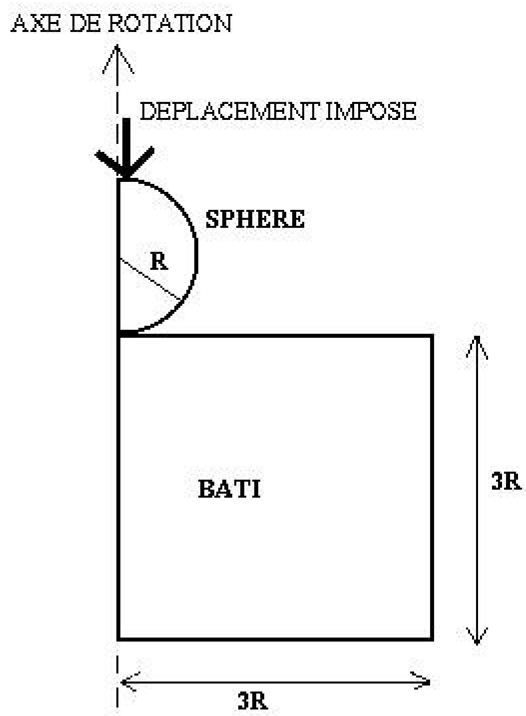
SSNV506 - Elastoplastic indentation of a block by an elastic spherical indentor

Summary:

This test relates to the modeling of the indentation of an elastic sphere on a half-plane with the elastoplastic behavior. The objective is to test the features related to the contact on an example comprising nona - linearity material.

1 Problem of reference

1.1 Geometry



Ray of the sphere	$R=500\,mm$
Imposed displacement	$100\,mm$

1.2 Properties of material

Two different modelings to represent the rigid sphere:

Material rigidification: $E=2{,}1\mathrm{E}9\,Mpa$ and $\nu=0{,}3$
Rigidification by conditions kinematics

Block: Steel, law of perfect behavior élasto - plastic.

Module Young	$E=210000\,MPa$
Poisson's ratio	$\nu=0{,}3$
Module of work hardening	$Et=0$
Yield stress	$\sigma_y=50\,MPa$

1.3 Boundary conditions and loadings

The deformations are axisymmetric and the block forming the plan is supposed to be embedded on its basis.

An imposed displacement is applied:

- Loading of 0 with $-100\,mm$ on the upper part of the sphere in the models A and D

- Loading of 0 with $-100\,mm$ on the surface of contact of the sphere in the models B and C

2 Reference solution

2.1 Méthode of calculation used for the reference solution

The results of reference result from the book quoted below [bib1].

$p_m = 3\sigma_0$ with p_m the contact pressure (page 171).

$Rjohnson = p_m a = 3\sigma_0 a$ if a is the surface of contact.

However in perfect plasticity $\delta = 0{,}368\, a^2 / R$ according to the analysis of Richmond (page 200)

Finally, one obtains:

$Rjohnson = 3\pi R \sigma_0 \delta / 0{,}368$

$Rjohnson$: Normal reaction of contact of the solid mass on the sphere
R : Ray of the sphere
δ : Déplacement top of the solid mass
σ_0 : Yield stress of the solid mass

This result is valid under the following assumptions:

- axisymmetric problem,
- perfectly plastic material (coefficient 0.368 is resulting from this assumption)
- small deformations
- rigid sphere.

2.2 Results of reference

The results of reference are got starting from the preceding formula. It is valid for the complete model in 3D.

Note:

In our study, Rjohnson depends only on displacement, one can write the relation in the following form thanks to the facts of the case: $Rjohnson = 640\,270\,\delta$ *with* $Rjohnson$ *in newton and* δ *in millimetre.* δ *is directly connected to the moment of calculation.*

The value of the normal resultant of contact coming from ASTER is given on a district of 1 radian of axisymmetric opening in 2D and on a district of $\pi/2$ for the model 3D (by symmetry, it is enough to model the quarter of the problem).

