SSNV191 – Validation of the conditions of Neumann with X-FEM in 2D and 3D

Summary

The purpose of this test is to validate the taking into account of boundary conditions of the Neumann type on elements of skin X-FEM in 2D and 3D.

This test brings into play a structure 3D or an equivalent in 2D comprising a horizontal plane interface cutting the structure in two parts (higher part a "above the interface, and "a lower" part below the interface). The interface is represented by level sets.

The loadings applied are several types: a loading with constant pressure on the side faces of the structure, a loading with a positive pressure on the higher side faces and a negative pressure on the lower side faces. One deals also with the same problem by replacing the pressures by surface forces in 3D and linear forces in 2D.

1 Problem of reference

1.1 Geometry

The structure 3D of dimensions $LX=1\text{m}$, $LY=2\text{m}$ and $LZ=3\text{m}$ comprise a horizontal plane interface being located at middle height (see [Figure 1.1-1]).

The interface is not with a grid, and the geometry is in fact a healthy structure without interface. The interface is then introduced by functions of levels (level sets) directly into the command file using the operator `DEFI_FISS_XFEM` [U4.82.08].

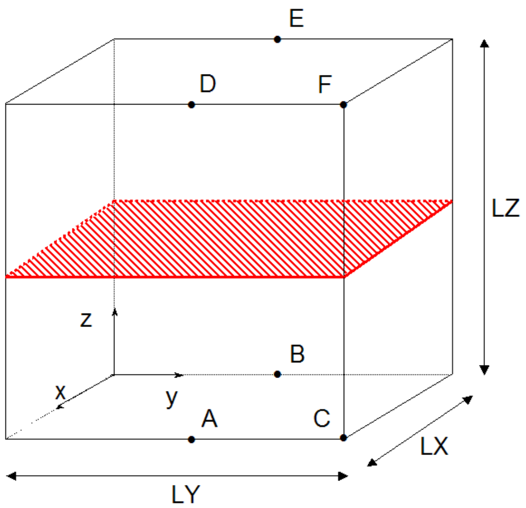

Figure 1.1-1 : Geometry of the structure 3D

The points are defined A, B, C, D, E and F who will be used to block the rigid modes.

	x	y	z
A	LX	$LY/2$	0
B	0	$LY/2$	0
C	LX	LY	0
D	LX	$LY/2$	LZ
E	0	$LY/2$	LZ
F	LX	LY	LZ

Table 1.1-1 : Coordinates of the cut-off points in 3D

One defines also the equivalent structure in 2D, of dimensions $LX=2\text{m}$, $LY=3\text{m}$ comprising a horizontal plane interface being located at middle height (see [Figure 1.1-2]).

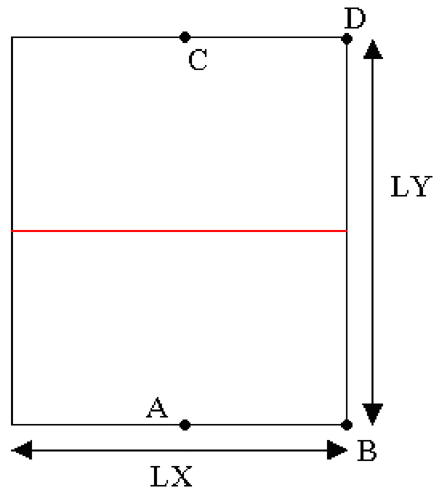

Figure 1.1-2 : Geometry of the structure 2D

The points are defined A , B , C and D who will be used to block the rigid modes.

	x	y
A	$LX/2$	0
B	LX	0
C	$LX/2$	LY
D	LX	LY

Table 1.1-2 : Coordinates of the cut-off points in 2D

1.2 Properties of material

Young modulus: $E=10000\,MPa$
Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

In 3D, the rigid modes are blocked in the following way:

