

SSNV221 – Hydrostatic test with a behavior DRUCK_PRAGER linear and parabolic

Summary:

The case test proposes a purely hydrostatic loading for the associated law Drucker-Prager [R7.01.16]. The formulation of this plastic law, often used for the grounds, is made at the same time on the deviatoric and hydrostatic part; nevertheless, surface criterion presents a singularity for a purely hydrostatic state of stress. This analytical CAS-test is used to check correct work hardening in this singularity.

The test is carried out on a material point with the order `SIMU_POINT_MAT`. One works with imposed deformations.

One makes a test with linear work hardening (modeling A) and another with parabolic work hardening (modeling B).

1 Problem of reference

1.1 Properties of material

Rubber bands :

$E = 3000 \text{ MPa}$ Young modulus
 $\nu = 0,25$ Poisson's ratio

DRUCK_PRAGER linear (modeling A):

$\alpha = 0,20$ Coefficient of dependence in pressure
 $p_{ultm} = 0,04$ Ultimate cumulated plastic deformation
 $\sigma_Y = 6 \text{ MPa}$ Plastic constraint
 $h = 100 \text{ MPa}$ Module of work hardening

DRUCK_PRAGER parabolic (modeling B):

$\alpha = 0,20$ Coefficient of dependence in pressure
 $p_{ultm} = 0,04$ Ultimate cumulated plastic deformation
 $\sigma_Y = 6 \text{ MPa}$ Plastic constraint
 $\sigma_Y^{ult} = 10 \text{ MPa}$ Ultimate plastic constraint

1.2 Loadings and boundary conditions

A voluminal deformation is imposed $\varepsilon_v = \text{tr}(\boldsymbol{\varepsilon})$. The loading is not monotonous: one charges initially until the voluminal deformation ε_{v1} , by exceeding the threshold of plasticization, then one discharges on a null level of deformation; then one still charges with the deformation ε_{v2} by thus exceeding the ultimate cumulated plastic deformation p_{ultm} , beyond which one finds a perfect plasticity; one still discharges with worthless constraint (deformation equal to the plastic deformation ε_{v2}^p) and one reloads while plasticizing later on until the deformation ε_{v3} . The time of loading (see Table 1.2-1) is fictitious because the plastic laws are independent of time.

t	ε_v
0	0
10	$\varepsilon_{v1} = 0,018$
14	0
26	$\varepsilon_{v2} = 0,045$
30	$\varepsilon_{v2}^p = 0,03667$
40	$\varepsilon_{v3} = 0,06$

Table 1.2-1: imposed voluminal deformation.

1.3 Initial conditions

All the components of the constraints and deformations are worthless at the beginning of the loading.

2 Reference solution

Modeling checks the behavior of the law with linear work hardening.

2.1 Method of calculating

The equations which interest us for analytical calculation are ($I_1 = \text{tr}(\boldsymbol{\sigma})$: trace of the tensor of the constraints, ε_v^p : voluminal plastic deformation):

- plastic constitutive law on the voluminal part:

$$I_1 = 3K(\varepsilon_v - \varepsilon_v^p) \quad (\text{éq 2.1-1})$$

- surface criterion, by posing worthless the constraint of Von Mises ($\sigma_{eq} = 0$) :

$$F(\boldsymbol{\sigma}, p) = \alpha I_1 - R(p) \quad (\text{éq 2.1-2})$$

- relation between the voluminal plastic deformation and the cumulated plastic deformation (variable intern of the plastic law):

$$\dot{\varepsilon}_v^p = 3\alpha \dot{p} \quad \text{thus while integrating: } \varepsilon_v^p = 3\alpha p \quad (\text{éq 2.1-3})$$

- expression of work hardening
 - linear:

$$\begin{aligned} R(p) &= \sigma_Y + h p & \text{si } p \leq p_{ult} \\ R(p) &= \sigma_Y + h p_{ult} = \sigma_Y^{ult} & \text{si } p > p_{ult} \end{aligned} \quad (\text{éq 2.1-4})$$

- parabolic:

$$\begin{aligned} R(p) &= \sigma_Y \left(1 - \left(1 - \sqrt{\frac{\sigma_Y^{ult}}{\sigma_Y}} \frac{p}{p_{ult}} \right)^2 \right) & \text{si } p \leq p_{ult} \\ R(p) &= \sigma_Y^{ult} & \text{si } p > p_{ult} \end{aligned} \quad (\text{éq 2.1-5})$$

It is observed that, as in the linear case, $R(p) = \sigma_Y$ if $p = 0$ and there is perfect plasticity if $p > p_{ult}$.