Thus, the values of reference are:

in 2D axisymmetric : $Rref = Rjohnson / 2\pi = 101902{,}1\,\delta$
in 3D : $Rref = Rjohnson / 4 = 160067{,}5\,\delta$

2.3 Uncertainties on the solution

Analytical solution.

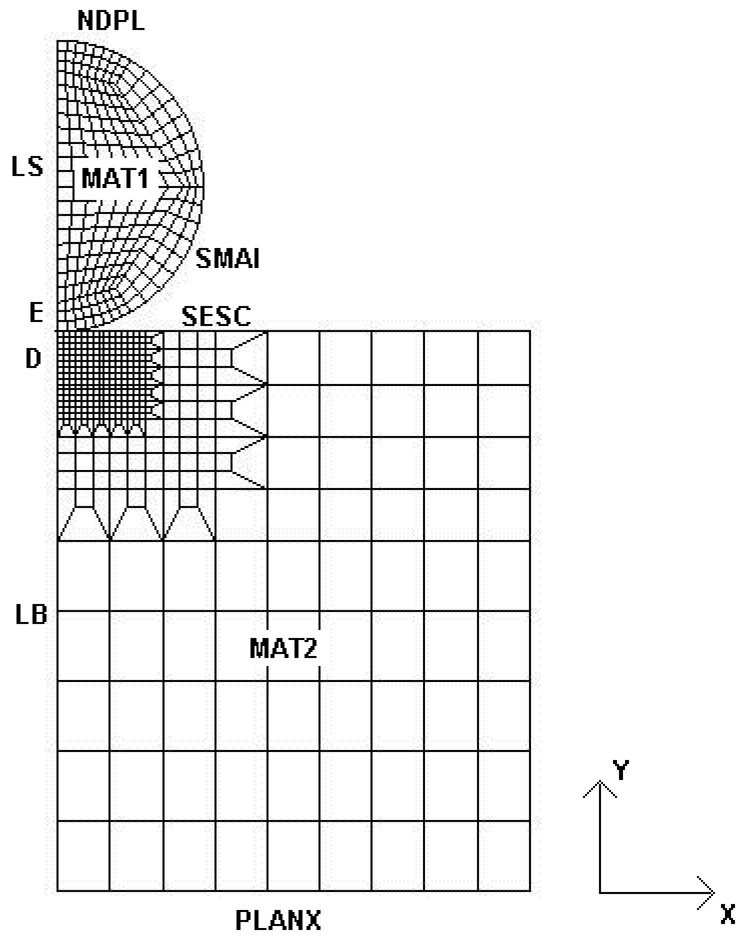
2.4 Bibliographical reference

1 “Contact Mechanics” - K.L. JOHNSON - Cambridge University Close - chapter 6 p. 153 - 201

3 Modeling A

3.1 Characteristics of modeling

The symmetry of revolution of the problem allows an axisymmetric modeling: The sphere and the block are represented respectively by a half disc and the cut of half of the block, with a grid with axisymmetric elements 2D.
A contact of the node-mesh type is defined between the two structures.
A loading in imposed displacement is applied to the upper part of the sphere rigidified by a high Young modulus.



Boundary condition:

- symmetry of revolution: nodes located on the axis Y (group of nodes *« LB »* and *« LS »*) are blocked according to the direction X (DX = 0),
- embedding of the base: nodes of the group *« PLANX »* are blocked according to the directions X and Y (DX = DY = 0),
- the rigid movements of body are removed by imposing a connection according to y between the node E belonging to the sphere and the node D belonging to the solid mass.