- Points A and D are blocked according to the 3 directions: $\begin{cases} DX^A=DX^D=0 \\ DY^A=DY^D=0 \\ DZ^A=DZ^D=0 \end{cases}$
- Points B and E are blocked along the axis Oz : $DZ^B=DZ^E=0$
- Points C and F are blocked along the axes Ox and Oz : $\begin{cases} DX^C=DX^F=0 \\ DZ^C=DZ^F=0 \end{cases}$

In 2D, the rigid modes are blocked in the following way:

- Points A and C are blocked according to the 2 directions: $\begin{cases} DX^A=DX^C=0 \\ DY^A=DY^C=0 \end{cases}$
- Points B and D are blocked along the axis Oy : $DY^B=DY^D=0$

In 3D, two loadings are considered:

- Pressure distributed constant ($p=10000\,Pa$) on the side faces (plans $y=0$ and $y=LY$), corresponding to a case of compression according to the axis Oy .
- Positive pressure ($p=10000\,Pa$) on the higher side faces and a negative pressure ($p=-10000\,Pa$) on the lower side faces, corresponding to a case of compression on the higher solid and a case of traction on the lower solid

In 2D, two loadings are considered:

- Pressure distributed constant ($p=10000\,Pa$) on the side segments ($x=0$ and $x=LX$), corresponding to a case of compression according to the axis Ox .
- Positive pressure ($p=10000\,Pa$) on the higher side edges and a negative pressure ($p=-10000\,Pa$) on the lower side edges, corresponding to a case of compression on the higher solid and a case of traction on the lower solid

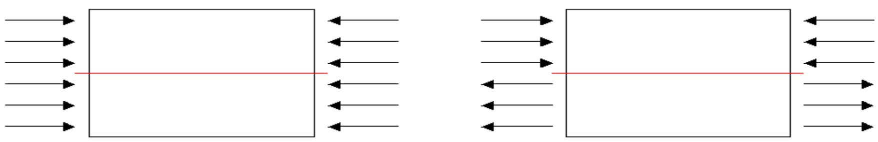

Figure 1.3-1 : Loadings in pressure

1.4 Solutions of the problem

One presents the solution of the 3D problem. That of the problem 2D is obtained while replacing y by x .

1.4.1 Loading in pure compression

The structure is in pure compression, the solution in displacement is commonplace. For the nodes of the side face of left, solution displacement is:

$$u_y=-\frac{LY}{2}\varepsilon_{yy}=-\frac{LY}{2}\frac{\sigma_{yy}}{E}=\frac{LY}{2}\frac{p}{E}=\frac{2.10^4}{2.10^{10}}=10^{-6}m$$

For the nodes of the side face of right-hand side, solution displacement is:

$$u_y=\frac{LY}{2}\varepsilon_{yy}=\frac{LY}{2}\frac{\sigma_{yy}}{E}=\frac{-LY}{2}\frac{p}{E}=-\frac{2.10^4}{2.10^{10}}=-10^{-6}m$$

When a node of the edge is enriched by the Heaviside function, its displacement is written like a combination of a continuous term and a discontinuous term. For this case of loading, there is no discontinuity through the interface, therefore the degrees of freedom nouveau riches are all worthless.

1.4.2 Loading in compression/traction

The higher structure is in pure compression and displacements of the nodes of the higher side face are the same ones as those of the case of preceding loading.

The lower structure is in pure traction. Only the signs of the values of displacement change. For the nodes of the lower side face of left, solution displacement is:

$$u_y=-\frac{LY}{2}\varepsilon_{yy}=\frac{LY}{2}\frac{p}{E}=-\frac{2.10^4}{2.10^{10}}=-10^{-6}m$$

For the nodes of the lower side face of right-hand side, solution displacement is:

$$u_y=\frac{LY}{2}\varepsilon_{yy}=\frac{LY}{2}\frac{p}{E}=\frac{2.10^4}{2.10^{10}}=10^{-6}m$$

In this case, displacement is discontinuous through the interface. The values of the discontinuous degrees of freedom can be easily given (see similar case treaty in [V6.04.173]).

