SSNV124 - Regularized limiting analysis. Law of Norton - Hoff

Summary

This test makes it possible to validate the operators used analyzes regularized limit of it. One calculates the limiting load by a kinematic approach regularized by the method of Norton-Hoff-Friaâ.

One considers a rectangular plate (modeling A) or a cube (modeling B) or an axisymmetric cylinder (modeling C). The constitutive material checks the criterion of resistance of von Mises and the structure is subjected to loadings on the edges. Calculation makes it possible to obtain the parameter of the limiting load in the direction of the loading.

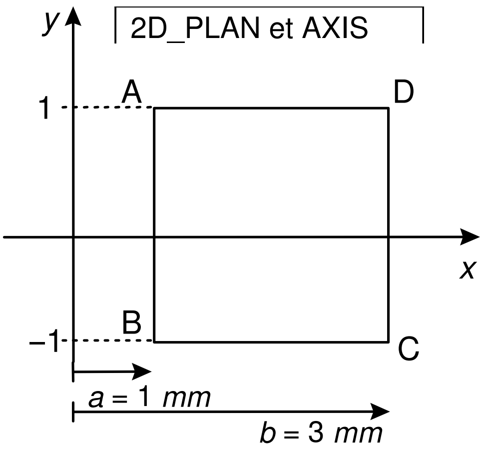
The structure is modelled by incompressible elements and the loading which one seeks the parameter of limiting load is standardized (unit power).

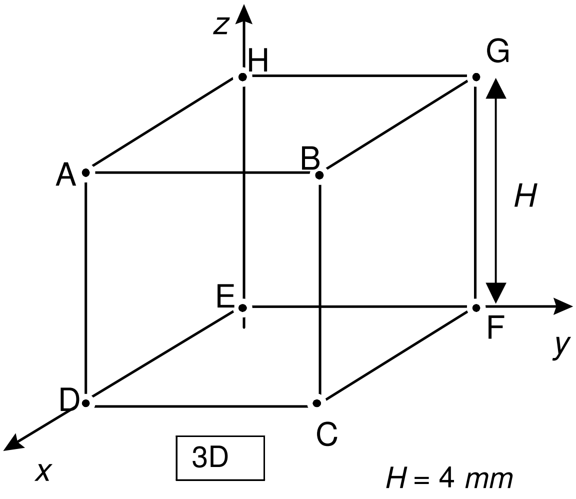
The resolution by the regularized method of Norton-Hoff-Friaâ is carried out by piloting in the order `STAT_NON_LINE`. A postprocessing in the order `POST_ELEM` allows to obtain the value of an upper limit of the limiting load, as well as an estimate by lower value, when there is no constant loading applied.

The reference solution is analytical and the results are in perfect agreement with the values of reference.

1 Problem of reference

1.1 Geometry





1.2 Material properties

Elastic limit: $\sigma_y = 10\,MPa$.

1.3 Boundary conditions and loadings

Limiting conditions in 2D:

- on AB : $DX = 0.$
- on BC : $DY = 0.$

Limiting conditions in 3D:

- $EFGH$ ($FACEXINF$): $DX = 0.$
- $ADEH$ ($FACEYINF$): $DY = 0.$
- $DCFE$ ($FACEZINF$): $DZ = 0.$

Limiting conditions in AXIS:

- on BC and AD : $DY = 0.$

The loading parameterized by λ is:

- in 2D:

  $FY = -1.$ on AD

- in 3D:

  $FX = -0.2$ on $ABCD$ ($FXSUP$)
  $FY = -0.8$ on $BCFG$ ($FYSUP$)

- in AXIS:

  $FX = 1.$ on AB .

2 Reference solution

2.1 Method of calculating used for the reference solution

The constitutive material checks the criterion of von Mises, with for threshold σ_y. The structure is subjected to pressures on the edges horizontal $-\alpha f$ and vertical $-(1-\alpha)f$ with $\alpha \geq 0.5$ ($\alpha = 1.$ in 2D, $\alpha = 0.8$ in 3D). In 2D plan, one considers two ways of making: on the one hand by amplifying the two pressures together, on the other hand by amplifying only the horizontal pressure, and by leaving the constant vertical pressure. Into axisymmetric, the solid is subjected to the pressure only interns $-\alpha f$. One obtains the exact limiting load and that by the method of regularization [R7.07.01] in this direction of loading, for the criterion of von Mises, with the threshold σ_y.

