SSNV143 - Biaxial traction with the law of behavior BETON_DOUBLE_DP

Summary:

This case of validation is intended to check the model of behavior 3D BETON_DOUBLE_DP formulated within the framework of thermoplasticity, for the description of the nonlinear behavior of the concrete, in traction, and compression, with the taking into account of the irreversible variations of the thermal and mechanical characteristics of the concrete, particularly sensitive at high temperature.
The description of cracking is treated within the framework of plasticity, using an energy equivalence, by identifying the density of energy of cracking in mode I, with the plastic work of a homogeneous medium are equivalent, where the plastic deformation is uniformly distributed, in an "elementary" zone. This approach preserves the continuity of the formulation of the model, on the whole of its behavior, and contributes to avoid the possible digital difficulties during the change of state of material.
The pathological sensitivity of the digital solution to the space discretization (grid), generated by the introduction of a softening behavior of the concrete in traction and compression, is partially solved by introducing an energy of cracking or rupture, dependent a characteristic length l_c, dependent in keeping with elements.

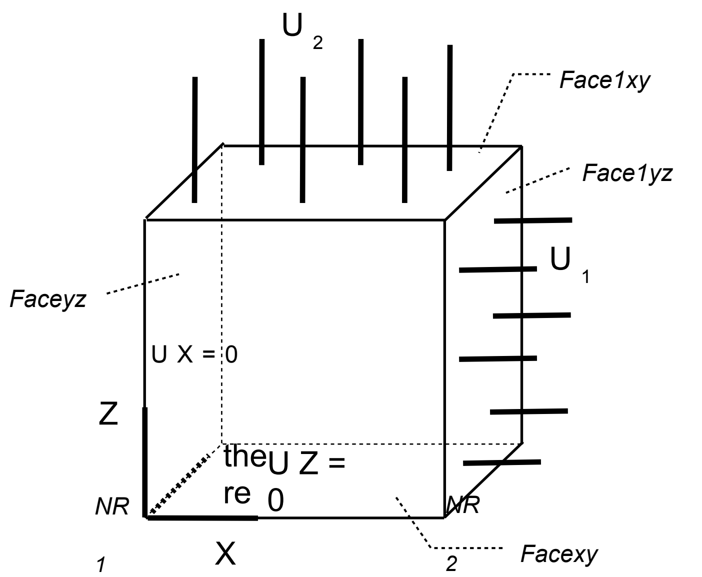
The case test understands two modelings 3D, the loading consists of a load followed by a discharge.

1 Problem of reference

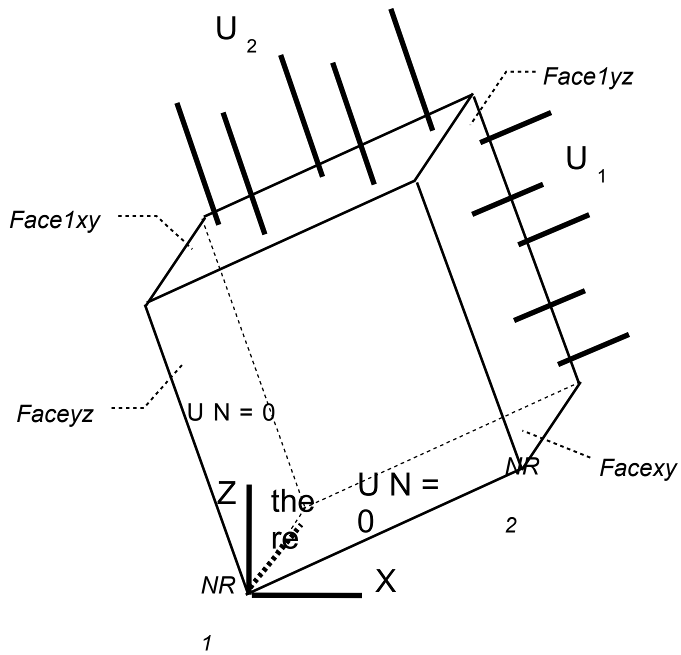
1.1 Geometry

It is about a cube with 8 nodes, whose two faces have a normal displacement no one, and the two opposite faces have an imposed normal displacement, different one from the other of a coefficient 2. The made cube $1\,mm$ on side. The cases tests are composed of a load, followed by a discharge. In modeling A, the cube is directed according to the reference mark $Oxyz$. In modeling B, it is turned of $30°$ by around the axis Oy .

