SSNV127 - Cylinder in a boring with contact and friction

Summary:

This problem corresponds to a quasi-static analysis of a problem of mechanics with contact and friction. A cylinder is compressed in a cylindrical boring of slightly higher diameter by a concentrated force applied to its axis.

This test, named “problem of Klang”, is rather largely used in the literature to validate modelings of contact with friction and was in particular used by P. Alart and A. Curnier [bib1] to validate their finite elements of contact and friction.

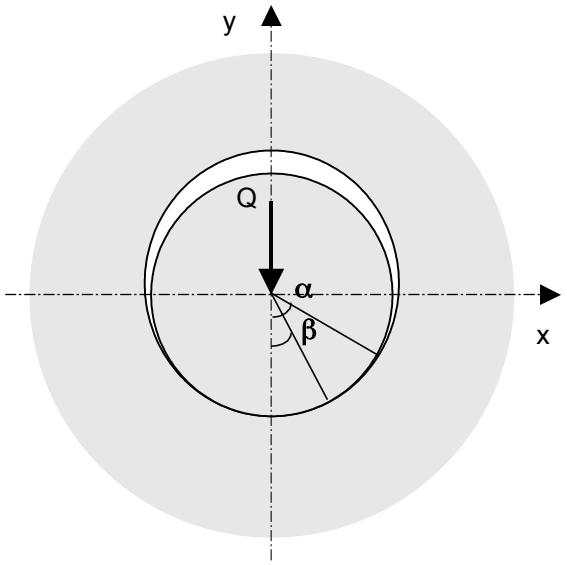
Three modelings 2D are proposed (D and H with linear elements and F with quadratic elements):
Modeling D tests the algorithm of the method “ PENALIZATION ” with master-slave pairing, the penalization relates to the contact and friction.
Modelings F and H test the algorithm of the method “ CONTINUOUS ” with quadratic elements (SEG3) and linear (SEG2) of contact, respectively.

The results are compared with an analytical solution given by Klang [bib2].
Instead of reproducing this relatively complicated analytical solution we will use the values of the pressures obtained from this one [bib1].

1 Problem of reference

1.1 Geometry

Contraintes planes



α = angle de contact
β = angle pour le glissement

Ray of the cylinder: $r=5.999\,cm$.

Ray of boring: $R=6.\,cm$.

Position of the points of reference on the surface of contact: a point all three degrees of angle starting from bottom, until 60°.

1.2 Material properties

Cylinder and boring:

Young modulus:	$E=2.1\,10^{11}\,N/m^2$
Poisson's ratio:	$\nu=0.3$
Coefficient of friction:	$\mu=0.4$

1.3 Boundary conditions and loadings

The solid mass containing boring is supposed infinite being, its displacements will be blocked (according to X and there) on a concentric circle with boring.

The cylinder is subjected to a force distributed Q according to the thickness (z) being worth: $Q=-1875\,10^3\,N/m$.
This force is applied in an increment.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is analytical [bib2].

2.2 Results of reference

Prediction on the zone of contact: 60 degrees.
Prediction on the beginning of the zone of slip: 26.2 degrees.

Efforts of pressure on the points of the surface of contact:

Identification		Reference
SIXX for an angle of	0°	– 1.7813E+07
SIXX	3°	– 1.7813E+07
SIXX	6°	– 1.7750E+07
SIXX	9°	– 1.7688E+07
SIXX	12°	– 1.7594E+07
SIXX	15°	– 1.7470E+07
SIXX	18°	– 1.7312E+07
SIXX	21°	– 1.7125E+07
SIXX	24°	– 1.6906E+07
SIXX	27°	– 1.6656E+07
SIXX	30°	– 1.6343E+07
SIXX	33°	– 1.5937E+07
SIXX	36°	– 1.5406E+07
SIXX	39°	– 1.4781E+07
SIXX	42°	– 1.4031E+07
SIXX	45°	– 1.3094E+07
SIXX	48°	– 1.1169E+07
SIXX	51°	–1.0593E+07

2.3 Bibliographical references

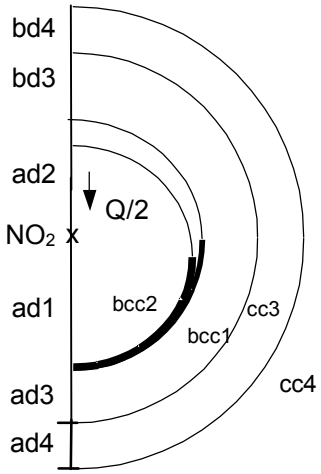
1) P. Alart, A. Curnier “with mixed formulation for frictional contact problems” Methods Computer in Applied Mechanics and Engineering (1991) p. 353-375

2) Mr. Klang “One interior contact under friction between cylindrical elastic bodies in contact” Thesis, Linköping University, Linköping, 1979.