2.2 Case plan

The structure is subjected to pressures on the edges horizontal: $-\alpha f$ and vertical: $-(1-\alpha)f$, with: $\alpha \geq 1/2$, and one exerts a blocking in z. One considers two ways of controlling the loading:

- case 1: the two pressures horizontal and vertical are parameterized by λ,
- case 2: the horizontal pressure is parameterized by λ, while the vertical pressure is constant $-(1-\alpha)f_0$, with $f_0 = \lambda_0 f$.

2.2.1 Solution analyzes limit of it

The solution is homogeneous (biaxées constraints σ : $\sigma_{xx} = \alpha f, \sigma_{yy} = (1-\alpha)f$, $\sigma_{xy} = 0$, plane deformations ε). One obtains [bib2] the limiting load in these directions of loading, for the criterion of von Mises, in plane deformations, with the threshold σ_y :

$$\text{case 1: } \lambda_{\lim} . f = \frac{2\sigma_y}{\sqrt{3}.|2\alpha - 1|} \qquad \text{éq 2.2.1-1}$$

$$\text{case 2: } \lambda_{\lim} . f = \frac{2\sqrt{3}\sigma_y}{3.|\alpha|} + \frac{1-\alpha}{|\alpha|}\lambda_0 . f \qquad \text{éq 2.2.1-2}$$

It is checked that if one takes $\lambda_0 = \lambda_{\lim}$ in the cas2, the cas1 then is found.

2.2.2 Solution analyzes regularized limit of it

The solution is homogeneous. The plane deformations are necessarily of the form:

$$\varepsilon(\mathbf{u}) = \gamma \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \sqrt{\varepsilon(\mathbf{u}).\varepsilon(\mathbf{u})} = |\gamma|\sqrt{2} \qquad \text{éq 2.2.2-1}$$

By the law of Norton-Hoff, the coefficient $m \in]1, 2[$ being given, the deviatoric constraints are obtained:

$$\sigma^D = A(m)\sqrt{2}^{m-2}|\gamma|^{m-2}\gamma. \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \|\sigma^D\|_{VM} = A(m)\sqrt{2}^{m-2}|\gamma|^{m-1}\sqrt{3} \qquad \text{éq 2.2.2-2}$$

The standardisation (unit power) of the loading which one seeks the parameter of limiting load, cf [R7.07.01] [§1.2], led to:

$$\text{case 1: } \gamma f = \frac{1}{H(b-a)(2\alpha - 1)} \qquad \text{éq 2.2.2-3}$$

case 2: $\gamma f = \frac{1}{H(b-a)\alpha}$ éq 2.2.2-4

Terms of the continuation $\hat{\lambda}_m$ approximations by excess of the limiting load in these two parameter settings of the loading are then:

case 1: $\hat{\lambda}_m . f = \frac{2\sqrt{3}\sigma_y}{3.|2\alpha-1|} \quad \forall m$ éq 2.2.2-5

case 2: $\hat{\lambda}_m . f = \frac{2\sqrt{3}\sigma_y}{3.|\alpha|} + \frac{1-\alpha}{|\alpha|}\lambda_0 . f \quad \forall m$ éq 2.2.2-6

Invariance according to m observed here (what is a typical case) results owing to the fact that one is in an isostatic situation. In case 1, one can also exploit the continuation of the approximations by default of the limiting load $\underline{\lambda}_m$:

case 1: $\underline{\lambda}_m . f = \frac{2\sqrt{3}\sigma_y}{3m.|2\alpha-1|}$ éq 2.2.2-7

One thus obtains the limiting load $\lambda_{\lim}$ exact when $m \to 1^+$.

In case 2, where the vertical pressure is constant, the power of this "permanent" loading in solution displacement is:

case 2: $\mathsf{L}_0(\mathbf{u}) = \frac{1-\alpha}{|\alpha|}.\lambda_0 f$ éq 2.2.2-8

2.3 Axisymmetric case

In 2D axisymmetric one considers the same geometry, but the solid, on which one imposes a complete axial blocking, is only subjected to a pressure on the internal wall: αf parameterized by λ.

