

SSNV246 - Application of a pressure distributed on the lips of an interface XFEM curves crossing a column

Summary:

It is about a test of validation of the facets of contact resulting from under elements from integration XFEM.

The facets of contact used by default do not result from under elements from integration and do not give the possibility of having quadratic facets of contact in 3D. One can activate the recovery of the facets of contact resulting from under elements from integration in the operator `MODI_MODELE_XFEM` with the keyword `DECOUPE_FACETTE=' SOUS_ELEMENTS'`. One then has linear and quadratic facets in 2D as in 3D which have the advantage of being in conformity with under elements of integration since they are selected among the sides of under elements of integration in 2D and the faces of under elements of integration in 3D. This mode of recovery also has the following advantage: the coordinates of the nodes of these facets are not recomputed as it is the case by default, because these nodes are recovered during the cutting which is carried out in `TOPOSE`.

In this test, one makes sure of the good performance of the recovery of these facets when a mechanical pressure is applied to the lips of the interface. The got results are compared with an analytical solution. The geometry is two-dimensional, an example of use of the facets of contact resulting from under elements from integration for a geometry 3D is presented in the case test `ssnv247 [v6.04.247]`. One voluntarily chooses an Iso-zero "Isn" curve representing the interface to appreciate the performance profit brought by the quadratic "facetisation".

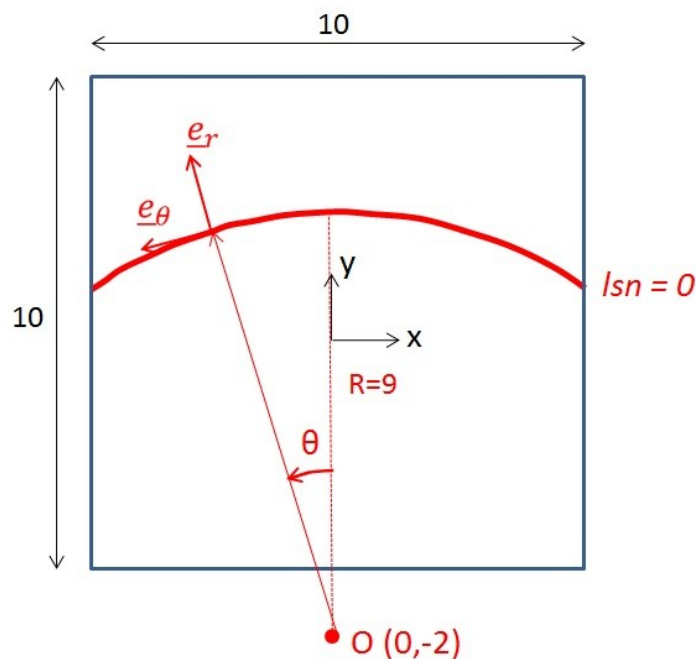
1 Problem of reference

1.1 Geometry of the problem

It is about a square on side $L=10\text{ m}$. This bar has a discontinuity of the type interfaces (interface nonwith a grid which is introduced into the model via the level-sets thanks to the operator `DEFI_FISS_XFEM`). The square is thus entirely crossed by discontinuity (on the level of the approximation of the field of displacements, one takes into account only Heaviside enrichment). Discontinuity is circular of center $O(0,-2)$ and of ray $R=9\text{ m}$.

One represents on the Figure 1.1-a geometry of the problem.

Figure 1.1-a: Geometry of the problem 2D



1.2 Properties materials

Parameters given in the Table 1.2-1, correspond to the parameters used for 4 modelings. The behavior is elastic ('ELAS').

Elastic parameters	Young modulus $E(\text{en MPa})$	5800
	Poisson's ratio ν	0
	Thermal dilation coefficient $\alpha(\text{en } K^{-1})$	0

Table 1.2-1 : Properties of material

1.3 Boundary conditions and loading

The following conditions of Dirichlet are applied:

- on the lower side of the square, displacements are blocked in all the directions ($u_x=0$ and $u_y=0$),
- on the higher side of the square, displacements according to x are blocked $u_x=0$ and one imposes a crushing of the following square y , $u_y = u_{y, \text{impo}} = -1.E^{-6}$.