3 Modeling D

3.1 Characteristics of modeling

This modeling is identical to modeling A (and in particular grid). Only the method of resolution differs which is here " PENALIZATION " with penalization on the contact and friction.

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CONTACT
METHOD = 'PENALIZATION
COULOMB = 0.4

APPARIEMENT=' MAIT_ESCL'
Master: bcc1
Slave: bcc2
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3.2 Values tested: SIGM_ELNO

One tests the efforts of normal pressure generated by contact-friction. These efforts defined in polar coordinates are expressed in Pa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	–1.7813E+07	– 1.867141E+07	4,819
SIXX	3°	– 1.7813E+07	– 1.972695E+07	10,745
SIXX	6°	– 1.7750E+07	– 1.999288E+07	12,636
SIXX	9°	– 1.7688E+07	– 1.988495E+07	12,421
SIXX	12°	– 1.7594E+07	– 1.971935E+07	12,080
SIXX	15°	– 1.7470E+07	– 1.954087E+07	11,854
SIXX	18°	– 1.7312E+07	– 1.916115E+07	10,681
SIXX	21°	– 1.7125E+07	– 1.880513E+07	9,811
SIXX	24°	– 1.6906E+07	– 1.824592E+07	7,926
SIXX	27°	– 1.6656E+07	– 1.782171E+07	6,999
SIXX	30°	– 1.6343E+07	– 1.713057E+07	4,819
SIXX	33°	– 1.5937E+07	– 1.640937E+07	2,964
SIXX	36°	– 1.5406E+07	– 1.558344E+07	1,152
SIXX	39°	– 1.4781E+07	– 1.467761E+07	– 0,699
SIXX	42°	– 1.4031E+07	– 1.366763E+07	– 2,590
SIXX	45°	– 1.3094E+07	– 1.276022E+07	– 2,549
SIXX	48°	– 1.1169E+07	– 1.241550E+07	11,160
SIXX	51°	–1.0593E+07	–1.086923E+07	2,608

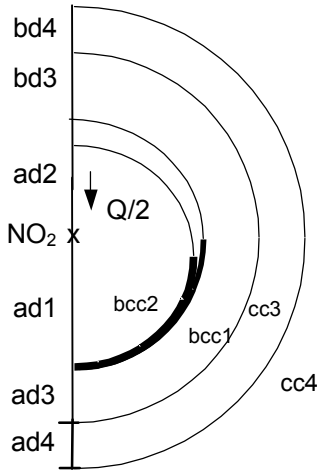
4 Modeling F

4.1 Characteristics of modeling

The symmetry of the problem makes it possible to model only half of it ($X \geq 0$).

60 finite elements SEG3 are laid out regularly on the initial surface of contact (3 degrees of angle for each one).

The cylinder and circular volume surrounding boring are with a grid with elements QUA8 and TRIA6.

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CONTACT
METHOD = 'CONTINUES'
COULOMB = 0.4

APPARIEMENT=' MAIT_ESCL'
Master: bcc1
Slave: bcc2
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Boundary condition:

- on GROUP_MA $CC4$: $DX=0$, $DY=0$.
- on GROUP_MA $AD1$, $AD2$, $AD3$, $AD4$, $BD3$, and $BD4$: $DX=0$.

Loadings:

The symmetry of the problem compared to the plan $x=0$ allows to model the force concentrated by a nodal force $Fy=-937.5\,10^3\,N$, equivalent to $Q/2$ for a cylinder length unit, applied to the group of nodes O_2 , center of the cylinder.
This force is applied into 1 increment.

4.2 Characteristics of the grid

Many nodes:	1603
Many meshs and types:	88 SEG3
	58 TRIA6
	456 QUAD8

4.3 Values tested: SIGM_ELNO

One tests the efforts of normal pressure generated by contact-friction. These efforts defined in polar coordinates are expressed in Pa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.88235E+07	5,673
SIXX	3°	– 1.7813E+07	– 1.87386E+07	5,196
SIXX	6°	– 1.7750E+07	– 1.85220E+07	4,349
SIXX	9°	– 1.7688E+07	– 1.84104E+07	4,084
SIXX	12°	– 1.7594E+07	– 1.81071E+07	2,917