2.1.1 Deformation in extreme cases elastic initial

This deformation is obtained for $\varepsilon_v^p = p = 0$.

If one poses $F(\boldsymbol{\sigma}, p) = 0$ (plastic evolution) one a:

$$\begin{aligned} I_1^{el} &= \frac{R(p)}{\alpha} = \frac{\sigma_Y}{\alpha} \\ \varepsilon_v^{el} &= \frac{I_1^{el}}{3K} \end{aligned}$$

2.1.2 Ultimate deformation

Ultimate deformation is called ε_v^{ult} that obtained for $p = p_{ult}$.

The trace of constraints easily is found I_1^{ult} and plastic deformation ε_v^{pult} corresponding:

$$I_1^{ult} = \frac{R(p)}{\alpha} = \frac{\sigma_Y^{ult}}{\alpha}$$

$$\varepsilon_v^{pult} = 3 \alpha p_{ult}$$

$$\varepsilon_v^{ult} = \frac{I_1^{ult}}{3K} + \varepsilon_v^{pult}$$

2.1.3 Deformation enters the yield stress and the ultimate deformation

One calculates initially the cumulated plastic deformation.

- By combining the equations (2.1-1), (2.1-2), (2.1-3) and (2.1-4) with $F(\sigma, p) = 0$ for **linear work hardening** one a:

$$p = \frac{3K A \varepsilon_{v1} - \sigma_Y}{9K \alpha^2 + h} \quad (\text{éq 2.1-6})$$

- By combining the equations (2.1-1), (2.1-2), (2.1-3) and (2.1-5) with $F(\sigma, p) = 0$ for **L'parabolic work hardening** one arrives at the equation of dismantled 2:

$$A_1 \bar{p}^2 + B_1 \bar{p} + C_1 = 0$$

$$A_1 = \sigma_Y (1 - \gamma)^2$$

$$B_1 = 9K \alpha^2 p_{ult} - 2\sigma_Y (1 - \gamma)$$

$$C_1 = \sigma_Y - 3K \alpha \varepsilon_{v1}$$

$$\gamma = \sqrt{\frac{\sigma_Y^{ult}}{\sigma_Y}}$$

$$\bar{p} = \frac{p}{p_{ult}} \quad (\text{eq 2.1-7})$$

One uses the equations then (2.1-3) (2.1-1) to find the plastic deformation ε_v^p and traces it constraints I_1 .

If one makes discharge elastic material of way until worthless constraint, one finds a residual deformation equal to the plastic deformation; it is on the other hand necessary to charge material in compression to obtain a worthless total deflection. This second branch is also elastic, because the material of Drucker-Prager cannot plasticize in a hydrostatic state of compression. In this last case, the trace of the constraints, negative, is:

$$I_1^c = -3K \varepsilon_v^p \quad (\text{éq 2.1-8})$$

2.1.4 Deformation higher than the ultimate deformation

All the quantities of interest are easily found, because the trace of constraints is known a priori and equal to I_1^{ult} .

$$\varepsilon_v^p = \varepsilon_v - \frac{I_1^{ult}}{3K}$$

$$p = \frac{\varepsilon_v^p}{3\alpha}$$

2.2 Sizes and results of reference

The module of compressibility K is:

$$K = \frac{E}{3(1-2\nu)} = 2000 \text{ MPa}$$

2.2.1 Deformation in extreme cases elastic

For two modelings one finds easily:

$$I_1^{el} = 30 \text{ MPa}$$

$$\varepsilon_v^{el} = 0,005$$

2.2.2 Ultimate deformation

For two modelings one finds:

$$I_1^{ult} = 50 \text{ MPa}$$

$$\varepsilon_v^{pult} = 0,024$$

$$\varepsilon_v^{ult} \approx 0,03233$$

2.2.3 Deformation equalizes to 0.018 and discharges with worthless deformation

This value of deformation $\varepsilon_{v1} = 0,018$ is higher than the yield stress ε_v^{el} and lower than ε_v^{ult} . One calculates initially the plastic deformation cumulated with the equations (2.1-7) and (2.1-8), then plastic deformation and the trace of the constraints:

