SSNV232 – Triaxial compression test drained with the law of Mohr-Coulomb

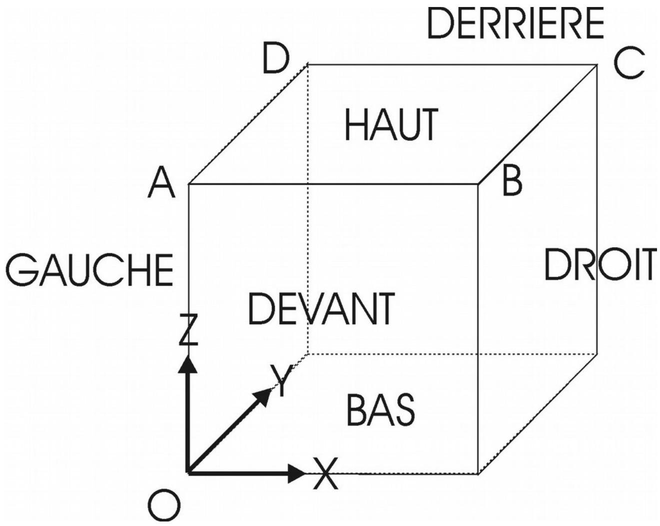
Summary

One is carried out *triaxial calculation in pure mechanics* (equivalent under drained hydraulic conditions) with *the law of Mohr-Coulomb*. The calculated solutions are compared with an analytical solution. This test comprises two modelings:

- a modeling on a material point (SIMU_POINT_MAT);
- a modeling 3D (STAT_NON_LINE);

1 Problem of reference

1.1 Geometry



The triaxial compression test is carried out on only one isoparametric finite element of cubic form $CUB8$. The length of each edge is worth 1. The various facets of this cube are named groups of meshs $HAUT$, BAS, $DEVANT$, $DERRIERE$, $DROIT$ and $GAUCHE$. The group of meshs *SYM* contains the groups of meshs in addition BAS, $DEVANT$ and $GAUCHE$; the group of meshs $COTE$ groups of meshs $DERRIERE$ and $DROIT$.

1.2 Material properties

The elastic properties are:

- isotropic module of compressibility: $K=516{,}2\,MPa$
- modulus of rigidity: $\mu=238{,}2\,MPa$

The parameters of the law of Mohr-Coulomb are:

- angle of friction: $\varphi=33\,°$
- angle of dilatancy: $\psi=27\,°$
- cohesion: $c_0=1\,kPa$

1.3 Boundary conditions and loadings

A triaxial compression test consists in imposing on the test-tube a vertical radial force all while keeping the side pressure constant. It can be drained (the pore water pressure of fluid does not vary during the test) or not-drained (one turns off the tap: the pore water pressure of fluid evolves in the sample). One is interested here in the case drained, simpler, because not utilizing the influence of the pore water pressure of the fluid and *modélisable of this fact by a pure mechanical calculation*.
In the model considered (case of modeling **B**), the cubic element represents a eighth of the sample. The limiting conditions are thus the following ones:

- Conditions of symmetry:
  - $u_z=0$ on the group of mesh BAS
  - $u_x=0$ on the group of mesh $GAUCHE$
  - $u_y=0$ on the group of mesh $DEVANT$
- Conditions of side pressure:
  - $P_n=1$ on the group of mesh $COTE$
- Conditions of loading:

- $P_n=1$ on the group of mesh $HAUT$ (phase 1)
- $u_z=-1$ on the group of mesh $HAUT$ (phase 2)

The loading is carried out in two phases:

- Initialization. Isotropic loading enters $t\in[-2;0]\,secondes$:

pressure p on the groups of meshs $COTE$ and $HAUT$ vary 0 with $p=P_0=50\,kPa$, isotropic pressure of preconsolidation to the state intial;

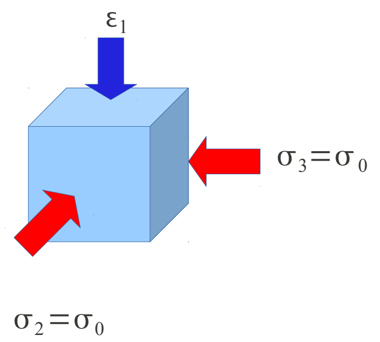
- triaxial test itself :

displacement imposed on the group of meshs $HAUT$ with t varying enters $t\in[0-30]\,secondes$ and u_z varying enters $u_z\in[0;-0{,}3]\,mm$. Vertical deformation ε_{zz} total is of $0{,}03\,\%$;