One applies in the crack "the pressure and the shearing of contact". I.e. that the efforts are applied that there would be on the level of the crack if the square were not fissured where if there were a perfect adherent contact. The conditions of Neuman are thus the following ones:

- on each lip of the interface one imposes a pressure distributed $p(\theta) = \sigma_{yy} * \cos^2(\theta)$ via AFFE_CHAR_MECA and of the keyword CRACK of PRES_REP.
- on each lip of the interface one imposes a shearing distributed $t(\theta) = -\sigma_{yy} * \cos(\theta)\sin(\theta)$ via AFFE_CHAR_MECA and of the keyword CRACK of CISA_2D.

2 Reference solution

2.1 Method of calculating

It is about an analytical solution. Taking into account the boundary conditions, displacements can be obtained starting from the analytical resolution of the conservation equation of the momentum.

The Poisson's ratio ν being null, the problem is unidimensional according to y . The tensor of the constraints is uniform in all the field: $\sigma = E \epsilon = E * \epsilon_{yy} e_y \otimes e_y$.

However $\text{Div}(\sigma) = 0$ thus $\frac{\partial \epsilon_{yy}}{\partial y} = 0$. According to the boundary conditions applied $\epsilon_{yy} = \frac{u_{y, \text{impo}}}{L}$

Finally, $\sigma = E * \frac{u_{y, \text{impo}}}{L} e_y \otimes e_y = \sigma_{yy} e_y \otimes e_y$.

On the level of the crack, $e_r = -\sin(\theta) e_x + \cos(\theta) e_y$ and $e_\theta = -\cos(\theta) e_x - \sin(\theta) e_y$.

On the level of a point of the crack of coordinates (R, θ) , if there were no crack, one would have:

$$\sigma \cdot e_r = (e_r \cdot \sigma \cdot e_r) e_r + (e_r \cdot \sigma \cdot e_\theta) e_\theta = \sigma_{yy} * [(e_y \cdot e_r)^2 e_r + (e_y \cdot e_r) \times (e_y \cdot e_\theta) e_\theta]$$

$$\sigma \cdot e_r = \sigma_{yy} * (\cos^2(\theta) e_r - \sin(\theta) \cos(\theta) e_\theta)$$

Finally the solution of the problem is:

$$\sigma = \sigma_{yy} e_y \otimes e_y$$

$$u(x, y) = \frac{u_{y, \text{impo}}}{L} * \left(\frac{L}{2} + y \right)$$

2.2 Sizes and results of reference

The value of the constraints is tested SIGMAXX and SIGMAYY on the whole of the column.

Sizes tested	Type of reference	Value of reference
SIGMAXX (MPa)	'ANALYTICAL'	0.0
SIGMAYY (MPa)	'ANALYTICAL'	-5800.0

2.3 Uncertainties on the solution

No, the solution is analytical.

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling `D_PLAN` using linear elements `HM-XFEM`.

3.2 Characteristics of the grid

The square on which one carries out modeling is divided into 16 `QUAD4`. The interface is nonwith a grid and cuts the square horizontally. The grid is represented Figure 3.2-a.