Modeling A



Modeling B



$U_2 = 2.U_1$

1.2 Material properties

To test the evolution of the mechanical characteristics in an irreversible way with the temperature, one applies a field of temperature decreasing. Certain variables depend on the temperature, others of drying. Lastly, one applies a coefficient of withdrawal of desiccation not no one, equal to the thermal dilation coefficient, to test "data-processing" operation. The thermal deformations thus equal and will be opposed to the deformations of withdrawal of desiccation. These dependences intervene only for checks purely data-processing, the mechanical characteristics can be regarded as constants.

For the usual linear mechanical characteristics:

Young modulus:	$E=32\,000\,MPa$	of	$0°C$ with $20°C$
	$E=15\,000\,MPa$	with	$400°C$ (linear decrease)
	$E=5\,000\,MPa$	with	$800°C$ (linear decrease)
Poisson's ratio:	$\nu=0.18$		
Thermal dilation coefficient:	$\alpha=10^{-5}$		
Coefficient of withdrawal of desiccation:	$\kappa=10^{-5}$		

For the nonlinear mechanical characteristics of the model `BETON_DOUBLE_DP` :

Resistance in uniaxial pressing:	$f'c=40\,N/mm^2$	of	$0°C$ with $400°C$
	f' C = 15 N/mm^2	with	$800°C$ (linear decrease)
Resistance in uniaxial traction:	$f't=4\,N/mm^2$	of	$0°C$ with $400°C$
	$f't=1.5\,N/mm^2$	with	$800°C$ (linear decrease)
Report of resistances in biaxial compression/uniaxial pressing:	$\beta=1.16$		
Energy of rupture in compression:	$Gc=10\,Nmm/mm$		
Energy of rupture in traction:	$\mathrm{Gt}=0.1\,Nmm/mm$		
Report of the limit elastic to resistance in uniaxial pressing:	30%		

1.3 Boundary conditions and loadings mechanical

Field of temperature decreasing of $20°C$ with $0°C$.

Lower face of the cube ($facexy$) :	blocked according to oz .
Higher face of the cube ($face1xy$) :	displacement $0.30\,mm$ imposed followed by a discharge of $0.1\,mm$
Left face of the cube ($faceyz$) :	blocked according to ox .
Right face of the cube ($face1yz$) :	displacement $0.15\,mm$ imposed followed by a discharge of $0.05\,mm$
Lower nodes front face (N_1 , N_2) :	blocked according to oy (Suppression of the movements of solid body).

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is calculated in an semi-analytical way, knowing that in traction, only the criterion of traction is activated. It is thus necessary to solve a system of an equation to an unknown factor, which makes it possible to obtain by dichotomy for example, the plastic deformation cumulated in traction. This one makes it possible to calculate strains and stresses then. This is possible, knowing displacement, and thus the deformation in the two imposed directions. Displacement in the third direction is then an unknown factor of the problem.
The reference solution is calculated only in traction. The solution is determined by a programme of resolution per dichotomy in independent FORTRAN. In compression, in discharge, the exact solution was not recomputed, and constitutes a solution of nonregression of the code, been dependent on version 5.02.14.
For modeling B, the results result by rotation from the tensor from constraint from modeling A, the intrinsic reference mark of the cube to the reference mark user, the stress field of the two configurations being identical in the intrinsic reference mark of the cube.

2.2 Calculation of the reference solution of reference

For more details on the notations and the setting in equation, one will refer to the reference document. Only, the principal equations are pointed out here.
One notes a, imposed displacement following the direction x, and $2.\mathrm{a}$ imposed displacement following the direction z. The tensor of deformation is form $(a, \varepsilon_y, 2.\mathrm{a}, 0., 0., 0.)$ by taking the usual notations of *Code_Aster* (three principal components, three components of shearing).
The tensor of constraint is form $(\sigma_x, 0., \sigma_z, 0., 0., 0.)$, in modeling A.
The criterion of traction is expressed in the form:

$$f_{trac} = \frac{\tau_{oct} + c.\sigma_{oct}}{d} - f_t(\lambda_t) = \frac{\sqrt{2}}{3d}\sigma^{eq} + \frac{c}{d}\sigma_H - f_t(\lambda_t)$$