Loadings:

An imposed displacement is applied to the upper part of the sphere (group of nodes *« NDPL »*) according to the direction Y : Loading of 0 with $-100.mm$

3.2 Characteristics of the grid

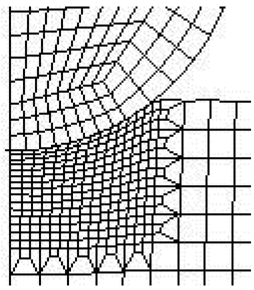
Many nodes: 916
Number of meshs and type: 625 QUAD4 and 289 SEG2

3.3 Values tested

Identification	Displacement (mm)	Reference	*Aster*	% tolerance
Reaction (N)	20	– 2.03804E+06	– 2.06806E+06	5
Reaction (N)	40	– 4.07608E+06	–4.04698E+06	5
Reaction (N)	60	– 6.11412E+06	– 5.82730E+06	5
Reaction (N)	80	– 8.15217E+06	–7.66632E+06	10
Reaction (N)	100	–1.01902E+07	–9.11899E+06	15

3.4 Remarks

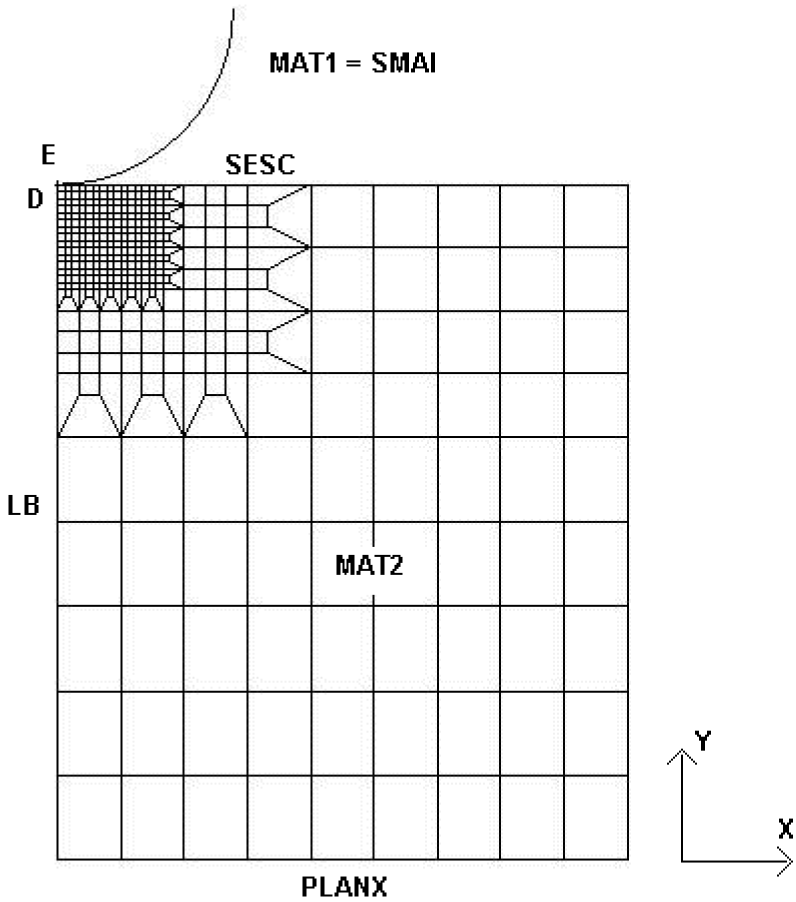
The most important error is for the last result. It remains acceptable nevertheless.
We illustrated the deformation of the solid mass to the step of final time:



4 Modeling B

4.1 Characteristics of modeling

The symmetry of revolution of the problem allows an axisymmetric modeling: The block is represented by the cut of its half and the sphere is represented by its surface potentially in contact, they are with a grid with axisymmetric elements 2D.
A contact of the node-mesh type is defined between the two structures.
A loading in imposed displacement is applied to all the meshs representing the sphere, rigidified by conditions kinematics.