- linear work hardening:

$$p_1 = \frac{3K A \varepsilon_{v1} - \sigma_Y}{9K \alpha^2 + h} \approx 0,019$$

$$\varepsilon_{v1}^p = 3 \alpha p_1 = 0,0114$$

$$I_1^1 = 3 K (\varepsilon_{v1} - \varepsilon_{v1}^p) \approx 39,51 \text{ MPa}$$

- parabolic work hardening:

$$p_1 \approx 0,0192$$

$$\varepsilon_{v1}^p = 3 \alpha p_1 \approx 0,0115$$

$$I_1^1 = 3 K (\varepsilon_{v1} - \varepsilon_{v1}^p) \approx 38,956 \text{ MPa}$$

The trace of the constraints with worthless deformation is:

- linear work hardening:

$$I_1^{1c} = -3 K \varepsilon_{v1}^p \approx -68,49 \text{ MPa}$$

- parabolic work hardening:

$$I_1^{1c} = -3 K \varepsilon_{v1}^p \approx -69,044 \text{ MPa}$$

Indeed, the difference between the parabolic and linear case is very weak.

2.2.4 Loading until deformation EGA to 0.045 and 0.06

One reloads material up to the values of deformation $\varepsilon_{v2} = 0,045$ and $\varepsilon_{v3} = 0,06$, higher than ε_v^{ult} .

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The results are the same ones for two modelings.

For $\varepsilon_{v2}=0,045$:

$$\varepsilon_{v2}^p = \varepsilon_{v2} - \frac{I_1^{ult}}{3K} \approx 0,03667$$

$$p_2 = \frac{\varepsilon_{v2}^p}{3\alpha} \approx 0,0611$$

Following the elastic discharge (until worthless constraint), one finds $\varepsilon_v = \varepsilon_{v2}^p$, $p = p_2$.

For $\varepsilon_{v3}=0,06$:

$$\varepsilon_{v3}^p = \varepsilon_{v3} - \frac{I_1^{ult}}{3K} \approx 0,051667$$

$$p_3 = \frac{\varepsilon_{v3}^p}{3\alpha} \approx 0,0861$$

2.2.5 Stress-strain curves

In the Figures (2.2.5-a) and (2.2.5-b) one represents the curve (ε_v, I_1) for linear and parabolic work hardening. In red are the points tested by the CAS-test.

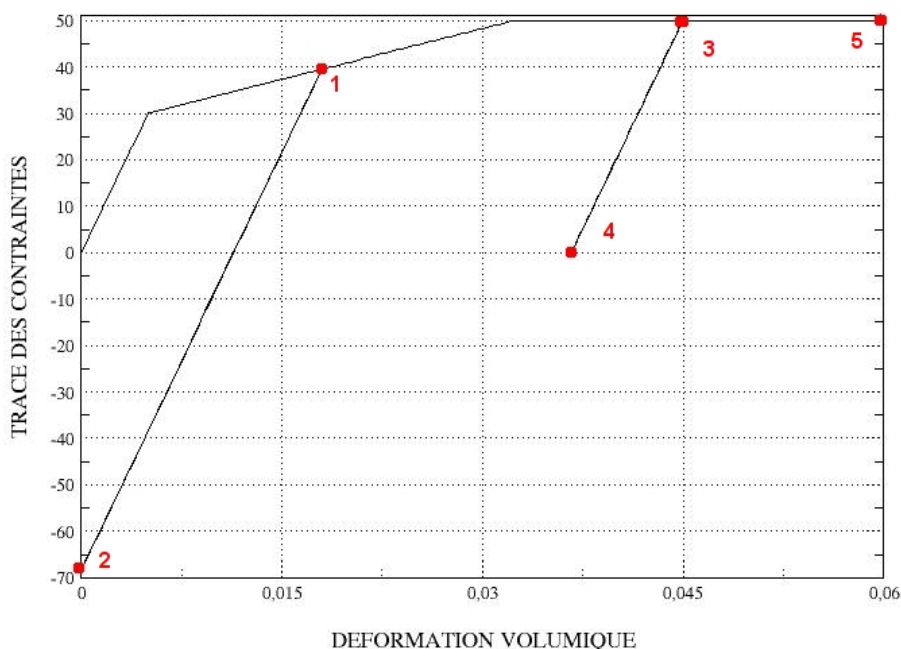


Figure 2.2.5-a: stress-strain curves for linear work hardening.