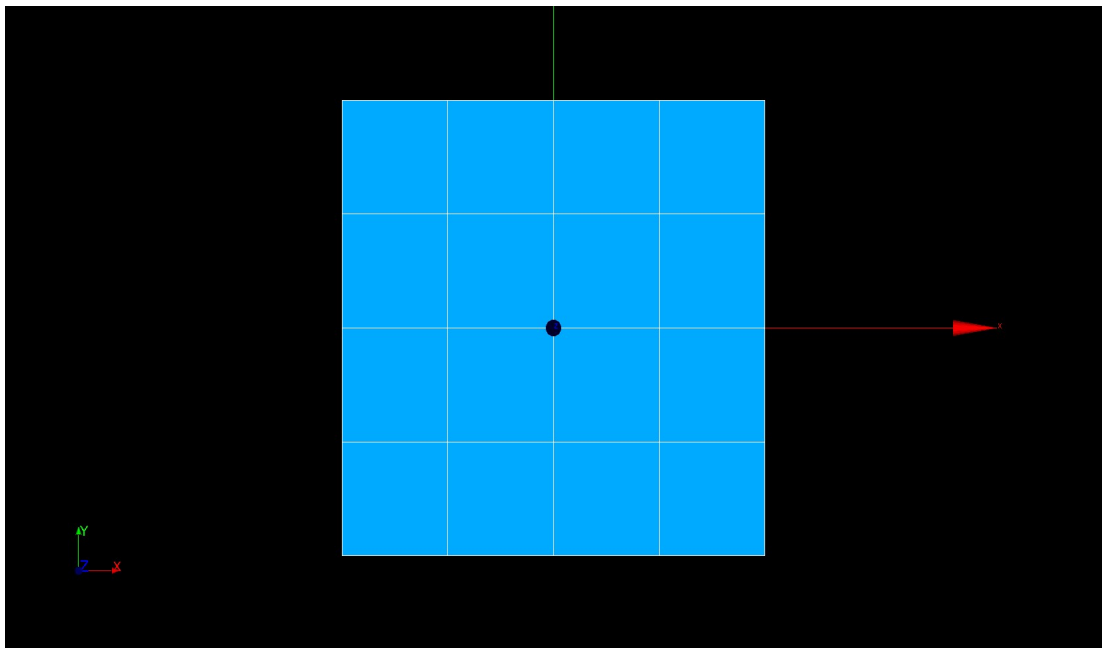


Figure 3.2-a: Grid 2D

3.3 Sizes tested and results

The results are got with `Code_Aster` (resolution with `STAT_NON_LINE`). The horizontal constraint is tested σ_{xx} supposed uniformly worthless and the vertical constraint σ_{yy} presumedly uniform of value $-E$. With this intention, it is tested MIN and it MAX of these two sizes in all the element. The got results are synthesized in the table below.

Sizes tested	Type of reference	Values of reference	Tolerance
SIGMAXX (MPa) MIN	'ANALYTICAL'	0.0	30
SIGMAXX (MPa) MAX	'ANALYTICAL'	0.0	45
SIGMAYY (MPa) MIN	'ANALYTICAL'	-5800.0	8 %
SIGMAYY (MPa) MAX	'ANALYTICAL'	-5800.0	4 %

Displacements according to y and the deformation are represented on the Figure 3.3-a. One observes well a linear compression of the square, little disturbed by the presence of the crack.