Boundary condition:

- Conditions of symmetry: nodes of the frame located on the axis Y (group of nodes « LB ») are blocked according to the direction X (DX = 0). All nodes belonging to the sphere (group of nodes « $MAT1$ ») are blocked according to direction X (DX = 0).
- Embedding of the base: nodes of « $PLANX$ » are blocked according to the directions X and Y (DX = DY = 0).
- The rigid movements of body are removed by imposing a rigid, following connection y, between the node E belonging to the sphere and the node D belonging to the solid mass.

Loadings:

An imposed displacement is applied to the part representing the sphere (group of node « $MAT1$ ») according to the direction Y: Loading of 0 with $-100.mm$

4.2 Characteristics of the grid

Many nodes: 458
Number of meshs and type: 419 QUAD4 and 171 SEG2.

4.3 Values tested

Identification	Displacement (mm)	Reference	*Aster*	% difference
Reaction (N)	$d=-20\,mm$	– 2.06771E+06	-2.0677082E+06	10
Reaction (N)	$d=-40\,mm$	– 4.04742E+06	-4.0474212E+06	10
Reaction (N)	$d=-60\,mm$	– 5.82779E+06	-5.8277879E+06	10
Reaction (N)	$d=-80\,mm$	– 7.66673E+06	-7.6667317E+06	10
Reaction (N)	$d=-100\,mm$	– 9.11942E+06	-9.1194226E+06	15

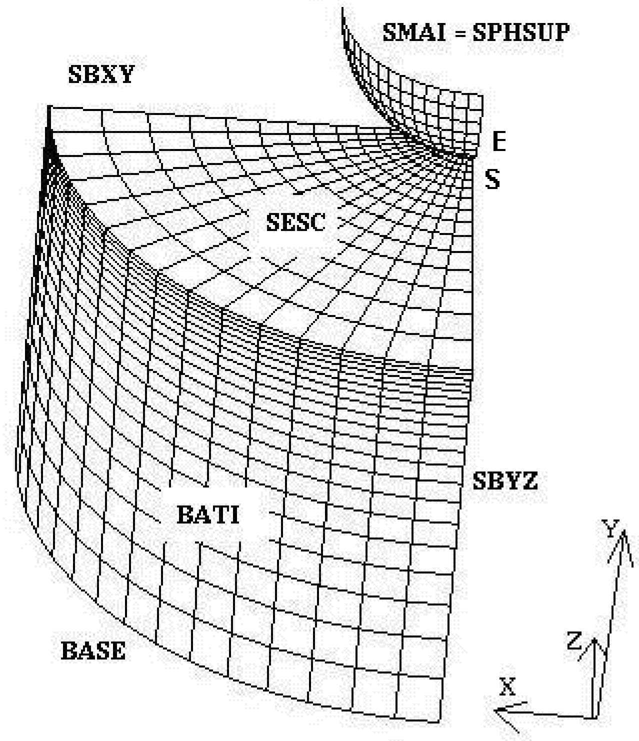
4.4 Remarks

The results are almost identical to those of modeling A.
One notices a computing time reduced by modelling only the surface of contact of the sphere rigidified by conditions kinematics.

5 Modeling C

5.1 Characteristics of modeling

The symmetry of the problem makes it possible to represent in 3D only one quarter of the model: the sphere and the block are represented respectively by the surface of contact of the sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.
A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to all the surface of the sphere rigidified by conditions kinematics.



Boundary condition:

- Conditions of symmetry: nodes located in the plan (O, y, z) (group of nodes *« SBYZ »*) are blocked according to the direction X (DX = 0),
nodes located in the plan (O, x, y) (group of nodes *« SBXY »*) are blocked according to direction Z (DZ = 0),
the nodes of the sphere (group of nodes “SPHSUP”) are blocked according to the directions X and Z (DX = DZ = 0)
- Embedding of the base: nodes of the group *« BASE »* (plan $Y=0.$) are blocked according to the directions X, Y, and Z (DX = DY = DZ = 0).
- The rigid movements of body are removed by imposing a connection following there enters the node E belonging to the sphere and the node S belonging to the solid mass.