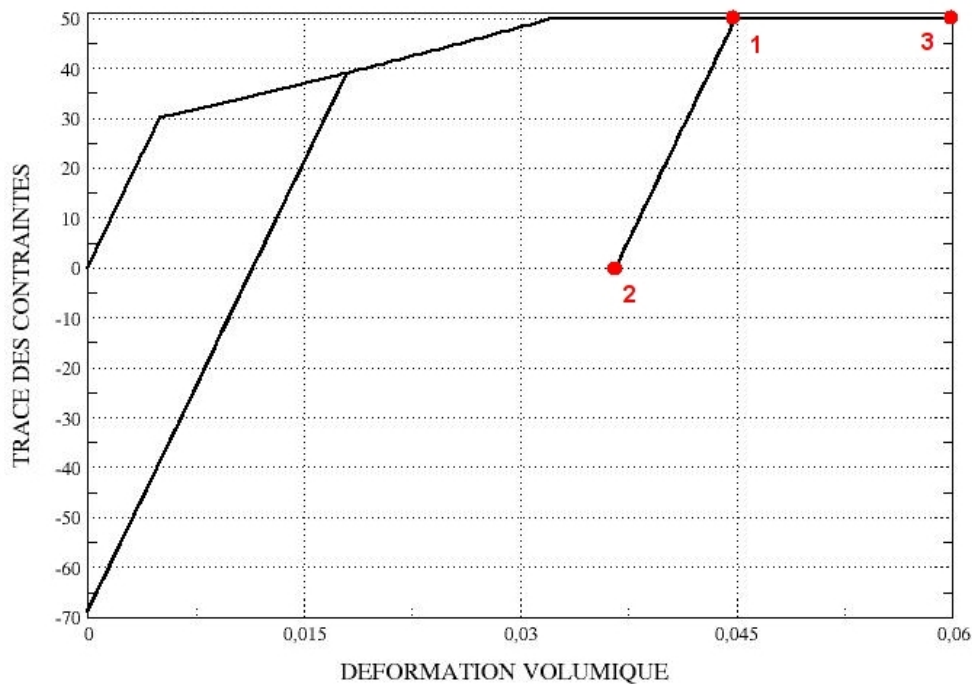


Figure 2.2.5-b: stress-strain curves for parabolic work hardening.

2.3 Uncertainties on the solution

The solution is analytical.

2.4 Bibliographical references

- [1] Document [R3.01.16], Integration of the elastoplastic mechanical behavior of Drucker-Prager DRUCK_PRAGER and postprocessings. Handbook of Code_Aster reference.

3 Modeling A

3.1 Characteristics of modeling

The test is carried out on a material point with the order SIMU_POINT_MAT . One works with imposed deformations.
Work hardening is linear.

3.2 Sizes and results of reference

Not on Figure 2.2.5-a	Checked quantity	Value of reference	Type of reference	Tolerance (relative)
1	Trace of the constraints	$I_1^1 = 39,51 \text{ MPa}$	ANALYTICAL	$10^{-6} \%$
2	Trace of the constraints	$I_1^{1c} = -68,49 \text{ MPa}$	ANALYTICAL	$10^{-6} \%$
3 or 4	Spherical part of the plastic deformation	$\varepsilon_{v2}^p = 0,03667$	ANALYTICAL	$10^{-6} \%$
3 or 5	Trace of constraints	$I_1^{ult} = 50 \text{ MPa}$	ANALYTICAL	$10^{-6} \%$
5	Spherical part of the plastic deformation	$\varepsilon_{v3}^p = 0,051667$	ANALYTICAL	$10^{-6} \%$

4 Modeling B

4.1 Characteristics of modeling

The test is carried out on a material point with the order `SIMU_POINT_MAT` . One works with imposed deformations.

Work hardening is parabolic.

4.2 Sizes and results of reference

Not on Figure 2.2.5-b	Checked quantity	Value of reference	Type of reference	Tolerance (relative)
1 or 2	Spherical part of the plastic deformation	$\varepsilon_{v2}^p = 0,03667$	ANALYTICAL	10^{-6} %
1 or 3	Trace of constraints	$I_1^{ult} = 50 MPa$	ANALYTICAL	10^{-6} %
3	Spherical part of the plastic deformation	$\varepsilon_{v3}^p = 0,051667$	ANALYTICAL	10^{-6} %

5 Summary of the results

The results of the CAS-test are satisfactory, *Code_Aster* reproduced the analytical results with a high precision.