1.4 Analytical solution

The loadings applied applied to the cubic sample are represented below:

- $\varepsilon_1=F(t)$
- $\sigma_2=\sigma_3=\sigma_0$



The behavior of the sample is governed by the law of Mohr-Coulomb, who expresses himself as follows:

$$\begin{cases} f(\sigma_1,\sigma_3)=|\sigma_1-\sigma_3|+(\sigma_1+\sigma_3)\sin\phi-2c_0\cos\phi\le 0 \\ \sigma_1\ge\sigma_2=\sigma_3 \end{cases} \quad \textbf{(1.4-1)}$$

Table 1.4-1

It is associated with the plastic potential of flow:

$$g(\sigma_1,\sigma_3)=|\sigma_1-\sigma_3|+(\sigma_1+\sigma_3)\sin\psi-2c_0\cos\psi \quad \textbf{(1.4-2)}$$

so that, while noting $t=\sin\psi$ and $\xi=signe(\sigma_1-\sigma_3)$, the law of flow is written:

$$\begin{cases} f(\sigma_1,\sigma_3)\ge 0 \\ \dfrac{\partial g}{\partial\sigma_1}=\dot{\varepsilon}_1^p=\dot{\lambda}(t+\xi) \\ \dfrac{\partial g}{\partial\sigma_2}=\dot{\varepsilon}_2^p=\dot{\varepsilon}_3^p=\dot{\lambda}(t-\xi) \end{cases} \quad \textbf{(1.4-3)}$$

where $\dot{\lambda}$ represent the plastic multiplier.

The two unknown factors of the problem are thus: σ_1 (or ε_3) and $\dot{\lambda}$.

1.4.1 Resolution in elastic mode

The equation **1.4-1** is satisfied. One has then:

$$\begin{cases} \sigma_1 = \overbrace{\left(K+\frac{4}{3}G\right)}^{E}\varepsilon_1^e + 2\left(K-\frac{2}{3}G\right)\varepsilon_3^e \\ \sigma_3 = \left(K-\frac{2}{3}G\right)\varepsilon_1^e + \left(K-\frac{2}{3}G\right)\varepsilon_3^e + \left(K+\frac{4}{3}G\right)\varepsilon_3^e \\ \quad = \underbrace{\left(K-\frac{2}{3}G\right)}_{C}\varepsilon_1^e + \underbrace{2\left(K+\frac{G}{3}\right)}_{D}\varepsilon_3^e \end{cases} \quad \textbf{(1.4.1-1)}$$

Knowing that $\sigma_3=\sigma_0$, one deduces directly from **1.4.1-1** that:

$$\dot{\varepsilon}_3^e = -\frac{C}{D}\dot{\varepsilon}_1^e \quad \textbf{(1.4.1-2)}$$

1.4.2 Resolution in plastic mode

The equation **1.4-3** is satisfied.
That is to say $\sigma^p=\sigma^e$ the elastic prediction given by the equations **1.4.1-1**, the final constraint σ^+ is written:

$$\sigma^+ = \mathbb{C}.\varepsilon^+ = \sigma^p - \dot{\lambda}\,\mathbb{C}.\vec{n}_g \quad \textbf{(1.4.2-1)}$$

where $\mathbb{C}=\begin{bmatrix} E & 2C \\ C & D \end{bmatrix}$ represent the linear tensor of elasticity and $\vec{n}_g=(t+\xi,\ \ t-\xi)$.

One calculates the plastic multiplier while writing $f(\sigma^+)=0$, that is to say:

$$\begin{aligned} &(s+\xi)\left(K+\frac{4}{3}G\right)\varepsilon_1^+ + 2(s+\xi)\left(K-\frac{2}{3}G\right)\varepsilon_3^+ \quad + \\ &(s-\xi)\left(K-\frac{2}{3}G\right)\varepsilon_1^+ + 2(s-\xi)\left(K+\frac{G}{3}\right)\varepsilon_3^+ \quad = 2c_0\cos\phi \\ &\Leftrightarrow \underbrace{2\left(Ks+G\left(\xi+\frac{s}{3}\right)\right)}_{A}\varepsilon_1^+ + \underbrace{2\left(2Ks-G\left(\xi+\frac{s}{3}\right)\right)}_{B}\varepsilon_3^+ = 2c_0\cos\phi \quad \textbf{(1.4.2-2)} \end{aligned}$$