Loadings:

An imposed displacement is applied to all surface representing the sphere (group of nodes *« SPHSUP »*) according to the direction Y : Loading of 0 with $-100.mm$

5.2 Characteristics of the grid

Many nodes: 6852
Number of meshs and type: 5326 HEXA8, 387 PENTA6 and 183 QUAD4.

5.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20\,mm$	– 3.201351E+06	-3.2100211E+06	1
Reaction (N)	$d=-40\,mm$	– 6.402702E+06	-6.1671049E+06	5
Reaction (N)	$d=-60\,mm$	– 9.604053E+06	-9.1689400E+06	5
Reaction (N)	$d=-80\,mm$	– 1.280540E+07	-1.1738899E+07	10
Reaction (N)	$d=-100\,mm$	– 1.600675E+07	-1.4244367E+07	12

5.4 Remarks

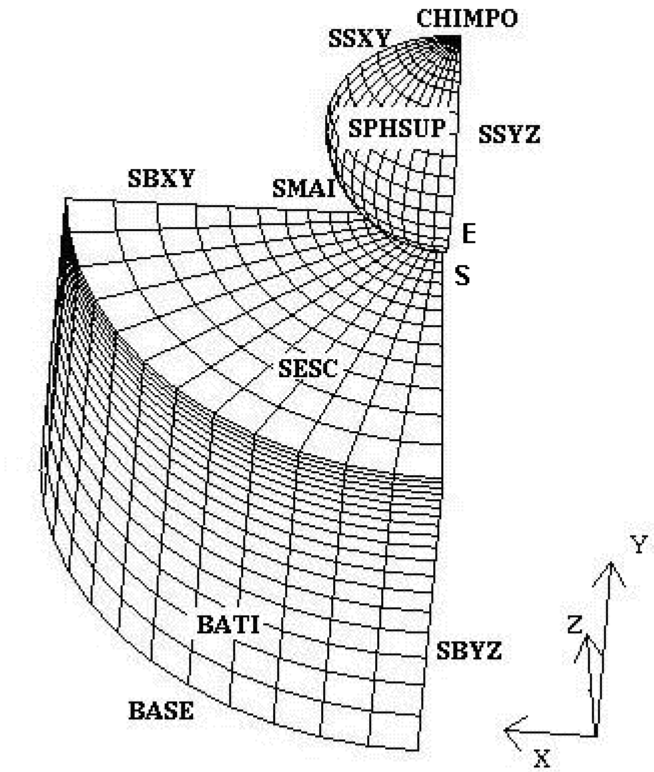
The results are less precise than those resulting from modelings 2D. The grid in 3D makes lose the exact character of the axisymmetric case. Moreover, for time-savers of calculation and memory capacity, the grid 3D is refined less than that in 2D.

6 Modeling D

6.1 Characteristics of modeling

The symmetry of the problem makes it possible to represent in 3D only one quarter of the model: The sphere and the block are represented respectively by a quarter of sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.

A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to the upper part of the sphere rigidified by a high Young modulus.



Boundary condition:

- Conditions of symmetry: nodes located in the plan (O, y, z) (groups of nodes *« SBYZ »* and *« SSYZ »*) are blocked according to the direction X (`DX = 0`),
nodes located in the plan (O, x, y) (groups of nodes *« SBXY »* and *« SSXY »*) are blocked according to direction Z (`DZ = 0`).
- Embedding of the base: nodes of *« BASE »* (plan $Y=0.$) are blocked according to the directions X, Y, and Z (`DX = DY = DZ = 0`).
- The rigid movements of body are removed by imposing a connection following there enters the node E belonging to the sphere and the node S belonging to the solid mass.