2 Modeling A

In this modeling, for the loading in pressure, the loading is applied using a pressure distributed constant or constant surface forces for the loading n°1, and of a pressure distributed or forces distributed functions of z for the loading n°2.
In this modeling, for the loading in surface force, the loading is applied using constant surface forces for the loading n°1, and of surface forces function of z for the loading n°2.

2.1 Characteristics of the grid

The structure is with a grid with hexahedrons with 8 nodes. The number of elements is smallest possible, that is to say an element following the axis Ox , 2 elements along the axis Oy (in order to be able to define the nodes in the medium plan in $y=LY/2$), and 5 elements along the axis Oz . Along the axis Oz , the number of elements is odd so that the interface does not coincide with the faces of the elements; the 3 layers of central elements use elements X-FEM, and the 2 layers of elements in top and bottom use classical elements.

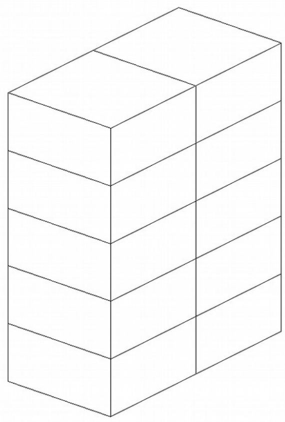

Figure 2.1-1 : Grid 3D

2.2 Features tested

The keyword PRES_REP of the operator AFFE_CHAR_MECA [U4.44.01] allows to apply a pressure distributed constant to meshs of skin. When the pressure is a function or a formula, the keyword is used PRES_REP of AFFE_CHAR_MECA_F [U4.44.01]). This functionality is tested with the loading n°2. Indeed, with X-FEM, one cannot define a higher and lower side surface as a group of meshs. In this case, only one group of meshs comprising all the side surface meshs is defined, and one applies a pressure function of z on this group of meshs.

The keyword FORCE_FACE of the operator AFFE_CHAR_MECA [U4.44.01] allows to apply a constant surface force to meshs of skin. When the surface force is a function or a formula, the keyword is used FORCE_FACE of AFFE_CHAR_MECA_F [U4.44.01]).

2.3 Sizes tested and results

The operator POST_MAIL_XFEM allows to net the cracks represented by method X-FEM. The operator POST_CHAM_XFEM, then allows to export results X-FEM on this new grid. These two operators are to be used only in a posterior way with calculation at sights of postprocessing. They make it possible to generate nodes right in lower part and with the top of the interface and to display their displacements.

For each side face of the structure ($y=0$ and $y=LY$), one tests displacements of the nodes located just at the top and right below the level set.

2.3.1 Loading in compression (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

2.3.2 Loading in compression/traction (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

2.3.3 Loading in compression (loading in surface force)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

2.3.4 Loading in compression/traction (loading in surface force)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

To test all the nodes in only once, one tests the minimum and the maximum of column.

3 Modeling B

In this modeling, for the loading in pressure, the loading is applied using a pressure distributed constant for the loading n°1, and of a pressure distributed function of y for the loading n°2.
In this modeling, for the loading in linear force, the loading is applied using constant linear forces for the loading n°1, and of linear forces functions of y for the loading n°2.

3.1 Characteristics of the grid

In 2D, the structure is with a grid with quadrangles with 4 nodes. The number of elements is smallest possible, are 2 elements along the axis Ox (in order to be able to define the nodes in the medium plan in $x=LX/2$), 5 elements along the axis Oy . Along the axis Oy , the number of elements is odd so that the interface does not coincide with the faces of the elements; the 3 layers of central elements use elements X-FEM, and the 2 layers of elements in top and bottom use classical elements.