The constitutive equations are written by distinguishing the isotropic part of the deviatoric part of the tensors of constraints and deformations.

$$\sigma_H = \frac{1}{3}tr(\sigma) \quad s = \sigma - \frac{1}{3}tr(\sigma)I \quad \varepsilon_H = \frac{1}{3}tr(\varepsilon) \quad \tilde{\varepsilon} = \varepsilon - \frac{1}{3}tr(\varepsilon)I$$
$$\sigma = s + \sigma_H I$$
$$\varepsilon = \tilde{\varepsilon} + \varepsilon_H I$$

The equivalent constraint is written then: $\sigma^{eq} = \sqrt{\frac{3}{2}tr(s)}$

In the case of an incremental formulation, and of a variable law of behavior, while noting with an exhibitor e the elastic components of the constraint and the deformation, one obtains:

$$s^e = \frac{\mu^+}{\mu^-}s^- + 2\mu^+\Delta\tilde{\varepsilon} \text{ and } {\sigma_H}^e = \frac{K^+}{K^-}{\sigma_H}^- + 3K^+\Delta\varepsilon_H$$

The criteria in compression and traction are expressed in the following way:

$$f_{comp}=\frac{\tau_{oct}+a.\sigma_{oct}}{b}-f_c(\lambda_c)=\frac{\sqrt{2}}{3b}\sigma^{eq}+\frac{a}{b}\sigma_H-f_c(\lambda_c)$$

$$f_{trac}=\frac{\tau_{oct}+c.\sigma_{oct}}{d}-f_t(\lambda_t)=\frac{\sqrt{2}}{3d}\sigma^{eq}+\frac{c}{d}\sigma_H-f_t(\lambda_t)$$

The plastic deformations in traction and compression are expressed:

$$\Delta\tilde{\varepsilon}^p{}_c=\frac{\Delta\lambda_c}{\sqrt{2}b}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_c}=\Delta\lambda_c\frac{a}{3b}$$

$$\Delta\tilde{\varepsilon}^p{}_t=\frac{\Delta\lambda_t}{\sqrt{2}d}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_t}=\Delta\lambda_t\frac{c}{3d}$$

One obtains for the constraint:

$$s=s^e-2\mu^+\left(\Delta\tilde{\varepsilon}^p{}_c+\Delta\tilde{\varepsilon}^p{}_t\right) \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\varepsilon_H{}^p{}_c+\Delta\varepsilon_H{}^p{}_t\right)$$

$$s=\left(1-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)\frac{1}{\sigma^{e\,eq}}\right)s^e \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\lambda_c\frac{a}{3b}+\Delta\lambda_t\frac{c}{3d}\right)$$

for the equivalent constraint:

$$\sigma^{eq}=\sigma^{e\,eq}-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)$$

The two criteria lead then to a system of two equations to two unknown factors $\Delta\lambda_c$ and $\Delta\lambda_t$ to solve:

$$\begin{cases}\dfrac{\sqrt{2}}{3b}\sigma^{e\,eq}+\dfrac{a}{b}\sigma_H{}^e-\Delta\lambda_c\left(\dfrac{2\mu^+}{3b^2}+\dfrac{K^+a^2}{b^2}\right)-\Delta\lambda_t\left(\dfrac{2\mu^+}{3bd}+\dfrac{K^+ac}{bd}\right)-f_c\left(\lambda_c^-+\Delta\lambda_c\right)=0\\ \dfrac{\sqrt{2}}{3d}\sigma^{e\,eq}+\dfrac{c}{d}\sigma_H{}^e-\Delta\lambda_c\left(\dfrac{2\mu^+}{3bd}+\dfrac{K^+ac}{bd}\right)-\Delta\lambda_t\left(\dfrac{2\mu^+}{3d^2}+\dfrac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t^-+\Delta\lambda_t\right)=0\end{cases}$$