Loadings:

An imposed displacement is applied to the upper part of the sphere (group of nodes *« CHIMPO »*) according to the direction Y : Loading of 0 with $-100.mm$

6.2 Characteristics of the grid

Many nodes: 6993
Number of meshs and type: 5544 HEXA8, 407 PENTA6 and 191 QUAD4

6.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20\,mm$	– 3.201351E+06	-3.82828724E+06	25
Reaction (N)	$d=-40\,mm$	– 6.402702E+06	-7.38942843E+06	20
Reaction (N)	$d=-60\,mm$	– 9.604053E+06	-1.06420713E+07	15
Reaction (N)	$d=-80\,mm$	– 1.280540E+07	-1.28912992E+07	10
Reaction (N)	$d=-100\,mm$	– 1.600675E+07	-1.56376456E+07	5

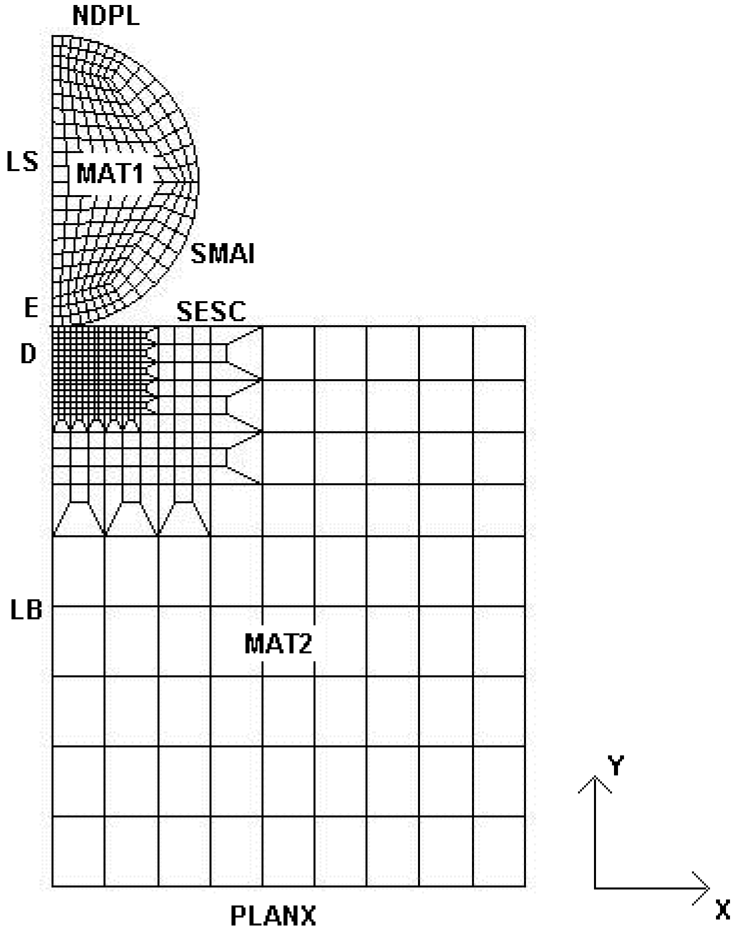
6.4 Remarks

The results are almost identical to those of modeling C. But calculation is even more tiresome because a quarter of the sphere is with a grid.

7 Modeling E

7.1 Characteristics of modeling

The symmetry of revolution of the problem allows an axisymmetric modeling: The block is represented by the cut of its half and the sphere is represented by its surface potentially in contact, they are with a grid with axisymmetric elements 2D.
A contact of the node-mesh type is defined between the two structures.
A loading in imposed displacement is applied to the upper part of the sphere rigidified by a high Young modulus.



Boundary condition:

- symmetry of revolution: nodes located on the axis Y (group of nodes $« LB »$ and $« LS »$) are blocked according to the direction X (DX = 0),
- embedding of the base: nodes of the group $« PLANX »$ are blocked according to the directions X and Y (DX = DY = 0),
- the rigid movements of body are removed by imposing a connection according to y between the node E belonging to the sphere and the node D belonging to the solid mass.

