SSNV148 - Models of Weibull and Rice-Tracey in 3D and discharge

Summary:

This test of nonlinear quasi-static mechanics makes it possible to validate the models of Weibull and Rice and Tracey in 3D for nonmonotonous cases of mechanical loadings (cf. POST_ELEM).

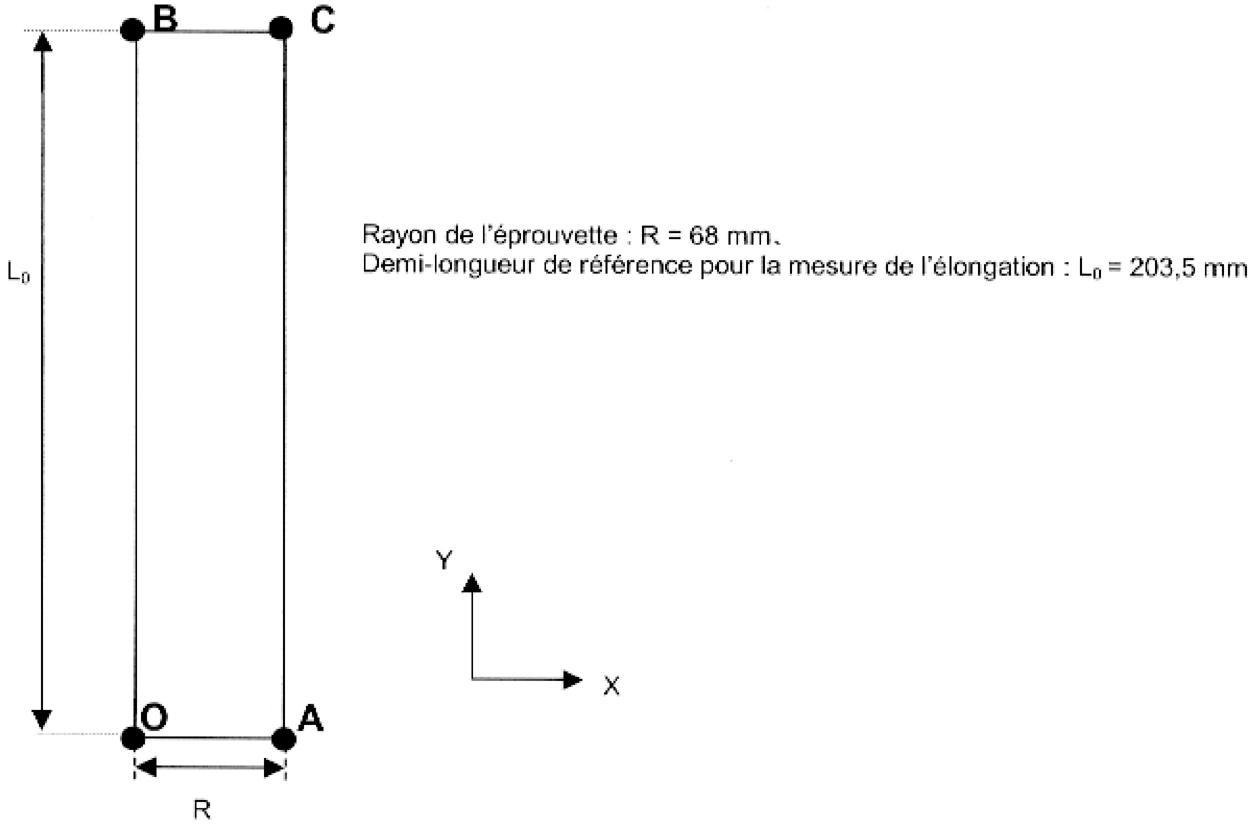
At the temperature of $-50°C$, a cylindrical test-tube smoothes is first of all deformed up to 10%. After having slightly discharged it, one maintains constant the level of deformation reaches while decreasing in a homogeneous way the temperature of the test-tube until $-150°C$. With this new temperature, one applies an additional deformation to reach 15% on the whole. The probability of rupture per cleavage as well as the growth rate of the cavities of the test-tube are calculated for the whole of the way of loading.

The modeling of the test-tube is carried out with elements 3D (HEXA20, PENTA15).

1 Problem of reference

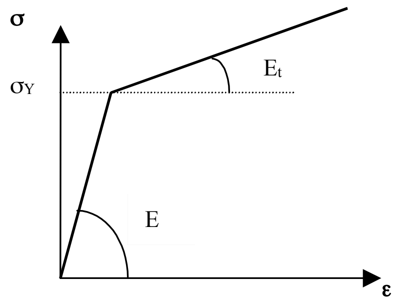
1.1 Geometry

One considers a half - cylindrical test-tube smooth.



1.2 Properties of material

One adopts an elastoplastic law of behavior of Von Mises with linear isotropic work hardening `'VMIS_ISOT_LINE'`. The deformations used in the relation of behavior are the linearized deformations.



The Young modulus E, the tangent module E_t as well as the Poisson's ratio do not depend on the temperature. One takes: $E=200 GPa$, $E_t=2000 MPa$ and $\nu=0,3$.

The evolution of the elastic limit with the temperature is given in the following table:

Temperature $[°C]$	**–150**	**–100**	**–50**
$\sigma_Y [MPa]$	750	700	650

Lastly, thermal dilation is neglected (thermal dilation coefficient taken equal to 0).