what gives, while replacing:

$$\begin{cases} \varepsilon_1^+ = \varepsilon_1^e - \dot{\lambda}(t+\xi) \\ \varepsilon_3^+ = \varepsilon_3^e - \dot{\lambda}(t-\xi) \end{cases} \textbf{(1.4.2-3)}$$

in **1.4.2-2**, one obtains:
$\dot{\lambda}\underbrace{[A(t+\xi)+B(t-\xi)]}_{BB} = -2c_0\cos\phi + A\varepsilon_1^e + B\varepsilon_3^e = f(\sigma^e)$, that is to say:

$$\dot{\lambda}=\frac{f\left(\sigma^{e}\right)}{BB}$$ **(1.4.2-4)**

1.4.3 Correction of imbalance

One must check $\sigma_3^+=\sigma_0$.
A small virtual variation is supposed δ solution. One seeks the expression of $\delta\sigma_3^+$, that is to say:

$$\delta\sigma_3^+=C\,\delta\varepsilon_1^e+E\,\delta\varepsilon_3^e-\delta\dot{\lambda}\,C\left(t+\xi\right)-\delta\dot{\lambda}\,E\left(t-\xi\right)$$ **(1.4.3-1)**

Knowing that $\dot{\lambda}$ is given by **1.4.2-4**, one a:

$$\delta\dot{\lambda}=\frac{A}{BB}\delta\varepsilon_1^e+\frac{B}{BB}\delta\varepsilon_3^e$$ **(1.4.3-2)**

that one defers in **1.4.3-1**. That gives then:

$$\delta\sigma_3^+=\left[C-A\frac{C\left(t+\xi\right)+E\left(t-\xi\right)}{BB}\right]\delta\varepsilon_1^e+\left[D-B\frac{C\left(t+\xi\right)+E\left(t-\xi\right)}{BB}\right]\delta\varepsilon_3^e$$ **(1.4.3-3)**

During the process of Newton, one searches a new value of $\delta\varepsilon_3$ knowing that $\delta\varepsilon_1=0$ (it is the imposed loading, which cannot be false). This value is such as imbalance $\sigma_0-\sigma_3^+$ either corrected, or:

$$\frac{\partial\sigma_3^+}{\partial\varepsilon_3}\delta\varepsilon_3=\sigma_0-\sigma_3^+$$

maybe, while using **1.4.3-3** :

$$\delta\varepsilon_3=\frac{\sigma_0-\sigma_3^+}{D-B\frac{C\left(t+\xi\right)+E\left(t-\xi\right)}{BB}}$$ **(1.4.3-4)**

1.4.4 Process of resolution of Newton

The process of resolution is written as follows:

With the step of time t such as $\varepsilon_1=F\left(t\right)$:

1 To calculate σ_1^p and σ_3^p using **1.4.2-2** and **1.4.2-1**

2 As long as $|\sigma_0-\sigma_3^+|>\epsilon$, to carry out:

| **3** If $f\left(\sigma_1^p,\sigma_3^p\right)<0$, then **OK** .
| $\sigma_1^+=\sigma_1^p$ and $\sigma_3^+=\sigma_3^p$ and outward journey with **5**
|
| **4** If $f\left(\sigma_1^p,\sigma_3^p\right)\geq 0$, then **NOOK** :
| To calculate $\dot{\lambda}$ using **1.4.2-4**
| To calculate ε_1^+ and ε_3^+ using **1.4.2-3**

	To calculate σ_1^+ and σ_3^+ using **1.4.1-1**
5	To calculate $\delta\varepsilon_3$ using **1.4.2-4**
	To update ε_3^+ , σ_1^+ and σ_3^+ and outward journey with **2**

Table 1.4.4-1 : Procedure of resolution for the triaxial test with the law of Mohr-Coulomb.

The calculation of the analytical solution is called by the function `Triaxial_DR` contained in the file `bibpyt/Contrib/essai_triaxial.py` .