Loadings:

An imposed displacement is applied to the upper part of the sphere (group of nodes $« NDPL »$) according to the direction Y : Loading of 0 with $-100. mm$

7.2 Characteristics of the grid

Many nodes: 688
Number of meshs and type: 625 QUAD4 and 241 SEG2.

7.3 Values tested

Identification	Displacement (mm)	Reference	*Aster*	% tolerance
Reaction (N)	20	– 2.03804E+06	-2.0892265E+06	5
Reaction (N)	40	– 4.07608E+06	-4.0928499E+06	5
Reaction (N)	60	– 6.11412E+06	-5.8467590E+06	5
Reaction (N)	80	– 8.15217E+06	-7.6820567E+06	10
Reaction (N)	100	– 1.01902E+07	-9.1299258E+06	15

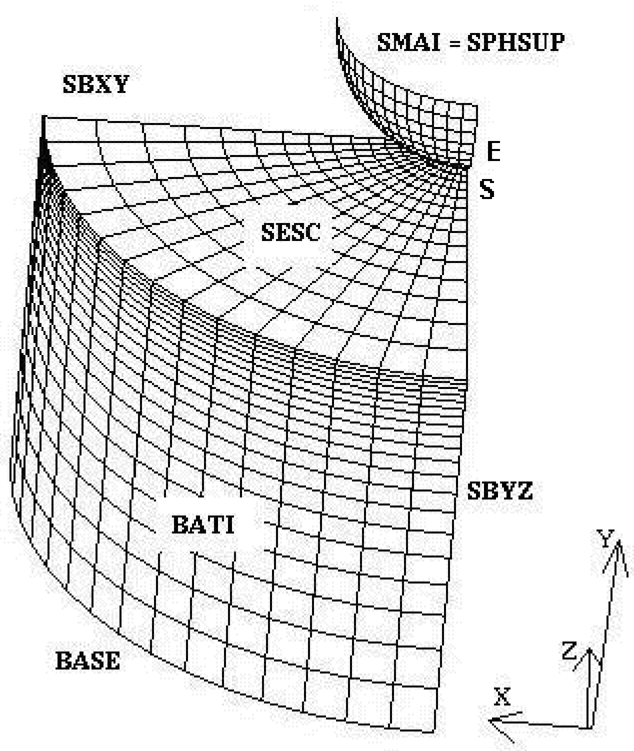
7.4 Remarks

The results are slightly better than those of modeling A.
One notices a computing time 5 times higher than the latter, using the method `CONSTRAINT`.

8 Modeling F

8.1 Characteristics of modeling

The symmetry of the problem makes it possible to represent in 3D only one quarter of the model: the sphere and the block are represented respectively by the surface of contact of the sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.
A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to all the surface of the sphere rigidified by conditions kinematics.



Boundary condition:

- Conditions of symmetry: nodes located in the plan (O, y, z) (group of nodes *« SBYZ »*) are blocked according to the direction X (`DX = 0`),
  nodes located in the plan (O, x, y) (group of nodes *« SBXY »*) are blocked according to direction Z (`DZ = 0`),
  nodes of the sphere (group of nodes *« SPHSUP »*) are blocked according to the directions X and Z (`DX = DZ = 0`)
- Embedding of the base: the nodes of the group “BASES” (plan $Y=0.$) are blocked according to the directions X, Y, and Z (`DX = DY = DZ = 0`).
- The rigid movements of body are removed by imposing a connection following there enters the node E belonging to the sphere and the node S belonging to the solid mass.

Loadings:

An imposed displacement is applied to all surface representing the sphere (group of nodes *« SPHSUP »*) according to the direction Y: Loading of 0 with $-100.mm$

8.2 Characteristics of the grid

Many nodes: 2236
Number of meshs and type: 1638 HEXA8, 126 PENTA6, 725 QUAD4, 27 TRIA3 and 26 SEG2.