2.3.1 Solution analyzes limit of it

One obtains [bib2] the limiting load in this direction of loading, for the criterion of von Mises, into axisymmetric and worthless axial deformations, with the threshold σ_y :

$$\lambda_{\lim} . \alpha f = \frac{2\sqrt{3}}{3}\sigma_y \ln\frac{b}{a}$$ éq 2.3.1-1

2.3.2 Solution analyzes regularized limit of it

The solution is homogeneous. Displacement being only radial, the isochoric deformations are necessarily of the form:

$$u_r(r) = \frac{\gamma}{r}; \quad \varepsilon(\mathbf{u}) = \frac{\gamma}{r^2}\begin{bmatrix}-1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}; \quad \sqrt{\varepsilon(\mathbf{u}).\varepsilon(\mathbf{u})} = \frac{|\gamma|}{r^2}\sqrt{2}$$ éq 2.3.2-1

By the law of Norton-Hoff, the coefficient $m \in]1, 2]$ being given, the deviatoric constraints are obtained:

$$\sigma^D = A(m)\sqrt{2}^{m-2}|\gamma|^{m-2}\gamma.r^{-2m+2}.\begin{bmatrix}-1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}; \|\sigma^D\|_{VM} = A(m)\sqrt{2}^{m-2}|\gamma|^{m-1}.r^{-2m+2}.\sqrt{3}$$ éq 2.3.2-2

The equilibrium equations axial and radial result in determining the average constraint:

$$\operatorname{tr}\sigma(r) = 3A(m)\sqrt{2}^{m-2}.\gamma.|\gamma|^{m-2}.r^{-2m+2}.\frac{2-m}{1-m} + 3\tau$$ éq 2.3.2-3

where τ is a constant, which is calculated starting from the boundary condition of worthless pressure in external wall. The components of the constraints then are obtained:

$$\begin{cases} \sigma_{rr}(r) = \beta\left(b^{-2m+2} - r^{-2m+2}\right) \\ \sigma_{zz}(r) = \beta\left(b^{-2m+2} - (2-m)\, r^{-2m+2}\right) \\ \sigma_{\theta\theta}(r) = \beta\left(b^{-2m+2} - (3-2m)\, r^{-2m+2}\right) \end{cases} \quad avec:\ \beta = \frac{A(m)\sqrt{2}^{m-2}}{(m-1)(\alpha f H)^{m-1}} \qquad \text{éq 2.3.2-4}$$

The standardisation (unit power) of the loading which one seeks the parameter of limiting load, cf [R7.07.01] [§1.2], led to: $\alpha f \gamma = \frac{1}{H}$.

Terms of the continuation $\hat{\lambda}_m$ approximations by excess of the limiting load for this loading are then:

$$\hat{\lambda}_m . \alpha f = \frac{2\sqrt{3}}{3}\sigma_y H \int_a^b \frac{|\gamma|}{r^2} r dr = \frac{2\sqrt{3}}{3}\sigma_y \ln\frac{b}{a} \quad \forall m \qquad \text{éq 2.3.2-5}$$

Terms of the continuation $\underline{\lambda}_m$ approximations by default of the limiting load for this loading are:

$$\underline{\lambda}_m . \alpha f = \frac{2\sqrt{3}}{3m}\sigma_y \int_a^b r^{-2m} . \left[\underset{]a,b[}{Max}\left(r^{-2m+2}\right)\right]^{-1} r dr = \frac{\sigma_y \sqrt{3}}{3m(1-m)} \frac{b^{-2m+2} - a^{-2m+2}}{a^{-2m+2}} \qquad \text{éq 2.3.2-6}$$

In $m \to 1^+$, one finds: $\underline{\lambda}_{1^+} . \alpha f = \frac{2\sqrt{3}}{3}\sigma_y \ln\frac{b}{a}$, i.e. the same value as $\hat{\lambda}_m$ and $\lambda_{\lim}$.