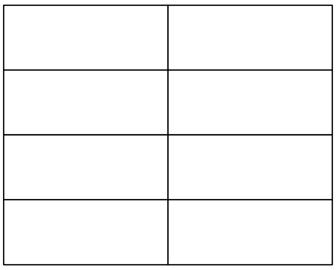

Figure 3.1-1 : Grid 2D

3.2 Features tested

The keyword `PRES_REP` of the operator `AFFE_CHAR_MECA` [U4.44.01] allows to apply a pressure distributed constant to meshs of skin. When the pressure is a function or a formula, the keyword is used `PRES_REP` of `AFFE_CHAR_MECA_F` [U4.44.01]). This functionality is tested with the loading n°2. Indeed, with X-FEM, one cannot define a higher and lower side edge in so much of group of meshs. In this case, only one group of meshs comprising all the side meshs 1D is defined, and one applies a pressure function of y on this group of meshs.

The keyword `FORCE_CONTOUR` of the operator `AFFE_CHAR_MECA` [U4.44.01] allows to apply a constant linear force to meshs of skin. When the linear force is a function or a formula, the keyword is used `FORCE_CONTOUR` of `AFFE_CHAR_MECA_F` [U4.44.01]).

3.3 Sizes tested and results

For each side face of the structure ($x=0$ and $x=LX$), one tests displacements of the nodes located just at the top and right below the level set.

3.3.1 Loading in compression (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}

DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

3.3.2 Loading in compression/traction (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

3.3.3 Loading in compression (loading in surface force)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

3.3.4 Loading in compression/traction (loading in surface force)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

To test all the nodes in only once, one tests the minimum and the maximum of column.

4 Modeling C

The only difference compared to modeling B is the fact that one chooses a modeling C_PLAN and either D_PLAN.

4.1 Sizes tested and results

For each side face of the structure ($x=0$ and $x=LX$), one tests displacements of the nodes located just at the top and right below the level set.

4.1.1 Loading in compression (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

4.1.2 Loading in compression/traction (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

4.1.3 Loading in compression (loading in surface force)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

4.1.4 Loading in compression/traction (loading in surface force)

Identification	Reference

DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DX for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

To test all the nodes in only once, one tests the minimum and the maximum of column.

5 Modeling D

In this modeling, one replaced the linear elements of modeling A by quadratic elements. All the rest is unchanged.

5.1 Characteristics of the grid

They is the same characteristics of modeling that for modeling A but one transformed the grid by linear elements into grid by quadratic elements by using the order CREA_MAILLAGE/LINE_QUAD. The structure is thus with a grid with hexahedrons with 20 nodes.

5.2 Features tested

They is the same features tested as those of modeling A.

5.3 Sizes tested and results

The operator POST_MAIL_XFEM allows to net the cracks represented by method X-FEM. The operator POST_CHAM_XFEM, then allows to export results X-FEM on this new grid. These two operators are to be used only in a posterior way with calculation at sights of postprocessing. They make it possible to generate nodes right in lower part and with the top of the interface and to display their displacements.

For each side face of the structure ($y=0$ and $y=LY$), one tests displacements of the nodes located just at the top and right below the level set.

5.3.1 Loading in compression (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

5.3.2 Loading in compression/traction (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

5.3.3 Loading in compression (loading in surface force)

Identification	Reference

DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	-10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

5.3.4 Loading in compression/traction (loading in surface force)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just with the top of the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just below the interface	10^{-6}
DY for all the nodes of the surface of right-hand side located just at the top of the interface	-10^{-6}

To test all the nodes in only once, the MIN and the MAX of the column are tested.

6 Summaries of the results

The goals of this test are achieved:

- To validate on a simple case the taking into account of the conditions of Neumann on elements of edge X-FEM nouveau riches with the Heaviside function.
- To validate various loadings: constant pressure, pressure function, constant surface force and surface force function in 3D, constant linear force and linear force function in 2D.