8.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20\,mm$	– 3.201351E+06	-3.6477118E+05	20

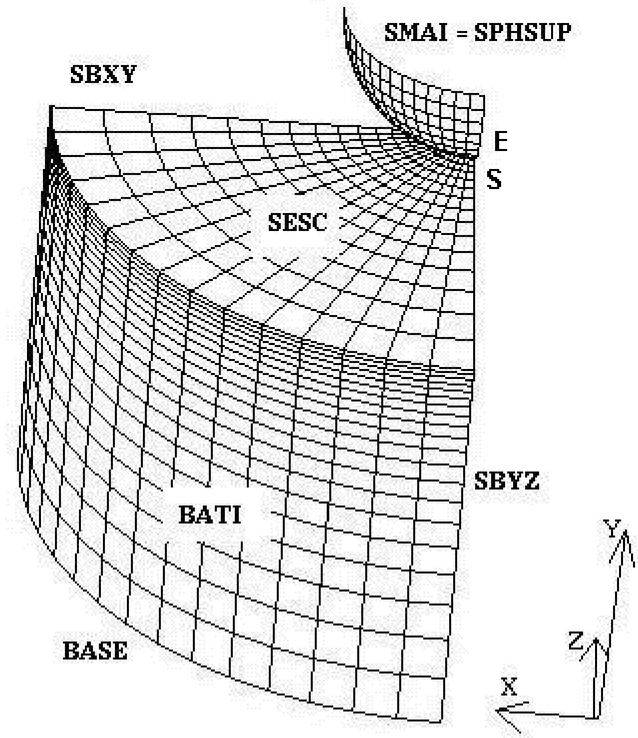
8.4 Remarks

The results are less precise than those resulting from modelings 2D. The grid in 3D makes lose the exact character of the axisymmetric case. Moreover, for time-savers of calculation and memory capacity, the grid 3D is refined less than that in 2D.

9 Modeling G

9.1 Characteristics of modeling

The symmetry of the problem makes it possible to represent in 3D only one quarter of the model: the sphere and the block are represented respectively by the surface of contact of the sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.
A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to all the surface of the sphere rigidified by conditions kinematics.



Boundary condition:

- Conditions of symmetry: nodes located in the plan (O, y, z) (group of nodes *« SBYZ »*) are blocked according to the direction X (`DX = 0`),
  nodes located in the plan (O, x, y) (group of nodes *« SBXY »*) are blocked according to direction Z (`DZ = 0`),
  nodes of the sphere (group of nodes *« SMAI »*) are blocked according to the directions X and Z (`DX = DZ = 0`)
- Embedding of the base: nodes of the group *« BASE »* (plan $Y=0.$) are blocked according to the directions X, Y, and Z (`DX = DY = DZ = 0`).
- The rigid movements of body are removed by imposing a connection following there enters the node E belonging to the sphere and the node S belonging to the solid mass.

Loadings:

An imposed displacement is applied to all surface representing the sphere (group of nodes *« SMAI »*) according to the direction Y : Loading of 0 with $-100.\,mm$

9.2 Characteristics of the grid

Many nodes: 2157
Number of meshs and type: 1496 HEXA8, 108 PENTA6, 988 QUAD4, 12 TRIA3 and 8 PYRAM5.

9.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20mm$	– 3.201351E+05	-3.6492888E+05	20

10 Summary of the results

The got results are good. However, a more important difference between the reference and the results 3D exist. It is possible to fill it by refining even more the grid but it should be paid in place memory and computing times.

The size of the elements is very important. If they are too large, one can see appearing on the curve of the reaction according to the displacement of the "waves" (loss of linearity of this curve). Each "vagueness" corresponds to the setting in contact of an element. Moreover, if the grid is not sufficiently refined, reaction given by *Aster* move away appreciably from that of reference.

Not to model that the sphere by its surface of contact rigidified by conditions kinematics allows a time-saver.