In a similar way, in the case of the only criterion of traction activated, **configuration of the case test** , one obtains a system of an equation to an unknown factor $\Delta\lambda_t$ to solve:

$$\left\{\dfrac{\sqrt{2}}{3d}\sigma^{e\,eq}+\dfrac{c}{d}\sigma_H{}^e-\Delta\lambda_t\left(\dfrac{2\mu^+}{3d^2}+\dfrac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t^-+\Delta\lambda_t\right)=0\right.$$

One thus seeks to solve this system, by using the particular shape of the tensors of constraints and deformations, uniforms on the structure.
On the basis of $\varepsilon = (a, \varepsilon_y, 2.a, 0., 0., 0.)$ *et de* $\sigma = (\sigma x, 0., \sigma z, 0., 0., 0.)$. one obtains:

The elastic tensor of constraint
$$\begin{cases} \sigma_x = a(\lambda + 2\mu) + \varepsilon_y \lambda + 2\, a.\lambda \\ \sigma_y = a.\lambda + \varepsilon_y (\lambda + 2\mu) + 2\, a.\lambda \\ \sigma_z = a.\lambda + \varepsilon_y \lambda + 2\, a.(\lambda + 2\mu) \end{cases}$$

The elastic diverter of constraint
$$\begin{cases} s_x = -\frac{2}{3}.\mu.\varepsilon_y \\ s_y = -2.\mu.a + \frac{4}{3}.\mu.\varepsilon_y \\ s_z = 2.\mu.a - \frac{2}{3}.\mu.\varepsilon_y \end{cases}$$

The elastic hydrostatic constraint $\sigma^e{}_H = (3\lambda + 2\mu)\left(a + \frac{1}{3}.\varepsilon_y\right)$

The elastic equivalent constraint $\sigma^e{}_{eq} = \mu\sqrt{4\varepsilon_y{}^2 - 12.a.\varepsilon_y + 12.a^2}$

In the case of a curve of linear work hardening post-peak in traction, the expression of the parameter of work hardening is the following:

$$f_t\left(\theta, \left\|\varepsilon_t^p\right\|\right) = \tau(\theta, \kappa) = f_t(\theta)\left(1 - \frac{\left\|\varepsilon_t^p\right\|}{\kappa_u(\theta)}\right) \text{ with } \kappa_u(\theta) = \frac{2.G_f(\theta)}{l_c . f_t(\theta)}$$

One thus seeks to solve the equation:

$$\left(\frac{\sqrt{2}}{3d}\sigma^{eeq} + \frac{c}{d}\sigma_H{}^e - \Delta\lambda_t\left(\frac{2\mu^+}{3d^2} + \frac{K^+ c^2}{d^2}\right) - f_t\left(1 - \Delta\lambda_t \frac{l_c . f_t}{2.G_t}\right)\right) = 0 \qquad \text{éq 2.2-1}$$

Knowing that the constraint in the direction y is worthless, one obtains one second equation:

$$\sigma_y = s_y + \sigma_H = 0 = \left(1 - \sqrt{2}\mu^+ \frac{\Delta\lambda_t}{d}\frac{1}{\sigma^{eeq}}\right)s^e{}_y + \sigma^e{}_H$$

$$\sigma_y = 0 = \left(1 - \sqrt{2}\mu^+ \frac{\Delta\lambda_t}{d}\frac{1}{\sigma^{eeq}}\right)\left(\frac{4}{3}\mu\varepsilon_y - 2.\mu.a\right) + \sigma^e{}_H - K\frac{c.\Delta\lambda_t}{d}$$

From where: $\Delta\lambda_t = \dfrac{\left(\frac{4}{3}\mu\varepsilon_y - 2.\mu.a\right) + \sigma^e{}_H}{\frac{\sqrt{2}\mu}{d.\sigma^{eeq}}\left(\frac{4}{3}\mu\varepsilon_y - 2.\mu.a\right) + K\frac{c}{d}}$ that one can substitute in the expression of the criterion [éq 2.2-1].

Knowing a , imposed displacement, one obtains a nonlinear equation with an unknown factor, which one can simply solve by dichotomy, and which makes it possible to calculate the deformation ε_y , then the whole of the unknown factors of the system.

2.3 Uncertainty on the solution

It is negligible, about the precision machine.