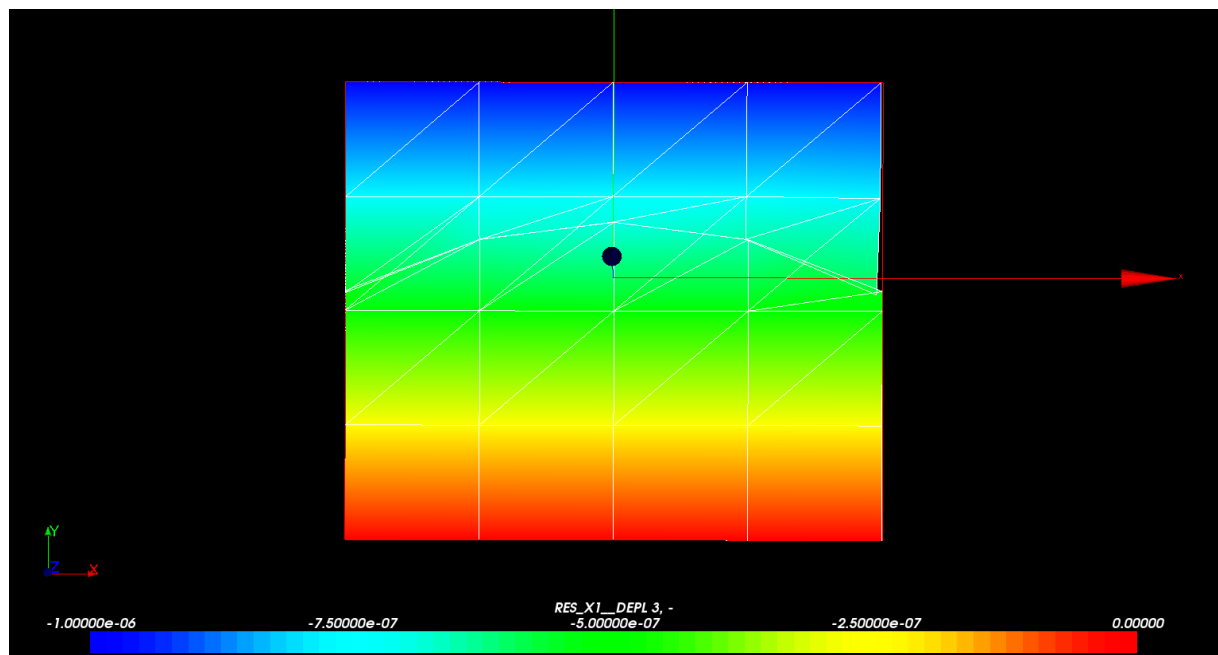


Figure 3.3-a: Field of displacements according to direction (OY)

On the other hand, on the Figures 3.3-b and 3.3-c, one observes the variations of the stress field compared to the analytical solution, in particular in the vicinity of the crack and close to the edges. The relative precision obtained can appear poor but it should be put in relation to the low number of elements used and the important curve of discontinuity.