2.4 Three-dimensional case

In 3D one considers the same geometry, but the solid, unit thickness, is free in the direction antiplane z. The solid is subjected to pressures on the walls horizontal: $-\alpha f$ and vertical: $-(1-\alpha) f$, with: $\alpha \geq 1/2$. The two pressures horizontal and vertical are parameterized by λ.

2.4.1 Solution analyzes limit of it

The solution is homogeneous (biaxées constraints σ : $\sigma_{xx} = \alpha f, \sigma_{yy} = (1-\alpha) f$, $\sigma_{xy} = 0$, $\sigma_{zz} = 0$, deformations ε). One obtains the limiting load in this direction of loading [bib2], for the criterion of von Mises, with the threshold σ_y :

$$\lambda_{\lim} . f = \frac{\sigma_y}{\sqrt{3\alpha^2 - 3\alpha + 1}} \qquad \text{éq 2.4.1-1}$$

2.4.2 Solution analyzes regularized limit of it

The solution is homogeneous. The isochoric deformations are necessarily of the form:

$$\varepsilon(\mathbf{u})=\gamma\begin{bmatrix}1&0&0\\0&\delta&0\\0&0&-1-\delta\end{bmatrix}\quad;\qquad\sqrt{\varepsilon(\mathbf{u}).\varepsilon(\mathbf{u})}=|\gamma|\sqrt{2(1+\delta+\delta^2)}$$ éq 2.4.2-1

By the law of Norton-Hoff, the coefficient $m\in]1,2[$ being given, the deviatoric constraints are obtained:

$$\sigma^D=\beta.\begin{bmatrix}1&0&0\\0&\delta&0\\0&0&-1-\delta\end{bmatrix}\quad;\qquad\left\|\sigma^D\right\|_{VM}=|\beta|\sqrt{3(1+\delta+\delta^2)}\ avec:\ \beta=A(m)\sqrt{2(1+\delta+\delta^2)}^{m-2}|\gamma|^{m-2}\gamma$$

éq 2.4.2-2

One deduces from $\sigma_{zz}=0$: $\mathrm{tr}\,\sigma=3\beta(1+\delta)$. From where constraints: $\sigma=\beta.\begin{bmatrix}2+\delta&0&0\\0&1+2\delta&0\\0&0&0\end{bmatrix}$.

The balance of the solid imposes that $\sigma_{xx}.(1-\alpha)=\sigma_{yy}.\alpha$. One from of deduced the parameter $\delta=\dfrac{3\alpha-2}{1-3\alpha}$.

The standardisation (unit power) of the loading which one seeks the parameter of limiting load, cf [R7.07.01] [§1.2], led to:

$$\gamma f=\frac{1}{H(b-a)(\alpha+\delta(1-\alpha))}$$ éq 2.4.2-3

Terms of the continuation $\hat{\lambda}_m$ hight delimiters of the limiting load in this case of loading are thus identical to:

$$\hat{\lambda}_m.f=\frac{2\sqrt{3}\sigma_y}{3}.\frac{\sqrt{2(1+\delta+\delta^2)}}{\alpha+\delta(1-\alpha)}=\frac{\sigma_y}{\sqrt{3\alpha^2-3\alpha+1}}$$ éq 2.4.2-4