2.4 Bibliographical references

The model was defined starting from the following theses:

1) J.F. GEORGIN, at the time of its thesis “Contribution to the digital modeling of the behavior of the concrete and the reinforced concrete structures under thermomechanical requests at high temperature”,

2) G. HEINFLING, at the time of its thesis “Contribution to the modeling of the concrete under fast request of dynamics. The taking into account of the effect speed by viscoplasticity”, and is described in the report of specification:

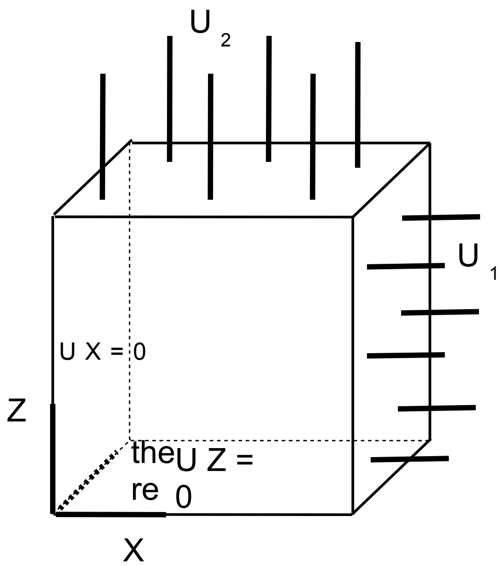
3) SCSA/128IQ1/RAP/00.034 Version 1.2, Development of a model of behavior 3D concrete with double criterion of plasticity in *Code_Aster* - Specifications “.

3 Modeling A

3.1 Characteristics of modeling

3D (HEXA8)

1 element, stress field and uniform deformation.



3.2 Characteristics of the grid

Many nodes: 8
Number of meshs and type: 1 HEXA8

3.3 Sizes tested and results

The nonworthless components of the stress field were tested SIGM_ELNO (component xx and zz), the component yy field of deformation EPSI_ELNO , which constitutes an unknown factor of the system (deformations in the two other directions being imposed), plastic deformation cumulated in traction (second variable internal, second component of the field VARI_ELNO), and finally, only for the fourth case of loading (discharge), the plastic deformation cumulated in compression, (first internal variable, first component of the field VARI_ELNO).

The first three loadings correspond to the load, and have results of reference. The fourth loading corresponds to the discharge, and constitutes a result of nonregression of the code.

Field SIGM_ELNO component SIXX

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.1235611	0.1235380	0,019
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	6.882374.10 $^{-2}$	6.878218.10 $^{-2}$	0,060
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	1.408764.10 $^{-2}$	1.402639.10 $^{-2}$	0,435
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	– 4.195092.10 $^{-5}$	-

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field SIGM_ELNO component SIZZ

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.239212	0.239174	0,016
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	0.133243	0.133165	0,059
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	2.727403.10 $^{-2}$	2.725569.10 $^{-2}$	0,434
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	– 4.959258.10 $^{-5}$	-

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field EPSI_ELNO component EPYY

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	$-3.419463.10^{-3}$	$-3.419464.10^{-3}$	2.10^{-7}
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	$-6.835813.10^{-3}$	$-6.835815.10^{-3}$	2.10^{-7}
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$-1.025216.10^{-2}$	$-1.025216.10^{-2}$	2.10^{-7}
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.357498.10^{-1}$	-

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field VARI_ELNO component VARI_2 (plastic deformation cumulated in traction)

Identification	Reference	*Aster*	% difference
For an imposed displacement $U_1=0.1$ and $U_2=0.05$	$1.085728.10^{-2}$	$1.085728.10^{-2}$	5.10^{-9}
For an imposed displacement $U_1=0.2$ and $U_2=0.10$	$2.171556.10^{-2}$	$2.171556.10^{-2}$	5.10^{-9}
For an imposed displacement $U_1=0.3$ and $U_2=0.15$	$3.257385.10^{-2}$	$3.257385.10^{-2}$	4.10^{-9}
For an imposed displacement $U_1=0.1$ and $U_2=0.05$	$3.257385.10^{-2}$	$3.257385.10^{-2}$	4.10^{-9}

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field VARI_ELNO component VARI_1 (plastic deformation cumulated in compression)

Identification	Reference	*Aster*	% difference
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$3.528401.10^{-1}$	-

