5   face AE
```

26.2 Characteristics of the grid

Many nodes: 175.
Many meshes and types: 20 QUAD8, 40 TRIA6.

26.3 Sizes tested and results

Displacements and the constraints are evaluated at the points *A* and *F*. Components of the field SIEQ_NOEU are tested at the point *A* only.

Identification	Type of reference	Reference	Tolerance
<i>A</i> u	ANALYTICAL	$6. 10^{-5}$	$1. 10^{-3}$
v	ANALYTICAL	0.	$1. 10^{-3}$
σ_{xx}	ANALYTICAL	-59.9955	$5. 10^{-3}$
σ_{yy}	ANALYTICAL	19.9326	$5. 10^{-3}$
σ_{zz}	ANALYTICAL	99.9566	$5. 10^{-4}$
σ_{xy}	ANALYTICAL	0.	$1. 10^{-6}$
$E(\log)_{xx}$	ANALYTICAL	0.00059946	0.003
$E(\log)_{yy}$	ANALYTICAL	-0.00060018	0.001
$E(\log)_{zz}$	ANALYTICAL	0,00060018	0.005
$E(\log)_{xy}$	ANALYTICAL	0	$1. 10^{-6}$
VMIS	ANALYTICAL	138.5226	$2. 10^{-3}$
TRESCA	ANALYTICAL	159.9521	$2. 10^{-3}$
PRIN_1	ANALYTICAL	-59.9955	$3. 10^{-3}$

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PRIN_2	ANALYTICAL	19.9326	$5 \cdot 10^{-3}$
PRIN_3	ANALYTICAL	99.9566	$5 \cdot 10^{-4}$
VMIS_SG	ANALYTICAL	138.5226	$2 \cdot 10^{-3}$

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$3 \cdot 10^{-5}$	$5 \cdot 10^{-4}$
v	ANALYTICAL	0.	$1 \cdot 10^{-3}$
σ_{xx}	ANALYTICAL	0.	0.03
σ_{yy}	ANALYTICAL	20.0	$3 \cdot 10^{-4}$
σ_{zz}	ANALYTICAL	40,006	$3 \cdot 10^{-4}$
σ_{xy}	ANALYTICAL	0.	$1 \cdot 10^{-6}$
$E(\log)_{xx}$	ANALYTICAL	-0.00015	0.002
$E(\log)_{yy}$	ANALYTICAL	0	$1 \cdot 10^{-6}$
$E(\log)_{zz}$	ANALYTICAL	+0.00015	0.0025
$E(\log)_{xy}$	ANALYTICAL	0	$1 \cdot 10^{-6}$

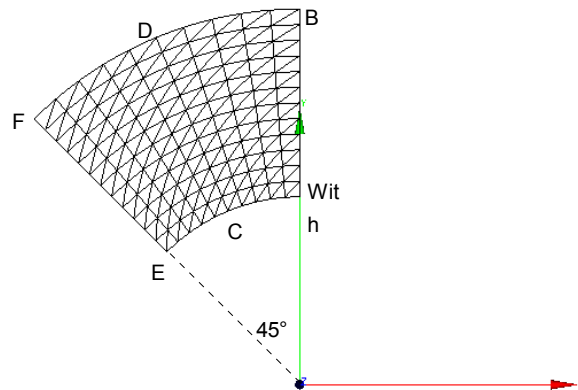
Test on the estimator in residue (ERME_ELEM and ERME_ELNO):

Identification	Type of reference	Place
ERME_ELEM NUEST	NON REGRESSION	M1 mesh
ERME_ELNO ERREST	NON REGRESSION	Node A

27 Modeling Y

27.1 Characteristics of modeling

Grid with elements 3D_INCO_UPGB (DEFORMATION='GDEF_LOG') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO = GROUP_NO = ' FACSUP' DZ = 0.
 GROUP_NO = ' FACINF' DZ = 0. faces *AEFD* ($z=0$ and $z=0.01$)
 GROUP_NO = ' FACEAB' DX = 0. face *AB*
 FACE_IMPO = GROUP_MA = ' FACEEF' DNOR = 0. face *EF*
 GROUP_MA = ' FACEAE' DNOR = -6.10^{-5} face *AE*

27.2 Characteristics of the grid

Many nodes: 1861
Many meshes: 480 PENTA15

27.3 Sizes tested and results

Displacements and the constraints are evaluated at the points *A* and *F*. Components of the field SIEQ_NOEU are tested at the point *A* only.

Identification	Type of reference	Reference	Tolerance
<i>A</i> u	ANALYTICAL	0.	1.10^{-5}
v	ANALYTICAL	6.10^{-5}	1.10^{-3}
σ_{xx}	ANALYTICAL	99.9566	0.03
σ_{yy}	ANALYTICAL	-59.9955	0.09
σ_{zz}	ANALYTICAL	19.9326	0.08
σ_{xy}	ANALYTICAL	0.	0.08
$E(\log)_{xx}$	ANALYTICAL	0.00060018	0.05
$E(\log)_{yy}$	ANALYTICAL	-0.00059946	0.05
$E(\log)_{zz}$	ANALYTICAL	0	1.10^{-6}
$E(\log)_{xy}$	ANALYTICAL	0	1.10^{-6}
VMIS	ANALYTICAL	138.5226	0.05
TRESCA	ANALYTICAL	159.9521	0.05

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

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Titre : SSNV112 - Cylindre creux en incompressible (grande[...]
Responsable : MICHEL-PONNELLE Sylvie

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PRIN_1	ANALYTICAL	-59.9955	0.09
PRIN_2	ANALYTICAL	19.9326	0.08
PRIN_3	ANALYTICAL	99.9566	0.03
VMIS_SG	ANALYTICAL	138.5226	0.05

Identification	Type of reference	Reference	Tolerance
F u	ANALYTICAL	$-2.1217 \cdot 10^{-5}$	$3 \cdot 10^{-3}$
v	ANALYTICAL	$2.1217 \cdot 10^{-5}$	$3 \cdot 10^{-3}$
σ_{xx}	ANALYTICAL	20,003	0.04
σ_{yy}	ANALYTICAL	20,003	0.17
σ_{zz}	ANALYTICAL	20,003	0.07
σ_{xy}	ANALYTICAL	20,003	0.01
$E(\log)_{xx}$	ANALYTICAL	+0.00015	0,002
$E(\log)_{yy}$	ANALYTICAL	-0.00015	0,002
$E(\log)_{zz}$	ANALYTICAL	0	$1 \cdot 10^{-6}$
$E(\log)_{xy}$	ANALYTICAL	+0.00015	0.01

28 Summary of the results

With a Poisson's ratio ν very near to 0.5 , one finds the results of the incompressible analytical solution in great deformations, with a completely correct precision.