1.3 Boundary conditions and loadings

While referring to the figure of [§1.1] the boundary conditions are the following ones:

- on surface $SSUP$ BC ($Y=L_0$) displacement l imposed according to the direction OY ,
- on surface $SINF$ OA ($Y=0$) displacements blocked according to the direction OY ,
- displacements of A blocked according to X and Z ,
- displacements of B blocked according to Z .

Evolution temporal of the temperature (presumedly homogeneous in the test-tube) and of lengthening l are deferred in the following table:

Time $[s]$	10	20	30	40
Temperature $[°C]$	– 50	– 50	– 150	– 150
Displacement $l-L_0$ $[mm]$	20.35	20.30	20.30	32.525

1.4 Initial conditions

Worthless constraints and deformations.

2 Reference solutions

2.1 Method of calculating

In simple traction and with the assumption of the small deformations, the tensile stress $\sigma(u)$ as well as the plastic multiplier $\dot{p}(u)$ at the moment u are given in the case considered by:

- if $0 \le u \le t_1^p$: $\sigma(u)=E\frac{l(u)-L_0}{L_0}$ $\dot{p}(u)=0$ $l(t_1^p)=L_0\left(1+\frac{\sigma_Y(-50°C)}{E}\right)$
- if $t_1^p \le u \le 10$: $\sigma(u)=E_t\left(\frac{l(u)-L_0}{L_0}\right)+\frac{E-E_t}{E}\sigma_Y(-50°C)$ $\dot{p}(u)=\left(1-\frac{E_t}{E}\right)\frac{\dot{l}(u)}{L_0}$,
- if $10 \le u \le 20$: $\sigma(u)=\sigma(u=10)-E\left(\frac{l(u=10)-l(u)}{L_0}\right)$ $\dot{p}(u)=0$,
- if $20 \le u \le 30$: $\sigma(u)=\sigma(u=20)$ $\dot{p}(u)=0$,
- if $30 \le u \le 40$: $\sigma(u)=\sigma(u=20)+E_t\left(\frac{l(u)-l(u=20)}{L_0}\right)$ $\dot{p}(u)=\left(1-\frac{E_t}{E}\right)\frac{\dot{l}(u)}{L_0}$

2.2 Weibull

Probability of cumulated rupture P_f at the moment t is given by (cf. POST_ELEM) :

$$P_f(t) = 1 - \exp\left[- \sum_{dV} \left[\max_{t^p \le u \le t} \left(\frac{\sigma_I(u)}{\sigma_u(\theta(u))} \right) \right]^m \frac{dV}{V_0} \right].$$

The summation relates to volumes of matter V_i plasticized (as from the moment t_p), $\sigma_I(u)$ and $\theta(u)$ indicating the maximum principal constraint and the temperature in each one of these volumes at the various moments (u). Here, volume V_0 of reference is equal to $50\,\mu m^3$. The module of Weibull m is equal to 24 while the constraint of cleavage σ_u depends on the temperature according to:

Temperature $[°C]$	**– 50**	**– 100**	**– 150**
$\sigma_u [MPa]$	2800	2700	2600

The probability of cumulated rupture varies according to ($\theta(t), l(t)$) according to:

$$P_f(t) = 1 - \exp\left[- \left[\max_{t^p \le u \le t} \left(\frac{\sigma(u)}{\sigma_u(\theta(u))} \right) \right]^m \frac{V}{V_0} \right].$$

2.3 Rice and Tracey

In simple traction, the Napierian logarithm of the growth rate of the cavities at the moment t is given by (cf. POST_ELEM) :

$$Log\left(\frac{R(t)}{R_0} \right) = 0{,}283 \times \exp(0{,}5) \times \int_0^t \dot{p}(u)\,du$$

2.4 Sizes and results of reference

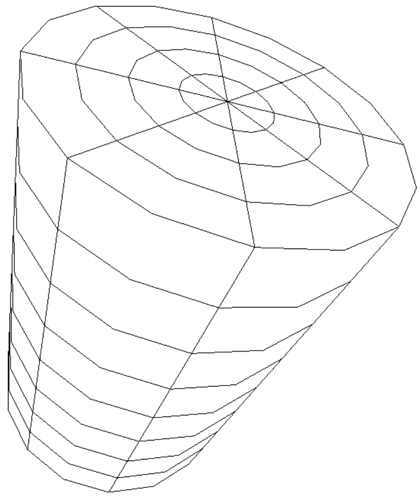
P_f and $\frac{R}{R_0}$ for the couples (temperature, displacements = $(l - l_0)$) following: $(-50{,}0\,°C, 20{,}35\,mm)$; $(-50{,}0\,°C, 20{,}30\,mm)$; $(-150{,}0\,°C, 20{,}30\,mm)$ and $(-150{,}0\,°C, 32{,}53\,mm)$.

2.5 Uncertainties on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of the grid



Many nodes: 1137
Many meshs and types: 64 (PENTA15), 192 (HEXA20)

3.2 Sizes tested and results

		Reference	*Code_Aster*		Reference	*Code_Aster*	
$T[°C]$	$l-L_0[mm]$	P_f	P_f	% diff.	$\frac{R}{R_0}$	$\frac{R}{R_0}$	% diff.
– 50	20.35	0.01465	0.01481	1.1	1.0447	1.0458	0.1
– 50	20.30	0.01465	0.01481	1.1	1.0447	1.0458	0.1
– 150	20.30	0.01465	0.01481	1.1	1.0447	1.0458	0.1
– 150	32.525	1.0	1.0	0.0	1.068	1.0701	0.2

4 Summary of the results

Results got by *Code_Aster* are very close to the analytical solutions of reference.