# SSNV234 - Elementary validation of the law ENDO_FISS_EXP and of piloting PRED_ELAS for modeling GRAD_VARI

**Summary:**

The purpose of this test is to validate the algorithm of integration of the law of behavior ENDO_FISS_EXP with gradient of internal variables as well as piloting PRED_ELAS available for this law. The studied problem corresponds to a request with imposed homogeneous deformation for which one can obtain an analytical solution.

Various treated modelings are following:

- **Modeling A** ( $2D$ ): Modeling is employed D_PLAN_GRAD_VARI.
- **Modeling B** ( $3D$ ): Modeling is employed 3D_GRAD_VARI.

# 1 Problem of reference

## 1.1 Geometry

According to modeling $2\mathrm{D}$ or $3\mathrm{D}$, one respectively considers a square or a cube on side 2 $\mathrm{mm}$.

## 1.2 Properties of material

The material obeys the law of elastic behavior fragile ENDO_FISS_EXP with gradient of damage (D_PLAN_GRAD_VARI and 3D_GRAD_VARI). The macroscopic data correspond to:

| | |
|---|---|
| $E=30\,000\,\mathrm{MPa}$ | Young modulus |
| $\nu=0.2$ | Poisson's ratio |
| $G_f=0.1\,\mathrm{N/mm^2}$ | Energy of cracking |
| $p=5$ | Parameter of form |
| $f_t=2.986\,\mathrm{MPa}$ | Limit in traction |
| $f_c=29.86\,\mathrm{MPa}$ | Limit in compression |
| $D=50\,\mathrm{mm}$ | Half-width of the band of damage |

The choice of these values of $F_T$ and $F_C$ in link with the criterion of François in ENDO_FISS_EXP corresponds to a limit in traction confined of $\Sigma^C$ = 3 MPa. In addition, the internal parameters of the model which intervene in the analytical solution are equal to:
$k=1.5\times10^{-3}\mathrm{MPa}$ ; $m=11.111$ ; $\gamma=9.534\times10^{3}$
In the test, they are calculated by the macro-order DEFI_MATER_GC.

## 1.3 Boundary conditions and loadings

Displacements are imposed in all the nodes of the structure, of kind to correspond to the desired homogeneous deformation. More precisely, displacement in a node of coordinates $X$ is worth: $u(x)=\varepsilon\cdot x$ . One starts by imposing a uniaxial deformation of traction then one discharges until imposing a deformation of compression to check the effect of the restoration of rigidity in compression.

## 1.4 Initial conditions

None.

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

This problem admits an analytical solution. During the phase of traction, one determines the relation which with the imposed deformation $\boldsymbol{\varepsilon}$ associate the homogeneous level of damage $a$ . One adopts here a uniaxial deformation of the form:

$\boldsymbol{\varepsilon}=\varepsilon\, \boldsymbol{n}\otimes\boldsymbol{n} \quad \text{où} \quad \|\boldsymbol{n}\|=1 \quad \text{et} \quad \varepsilon>0$

It is thus about a problem in confined uniaxial traction. The problem being homogeneous, the damage (in load) and the deformation are bound by the relation of coherence:

$$A'(a)\Gamma(\boldsymbol{\varepsilon})+\omega'(a)=0 \qquad \text{with} \quad \omega(a)=ka\,, \quad A(a)=\frac{(1-a)^2}{(1-a)^2+ma(1+pa)}$$

The function $\Gamma$ be based on the form of the criterion which defines the field of elasticity, cf [R5.03.28] or, in more detailed way, [Lorentz, 2016]. In this case of a traction/confined uniaxial pressing, she is written simply:

$$\Gamma(\boldsymbol{\varepsilon})=\frac{1}{2}E_c\varepsilon^2 \quad \text{where} \quad E_c=\lambda+2\mu$$

One of thus deduced simply the relation enters $\varepsilon$ and $a$ :

$$\varepsilon=\sqrt{\frac{-2k}{E_c A'(a)}}$$

For the CAS-test, the deformation will grow up to a level such as the level of damage is worth $a=0.2$ and one will test that the damage reached well his target value.

