Elastic SSNV152- Traction. Calculation of the constraints of Cauchy

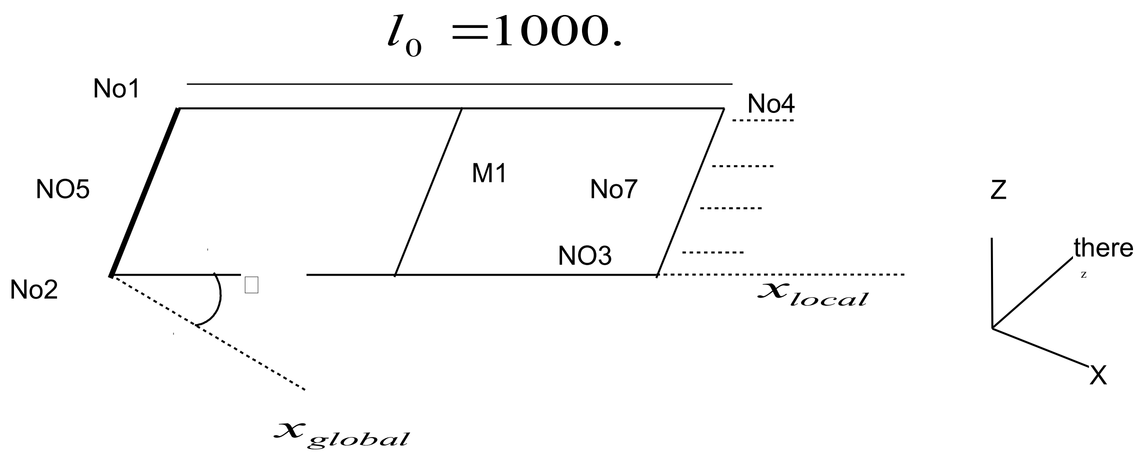
Summary

The goal of this test is to validate the calculation of the constraints of Cauchy by the option SIGM_ELNO.

1 Problem of reference

1.1 Geometry

The geometry of this test is a square plate in the plan (x, y) round of $30°$ compared to x around z.



One calls l the length of the deformed plate, one will note x, y, z coordinates of the deformed configuration and X, Y, Z coordinates of the initial configuration

1.2 Properties of materials

One takes $E = 200\,000. MPa$ et $\nu = 0$

1.3 Boundary conditions and loadings mechanical

The nodes are blocked $No1$, $No5$ and $No2$ so that $DX = DY = DZ = DRX = DRY = DRZ = 0$, and a local displacement is imposed $Dx = 100.$ on the nodes $No3$, $No4$ and $No7$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is analytical.

Passage de l' initial state in a deformed state:

$$x = \frac{1}{l_0} X,\ \ y = \frac{a}{a_0} Y,\ \ z = \frac{b}{b_0} Z$$

where

- a is the length of the deformation of the plate according to Y,
- a_0 is the initial length of the plate,
- b is the thickness of the deformed plate,
- b_0 is the initial thickness of the plate.

Owing to the fact that $\nu = 0$ and of the assumptions of hull, one has $a = a_0,\ b = b_0$

Tensor Green-Lagrange:

By definition of the tensor of Green-Lagrange, one has $E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \left(\frac{\partial u_k}{\partial X_i}\right)\left(\frac{\partial u_k}{\partial X_j}\right)\right)$

With $u = x - X = \frac{l - l_0}{l_0} X$, one thus has $E_{11} = \frac{1}{2}\left(\frac{l - l_0}{l} + \frac{l - l_0}{l} + \frac{(l - l_0)^2}{l_0^2}\right) = \frac{1}{2}\left(\frac{l^2 - l_0^2}{l_0^2}\right)$

While replacing, one has $E_{11} = \frac{1}{2}\left(\frac{1100^2 - 1000^2}{1000^2}\right) = 0.105$

Gradient of deformation:

By definition:

$$F = \begin{pmatrix} \frac{dx}{dX} & \frac{dx}{dY} & \frac{dx}{dZ} \\ \frac{dy}{dX} & \frac{dy}{dY} & \frac{dy}{dZ} \\ \frac{dz}{dX} & \frac{dz}{dY} & \frac{dz}{dZ} \end{pmatrix} = \begin{pmatrix} \frac{l}{l_0} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

That is to say $J = \det F = \frac{l}{l_0}$

Constraints of Piola-Kirchhoff of second species:

That is to say S the constraint of $PK2$, in our case, $S_{11}=E.E_{11}=200000\times 0.105=21000$

Constraint of Cauchy

That is to say s the tensor of constraints of Cauchy, one has the relation $s=\left(\frac{1}{\det F}\right)\left(F.S.F^T\right)$, one from of deduced whereas $s_{xx}=\left(\frac{1}{\frac{l}{l_0}}\right)\frac{l}{l_0}.S_{11}.\frac{l}{l_0}=\frac{l}{l_0}.S_{11}=\frac{1100}{1000}.21000=23100$

2.2 Results of reference

Displacements are calculated DX and DY with the node $NO3$, constraints of $PK2$ and constraints of Cauchy on the mesh $M1$.

2.3 Uncertainty on the solution

Analytical result.

2.4 Bibliographical references

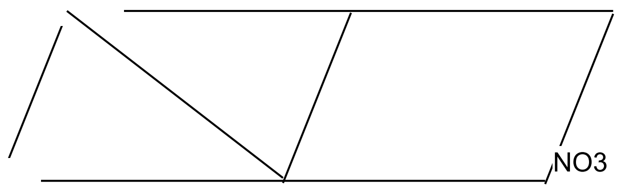
Nothing.

3 Modeling A

3.1 Characteristics of modeling

Elements are used COQUE_3D

3.2 Characteristics of the grid



Coordinates of the principal nodes:

Node	$Coor_x$	$Coor_y$	$Coor_z$
N01	– 500	866,025	0.
N02	0	0	0.
N03	866,025	500	0.
N04	366,025	1366.025	0.

The meshs used are:
1 mesh QUAD9
2 meshs TRIA7

3.3 Sizes tested and results

Identification	Reference	*Aster*	Difference
DX ($No4$)	8.66025 E+01	8.66025 E+01	4.66 E-05%
DY ($No4$)	50.0	50.0	0%
$PK2$ - $SIXX$ ($M1$)	21000.	21000.	2.04 E-08%
Cauchy- $SIXX$ ($M1$)	23100.	23100.	2.14 E-08%

4 Summary of the results

The found results are in agreement with the analytical solution.