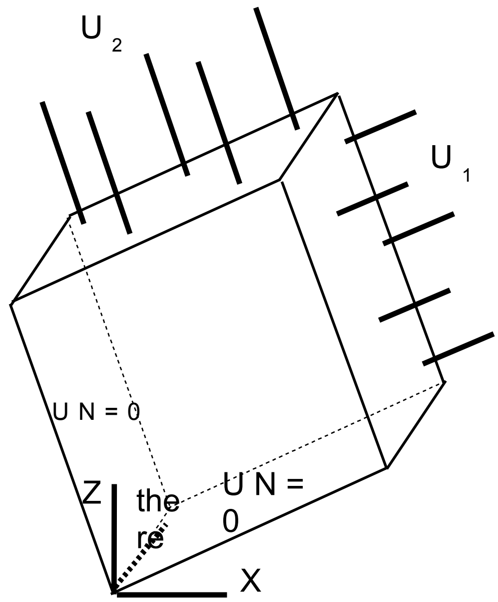
(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

4 Modeling B

4.1 Characteristics of modeling

3D (HEXA8)

1 element, stress field and uniform deformation.



4.2 Characteristics of the grid

Many nodes: 8
Number of meshs and type: 1 HEXA8

4.3 Sizes tested and results

The nonworthless components of the stress field were tested `SIGM_ELNO` (component xx , zz and xz), plastic deformation cumulated in traction (second variable internal, second component of the field `VARI_ELNO`), and finally, only for the fourth case of loading (discharge), the plastic deformation cumulated in compression, (first internal variable, first component of the field `VARI_ELNO`).

The first three loadings correspond to the load, and have results of reference. The fourth loading corresponds to the discharge, and constitutes a result of nonregression of the code.

Field `SIGM_ELNO` component `SIXX`

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.152474	0.1524472	0,018
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	$8.492877.10^{-2}$	$8.487797.10^{-2}$	0,060
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$1.732484.10^{-2}$	$1.730871.10^{-2}$	0,434
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.386134.10^{-5}$	-

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field `SIGM_ELNO` component `SIZZ`

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.210300	0.210265	0,016
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	0.117138	0.117069	0,059
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$2.397743.10^{-2}$	$2.387336.10^{-2}$	0,434
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.768217.10^{-5}$	-

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field `SIGM_ELNO` component `SIXZ`

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	– 5.007871.10 $^{-2}$	– 5.007226.10 $^{-2}$	0,013
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	– 2.789472.10 $^{-2}$	– 2.787871.10 $^{-2}$	0,057
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	– 5.709873.10 $^{-3}$	– 5.685155.10 $^{-3}$	0,433
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	– 3.308936.10 $^{-6}$	-

(*) discharges some, one carries out a test of nonregression. There is no analytical solution.

Field `VARI_ELNO` component `VARI_2` (plastic deformation cumulated in traction)

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	1.085728.10 $^{-2}$	1.085728.10 $^{-2}$	5.10 $^{-9}$
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	2.171556.10 $^{-2}$	2.171556.10 $^{-2}$	5.10 $^{-9}$
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	3.257385.10 $^{-2}$	3.257385.10 $^{-2}$	4.10 $^{-9}$
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	3.257385.10 $^{-2}$	3.257385.10 $^{-2}$	4.10 $^{-9}$

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

Field `VARI_ELNO` component `VARI_1` (plastic deformation cumulated in compression)

Identification	Reference	*Aster*	% difference
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	3.528401.10 $^{-1}$	-

(*) discharges some, one carries out a test of nonregression. There is no calculated analytical solution.

5 Summary of the results

This case test offers satisfactory results compared to the results of reference, lower than 0.06% for the first two cases of loading, more important for the third, which is explained by a relatively low level of constraint (one reaches the end of the curve of work hardening in traction).

The test discharges some (fourth loading) allows to check to it not regression of the code.

The iteration count is relatively important with the first step of calculation, about 13, then drops to 7.4 and 1, which is explained by the passage of the plastic threshold to the first step of calculation, to reach a quasi linear behavior thereafter (curved post-peak linear).
One obtains also a more significant number of iterations to step 31 (beginning of the fourth case of loading), then an iteration count dropping up to 1, because of the passage in discharge, with a behavioral change, follow-up of a quasi linear behavior thereafter (curved post-peak linear).