In the second time, one slackens the deformation then one imposes a compression up to a level $\varepsilon_{comp}=-2\sigma^c/E_c$ . The damage does not evolve any more. The relation stress-strain is written:

$$\sigma=A(a)E_c\varepsilon+\frac{1}{2}(1-A(a))E_c S'(\varepsilon) \quad \text{where} \quad S(\varepsilon)=\langle-\varepsilon\rangle^2\exp\left(\frac{1}{\gamma\varepsilon}\right)$$

The discharge is thus carried out linearly with a secant module of rigidity $A(a)E_c$ . Then, in phase of compression, the model gradually restores rigidity according to the relation above. One checks in the test that the level of constraint is well that corresponding to the analytical relation. In practice, rather than to project the tensor of constraint on the direction of request, one makes sure that the deformation energy obtained via order POST_ELEM (keyword factor TRAV_EXT, component TRAV_ELAS) is quite equal to the expected value, that is to say:

$\frac{vol(\Omega)}{2}\boldsymbol{\sigma}:\boldsymbol{\varepsilon}=\frac{vol(\Omega)}{2}\sigma\epsilon$ where $vol(\Omega)$ indicate the volume of the element (4 mm$^2$ or 8 mm$^3$)

## 2.2 Results of reference

In plane deformations, one adopts a direction of request $\boldsymbol{n}=(1/\sqrt{5},\ 2/\sqrt{5})$ . In 3D, it is worth $\boldsymbol{n}=(1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14})$ . One sets like target a damage $a=0.2$ ; that corresponds to an intensity of request $\varepsilon=2.86\times10^{-4}$ according to the reference solution above.

The loading is applied with the help of the technique of piloting `PRED_ELAS` in which one fixes the maximum terminal of kind to reach the level of deformation $\varepsilon$ above. It will be checked that the damage corresponding reached well 0.2.

In compression, the level of imposed deformation rises with $\varepsilon_{comp}=-1.8\times10^{-4}$ for a constraint of $\sigma_{comp}=-4.537\,\mathrm{MPa}$ .

## 2.3 Uncertainties on the solution

Nothing.

## 2.4 Bibliographical references

Lorentz E. (2016) A nonlocal ramming model for lime pit concrete consist with cohesive fracture. Submitted to J. Mech. Phys. Solids.

# 3 Modeling A

## 3.1 Characteristics of modeling

A modeling D_PLAN_GRAD_VARI with a single mesh, element QUAD8 .
Loading in the direction $\boldsymbol{n}=(1/\sqrt{5},\ 2/\sqrt{5})$ .

## 3.2 Characteristics of the grid

Many nodes: 8
Number and types of meshs: 1 QUAD8, 4 SEG3

## 3.3 Sizes tested and results of modeling A

One tests the damage in three nodes of the mesh, the value with the nodes being obtained by extrapolation (CHAM_NO 'VARI_NOEU', component V1) as well as the deformation energy after the phase of compression.

| Identification | Reference | Type | Tolerance |
|---|---|---|---|
| V1 ( $X=2$ , $Y=0$ ) | 0.2 | ANALYTICAL | RELATIVE – 0.1% |
| V1 ( $X=2$ , $Y=1$ ) | 0.2 | ANALYTICAL | RELATIVE – 0.1% |
| V1 ( $X=2$ , $Y=2$ ) | 0.2 | ANALYTICAL | RELATIVE – 0.1% |
| TRAV_ELAS | $1.633\times10^{-3}$ | ANALYTICAL | RELATIVE – 0.1% |

# 4 Modeling B

## 4.1 Characteristics of modeling

A modeling 3D_GRAD_VARI with a single mesh, element HEXA20 .
Loading in the direction $\boldsymbol{n}=(1/\sqrt{14},\ \ 2/\sqrt{14},\ \ 3/\sqrt{14})$ .

## 4.2 Characteristics of the grid

Many nodes: 20
Number and types of meshs: 1 HEXA20 , 6 QUAD8, 8 SEG3

## 4.3 Sizes tested and results of modeling B

One tests the damage in three nodes of the mesh, the value with the nodes being obtained by extrapolation (CHAM_NO 'VARI_NOEU', component V1) as well as the deformation energy after the phase of compression.

| Identification | Reference | Type | Tolerance |
|---|---|---|---|
| V1 ( $X=2$ , $Y=0$ ) | 0.6 | ANALYTICAL | RELATIVE – 0.1% |
| V1 ( $X=2$ , $Y=1$ ) | 0.6 | ANALYTICAL | RELATIVE – 0.1% |
| V1 ( $X=2$ , $Y=2$ ) | 0.6 | ANALYTICAL | RELATIVE – 0.1% |
| TRAV_ELAS | $3.267\times10^{-3}$ | ANALYTICAL | RELATIVE – 0.1% |

# 5 Summary of the results

This CAS-tests is realized on only one nets, consequently it is the homogeneous answer of damage which is found numerically. The reference solution is obtained while being placed on the threshold of damage. One notes very a good agreement between the modeling and the reference solution. In compression, one tests the restoration of rigidity, there too satisfactorily. On the other hand, the not-local part of the law is not tested here.