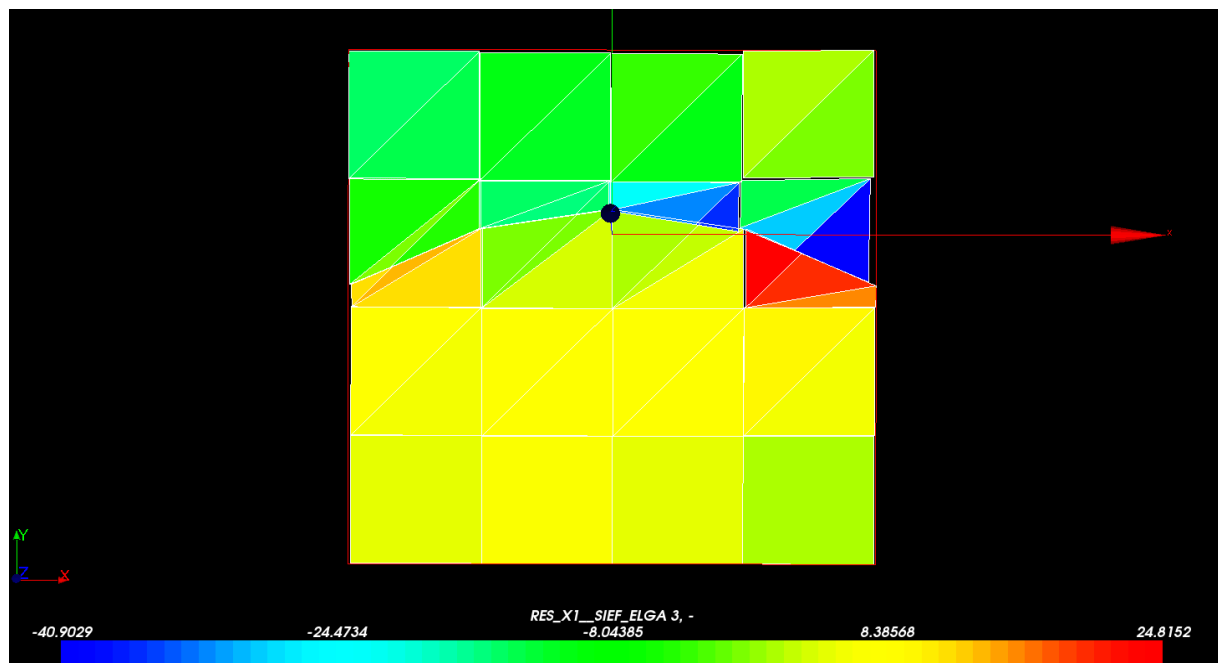


Figure 3.3-b: Stress field σ_{xx}

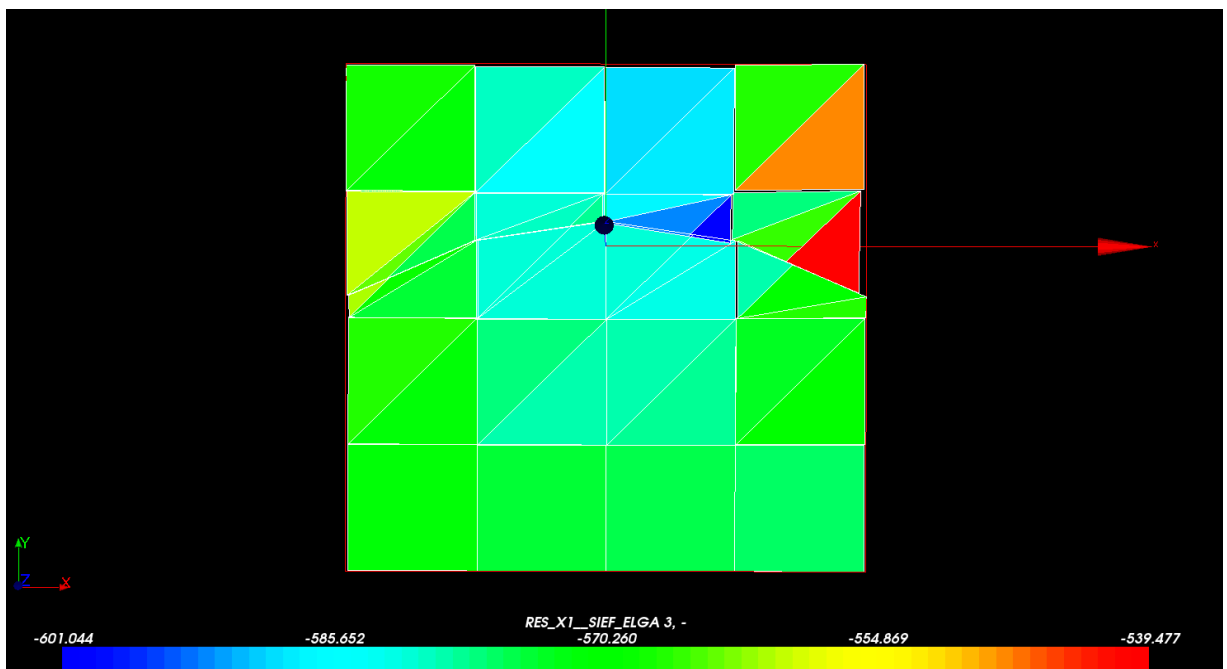


Figure 3.3-c: Stress field σ_{yy}

4 Modeling B

4.1 Characteristics of modeling

This modeling is strictly identical to the preceding one, except for the elements used which are quadratic.

4.2 Characteristics of the grid

The square on which one carries out modeling is divided into 16 QUAD8. The interface is nonwith a grid and cuts the square horizontally.

4.3 Sizes tested and results

The results are got with Code_Aster (resolution with STAT_NON_LINE). The horizontal constraint is tested σ_{xx} supposed uniformly worthless and the vertical constraint σ_{yy} presumedly uniform of value $-E$. With this intention, it is tested MIN and it MAX of these two sizes in all the element. The got results are synthesized in the table below.

Sizes tested	Type of reference	Values of reference	Tolerance
SIGMAXX (MPa) MIN	'ANALYTICAL'	0.0	0.01
SIGMAXX (MPa) MAX	'ANALYTICAL'	0.0	0.02
SIGMAYY (MPa) MIN	'ANALYTICAL'	-5800.0	0,002 %
SIGMAYY (MPa) MAX	'ANALYTICAL'	-5800.0	0,001 %

Displacements according to y and the deformation are represented on the Figure 4.3-a. One observes well a linear compression of the square, which is carried out as if the crack were not present.