SIXX	15°	– 1.7470E+07	– 1.78019E+07	1,900
SIXX	18°	– 1.7312E+07	– 1.73913E+07	0,458
SIXX	21°	– 1.7125E+07	– 1.68233E+07	1,762
SIXX	24°	– 1.6906E+07	– 1.65322E+07	– 2,211
SIXX	27°	– 1.6656E+07	– 1.63678E+07	– 1,730
SIXX	30°	– 1.6343E+07	– 1.60549E+07	– 1,763
SIXX	33°	– 1.5937E+07	– 1.56070E+07	– 2,071
SIXX	36°	– 1.5406E+07	– 1.48829E+07	– 3,395
SIXX	39°	– 1.4781E+07	– 1.41776E+07	– 4,082
SIXX	42°	– 1.4031E+07	– 1.29292E+07	– 7,852
SIXX	45°	– 1.3094E+07	– 1.19737E+07	– 8,555
SIXX	48°	– 1.1169E+07	– 1.06929E+07	– 4,263
SIXX	51°	–1.0593E+07	–0.96396E+07	– 8,999

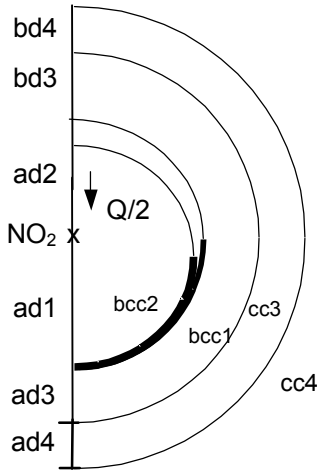
5 Modeling H

5.1 Characteristics of modeling

The symmetry of the problem makes it possible to model only half of it ($X \geq 0$).

60 finite elements SEG2 are laid out regularly on the initial surface of contact (3 degrees of angle for each one).

The cylinder and circular volume surrounding boring are with a grid with elements QUA4 and TRIA3.

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CONTACT
METHOD = 'CONTINUES'
COULOMB = 0.4

APPARIEMENT=' MAIT_ESCL'
Master: bcc1
Slave: bcc2
```

Boundary condition:

- on GROUP_MA $CC4$: $DX=0$, $DY=0$.
- on GROUP_MA $AD1$, $AD2$, $AD3$, $AD4$, $BD3$, and $BD4$: $DX=0$.

Loadings:

The symmetry of the problem compared to the plan $x=0$ allows to model the force concentrated by a nodal force $Fy=-937.5\,10^3\,N$, equivalent to $Q/2$ for a cylinder length unit, applied to the group of nodes O_2 , center of the cylinder.
This force is applied into 1 increment.

5.2 Characteristics of the grid

Many nodes:	1282
Many meshs and types:	128 SEG2
	162 TRIA3
	1106 QUAD4

5.3 Values tested: SIGM_ELNO

One tests the efforts of normal pressure generated by contact-friction. These efforts defined in polar coordinates are expressed in Pa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.79054E+07	0,519
SIXX	3°	– 1.7813E+07	– 1.88551E+07	5,850
SIXX	6°	– 1.7750E+07	– 1.89952E+07	7,015
SIXX	9°	– 1.7688E+07	– 1.88212E+07	6,407
SIXX	12°	– 1.7594E+07	– 1.85900E+07	5,661

SIXX	15°	– 1.7470E+07	– 1.83361E+07	4,958
SIXX	18°	– 1.7312E+07	– 1.78879E+07	3,327
SIXX	21°	– 1.7125E+07	– 1.74676E+07	2,001
SIXX	24°	– 1.6906E+07	– 1.67457E+07	– 0,948
SIXX	27°	– 1.6656E+07	– 1.60969E+07	– 3,357
SIXX	30°	– 1.6343E+07	– 1.56549E+07	– 4,211
SIXX	33°	– 1.5937E+07	– 1.57323E+07	– 1,284
SIXX	36°	– 1.5406E+07	– 1.55859E+07	1,168
SIXX	39°	– 1.4781E+07	– 1.51202E+07	2,295
SIXX	42°	– 1.4031E+07	– 1.42668E+07	1,680
SIXX	45°	– 1.3094E+07	– 1.33861E+07	2,230
SIXX	48°	– 1.1169E+07	– 1.24518E+07	11,485
SIXX	51°	– 1.0593E+07	– 1.11524E+07	5,280

6 Summary of the results

The results are of good quality if it is considered that they are compared with an analytical solution and that the grids used are not particularly fine.

The best solution is given by the modeling F (continuous algorithm) which uses quadratic elements with quadratic projection (without using this projection, the results are false). Then the modeling H comes which, like the preceding one, uses the continuous algorithm but with linear elements and master-slave pairing. Finally the modeling D which uses the algorithm of penalization gives acceptable results. Let us note that convergence with this algorithm is much slower and difficult.

If one examines the extent of the surface of contact using the impression of the structure of data CONT_NOEU (see file of results), it is noted that it is given very precisely (60°). With regard to adherent surface, the algorithms analytically return an acceptable value about 30° instead of 26.2°.