2.5 Results of reference

Modeling	case	$\lambda_{\lim}$	$\lambda_{\lim}^{\sup}=\hat{\lambda}_m$	$\lambda_{\lim}^{estimée}=\bar{\lambda}_m$	power $\mathrm{L}_0(\mathbf{u})$
With (case 1)	2D plan	$\lambda_{\lim}=\frac{2}{\sqrt{3}\lvert 2\alpha-1\rvert}.\frac{\sigma_y}{f}$	$\frac{2}{\sqrt{3}\lvert 2\alpha-1\rvert}.\frac{\sigma_y}{f}$	$\frac{2}{\sqrt{3}\lvert 2\alpha-1\rvert}.\frac{\sigma_y}{mf}$	0
Abis (cas2)	2D plan	$\frac{2\sqrt{3}}{3\lvert\alpha\rvert}\frac{\sigma_y}{f}+\frac{1-\alpha}{\lvert\alpha\rvert}\lambda_0 f$	$\frac{2\sqrt{3}}{3\lvert\alpha\rvert}\frac{\sigma_y}{f}+\frac{1-\alpha}{\lvert\alpha\rvert}\lambda_0 f$	nothing	$\frac{1-\alpha}{\lvert\alpha\rvert}.\lambda_0 f$
B	3D	$\lambda_{\lim}=\frac{1}{\sqrt{3\alpha^2-3\alpha+1}}.\frac{\sigma_y}{f}$	$\frac{1}{\sqrt{3\alpha^2-3\alpha+1}}.\frac{\sigma_y}{f}$	$\frac{1}{\sqrt{3\alpha^2-3\alpha+1}}.\frac{\sigma_y}{mf}$	0
C $\alpha=1$	2D AXIS	$\lambda_{\lim}=\frac{2\sqrt{3}}{3}.\frac{\sigma_y}{f}.\ln\frac{b}{a}$	$\frac{2\sqrt{3}}{3}.\frac{\sigma_y}{f}.\ln\frac{b}{a}$	$\frac{\sigma_y\sqrt{3}\left((b/a)^{-2m+2}-1\right)}{3m(1-m)}$	0

Modeling	case	$\lambda_{\lim}$	$\lambda_{\lim}^{\sup}$	$\lambda_{\lim}^{estimée}$ ($m=1{,}2$)	$\lambda_{\lim}^{estimée}$ ($m=1{,}0001$)	power $L_0(\mathbf{u})$
With	2D plan	11,547	11,547	9.6225	11.5458	0
Abis	2D plan	14.6837	14.6837	nothing	nothing	0.25
B ($\alpha=0{,}8$)	3D	13.8675	13.8675	11.5562	13.8661	0
C	2D AXIS	12.6857	12.6857	8.5545	12.6830	0

Note:

Code_Aster calculates the opposite value of the power of the "permanent" loading in solution displacement $L_0(\mathbf{u})$ *.*

2.6 Bibliographical references

1) F.VOLDOIRE, E.LORENTZ, J.M.PROIX, E.VISSE: Calculation of load limits by the method of Norton-Hoff-Friaâ. [R7.07.01].

2) F.VOLDOIRE, Design the collapse and analyze limit of the structures, notes EDF HI-74/93/082.

3 Modeling A

3.1 Characteristics of modeling

One considers a rectangular plate modelled by element QUAD8 of an incompressible type: miplqu8. The two cases are studied: the first with the two amplified loads, the second with the amplified horizontal pressure and the constant vertical.

3.2 Characteristics of the grid

Many nodes: 8

Many meshs and types: 1 mesh of the type QUAD8, incompressible finite element.

3.3 Values tested

Identification	Case	Reference
Load limits higher	With Abis	11,547 14.6837
Estimated limiting load ($m=1,2$)	With Abis	9.6225 nothing
Power permanent loading	With Abis	0 – 0.25
`NOEUD=' N3' EPSI_ELNO: EPXX`	With	– 0.3125
`NOEUD=' N3' EPEQ_ELNO: INVA_2`	With	0.360844

4 Modeling B

4.1 Characteristics of modeling

One considers a cube modelled by an element HEXA20 of incompressible type: minc_hexa20.

4.2 Characteristics of the grid

Many nodes: 20.

Many meshs and types: 1 mesh of the type HEXA20 incompressible finite element.

4.3 Values tested

Identification	Reference
Load limits higher	13.867505
Estimated limiting load ($m = 1,2$)	11.5562
NOAHUD=' N1' EPSI_ELNO: EPXX	0.0407692

5 Modeling C

5.1 Characteristics of modeling

One considers a cylinder modelled by axisymmetric elements QUAD8 of incompressible type: miaxqu8, according to a regulated grid.

5.2 Characteristics of the grid

Many nodes: 96

Many meshs and types: 25 meshs of the type QUAD8 incompressible finite element.

5.3 Values tested

Identification	Reference
Load limits higher	12.6857
Estimated limiting load ($m=1,2$)	8.5545

6 Summary of the results

The digital results are in perfect agreement with the values of reference. In the axisymmetric case, the light differences are explained by the fact why displacement is in $1/r$ in the analytical solution, which is not understood in the base of the selected finite elements.