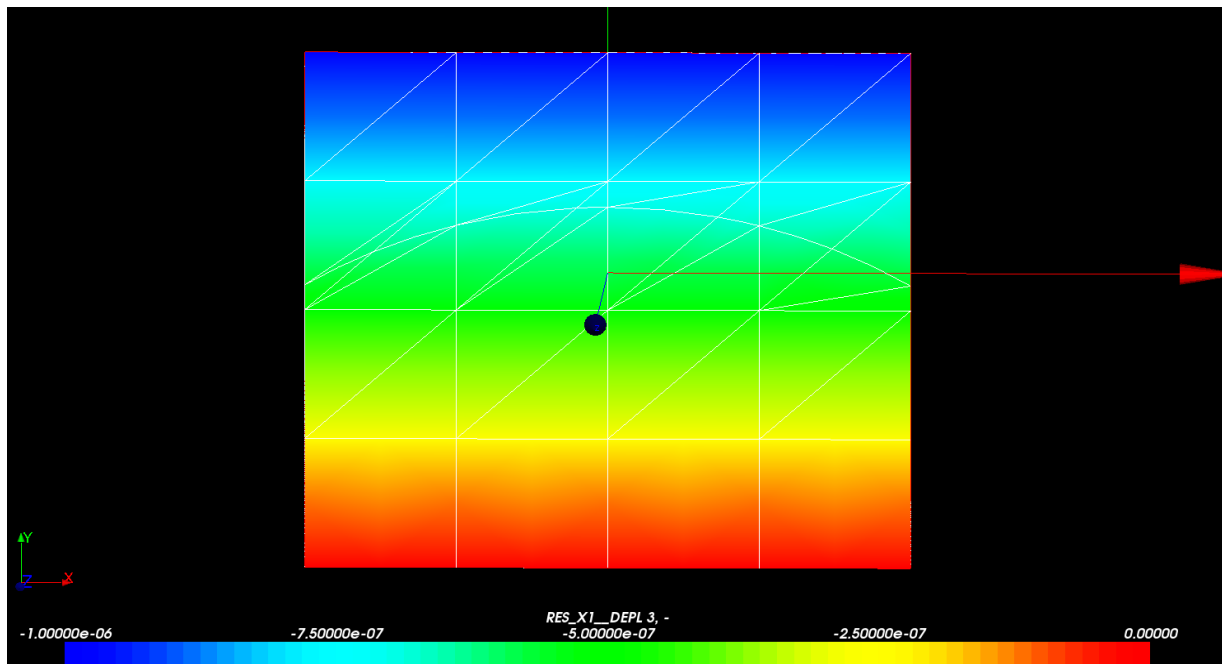


Figure 4.3-a: Field of displacements according to direction (OY)

On the Figures 4.3-b and 4.3-c, it is confirmed that the variations in constraints compared to the analytical solution are reduced in a consequent way compared to preceding linear modeling. The

relative error decreases by a factor 10^5 . Very light variations are always observed in the vicinity of the crack and the level of the edges.