1.4.5 Coherent tangent matrix

At ends of checks, it is possible to display the coherent tangent matrix of the problem.
In complement with the equation 1.4.3-3, one also obtains for $\delta\sigma_1^+$ the following expression:

$$\delta\sigma_1^+=\left[E-A\frac{E(t+\xi)+C(t-\xi)}{BB}\right]\delta\varepsilon_1^e+\left[C-B\frac{E(t+\xi)+C(t-\xi)}{BB}\right]\delta\varepsilon_3^e \quad \textbf{(1.4.5-1)}$$

The form of the coherent tangent matrix $DSDE$ is thus written:

$$DSDE=\begin{bmatrix} E-A\frac{E(t+\xi)+C(t-\xi)}{BB} & C-B\frac{E(t+\xi)+C(t-\xi)}{BB} \\ C-A\frac{C(t+\xi)+E(t-\xi)}{BB} & E-B\frac{C(t+\xi)+E(t-\xi)}{BB} \end{bmatrix} \quad \textbf{(1.4.5-2)}$$

1.5 Results

The solutions post-are treated with the point C for the terms of constraints vertical σ_{zz} and horizontal σ_{xx}, like those of plastic voluminal deformation ε_v^p and of plastic deviatoric deformation $|\varepsilon_d^p|=\sqrt{\frac{3}{2}\left(\varepsilon-\frac{\varepsilon_v^p}{3}I\right):\left(\varepsilon-\frac{\varepsilon_v^p}{3}I\right)}$.

2 Modeling A

2.1 Characteristics of modeling

Modeling **With** is carried out on a material point, using SIMU_POINT_MAT.

The step of time is of $\Delta t=0{,}1\,sec$, that is to say 300 temporal increments.

A light dissymmetry of $\epsilon=10^{-6}$ is introduced on the horizontal loading in order to avoid a too marked singularity of the tangent matrix at the time of the entry in plasticity:

$$\begin{cases}\sigma_{xx}=\sigma_0\\ \sigma_{yy}=\sigma_0\left(1+10^{-6}\right)\end{cases}$$

2.2 Sizes tested and results

2.2.1 Values tested

The solutions are calculated at the point C and compared with the analytical solution at the final moment $t=30\,sec$. They are given in terms of constraints vertical σ_{zz} and horizontal σ_{xx} , of plastic voluminal deformation ε_v^p and of plastic deviatoric deformation $|\varepsilon_d^p|=\sqrt{\frac{3}{2}\left(\varepsilon-\frac{\varepsilon_v^p}{3}I\right):\left(\varepsilon-\frac{\varepsilon_v^p}{3}I\right)}$, and recapitulated in the following table:

$t=30\,sec$	Analytical solution	Acceptable relative error [%]
σ_{zz}	−1,732895416041E+5	1,E-1
σ_{xx}	50000.	1,E-1
ε_v^p	1,6784502547224E-4	1,E-1
$\lvert\varepsilon_d^p\rvert$	3,3099447556585E-4	1,E-1

Table 2.2.1-1 : Validation of the results for modeling A

2.2.2 Comments

The variation with the analytical solution is very weak (lower than $10^{-3}\%$).

3 Modeling B

3.1 Characteristics of modeling

Modeling **B** is carried out in 3D, using `STAT_NON_LINE`.

The step of time is of $\Delta t=0,1\,sec$, that is to say 300 temporal increments. The automatic recutting of the step of time is activated.

A light dissymmetry of $\epsilon=10^{-3}$ is introduced on the horizontal loading in order to avoid a too marked singularity of the tangent matrix at the time of the entry in plasticity:

$$\begin{cases}\sigma_{xx}=\sigma_0\\ \sigma_{yy}=\sigma_0\left(1+10^{-3}\right)\end{cases}$$

3.2 Sizes tested and results

3.2.1 Values tested

The solutions are calculated at the point C and compared with the analytical solution at the final moment $t=30\,sec$. They are given in terms of constraints vertical σ_{zz} and horizontal σ_{xx} , of plastic voluminal deformation ε_v^p and of plastic deviatoric deformation $|\varepsilon_d^p|=\sqrt{\frac{3}{2}\left(\varepsilon-\frac{\varepsilon_v^p}{3}I\right):\left(\varepsilon-\frac{\varepsilon_v^p}{3}I\right)}$, and recapitulated in the following table:

$t=30\,sec$	Analytical solution	Acceptable relative error [%]
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ε_v^p	1,6784502547224E-4	1,E-1
$\lvert\varepsilon_d^p\rvert$	3,3099447556585E-4	1,E-1

Table 3.2.1-1 : Validation of the results for modeling B

3.2.2 Comments

The variation with the reference solution is very weak (lower than $10^{-5}\%$).