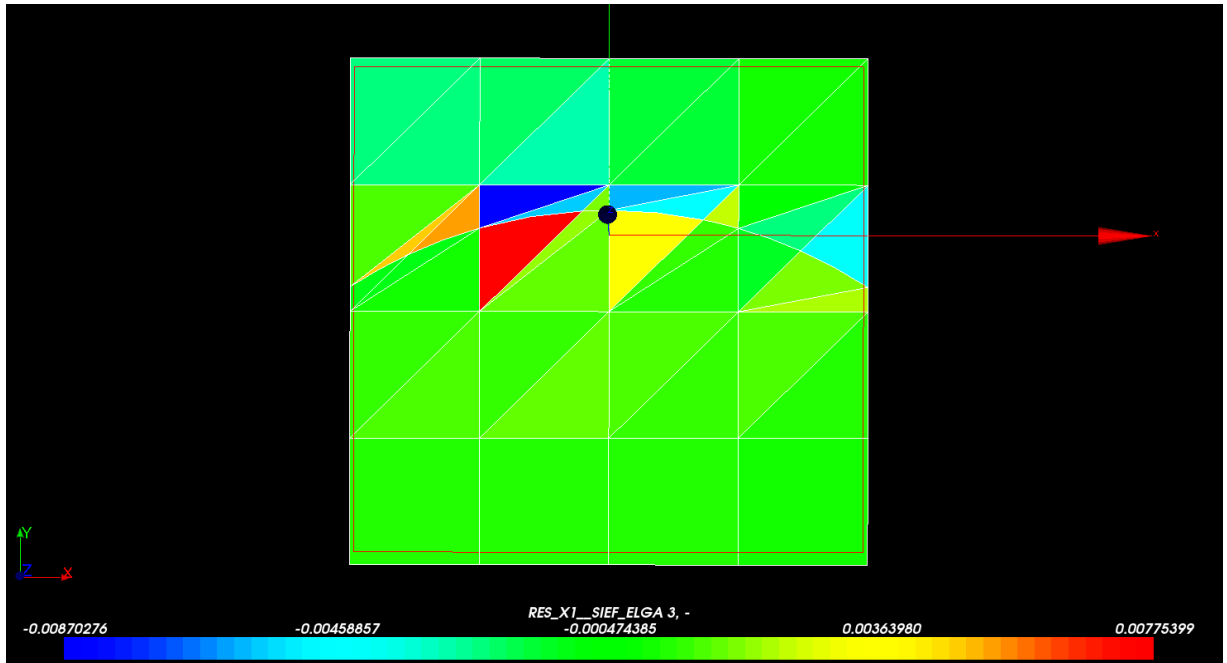


Figure 4.3-b: Stress field σ_{xx}

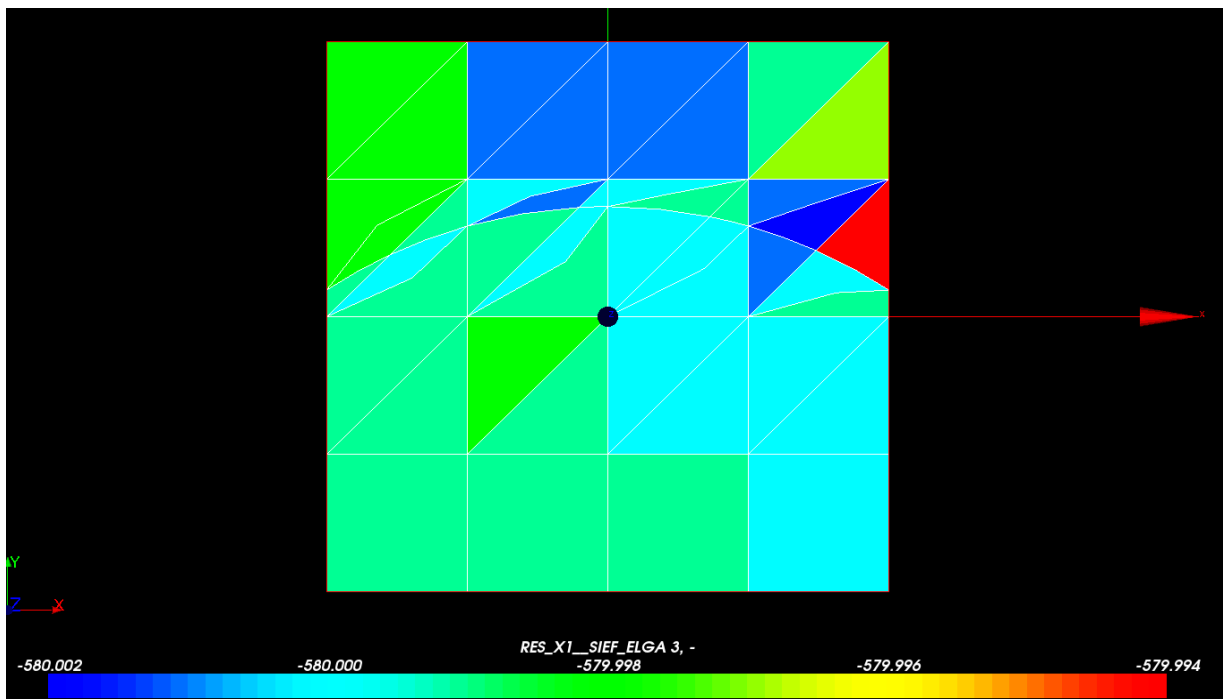


Figure 4.3-c: Stress field σ_{yy}

Two preceding modelings validate the use of the facets of contact resulting from under elements from integration for the application from a mechanical pressure and a shearing on the lips from crack for modelings 2D, linear and quadratic. One notes in particular the important profit in precision when quadratic elements are used. Even if discontinuity is curved, one obtains an excellent precision with very few elements.

5 Conclusion

Two modelings validate the use of the facets of contact resulting from under elements from integration XFEM for the application from mechanical efforts from pressure and shearing in 2D on the lips from a crack nonwith a grid. They also illustrate the profit of precision obtained with quadratic elements compared to linear elements.