SSNV509 – Chain in contact rubbing with quadratic X FEM

Summary:

The purpose of this test is to validate the taking into account of the contact on the lips of a curved crack, while being limited if the crack crosses completely the structure (interface). One places oneself within the framework of the continuous method adapted to the method X-FEM with resolution by Lagrangian increased. This test brings into play a parallelepipedic block in compression. The interface is represented by one *level set* circular crossing of the elements with variable angles. It utilizes the elements X-FEM P2 in displacement and P1 in pressure which has degrees of freedom of displacement in each node and degrees of freedom of contact-friction on the nodes tops.

1 Problem of reference in 2D

1.1 Geometry

The structure is a healthy square plate on side $L=10\text{m}$.

The interface is represented by a circular level set introduced into the command file using the operator DEFI_FISS_XFEM [U4.82.08] data in its analytical form: $(x-5)^2+(y-11,77)^2=64,27$. It was selected in manner has to cross the structure completely but is not identified with an inclusion [V6.04.507].

The position of the points of reference east:

	x	y
A	0	5.5
B	5	3.75
C	10	5.5

1.2 Material properties

Poisson's ratio: 0
Young modulus: $1.10^6 N/m^2$
Coefficient of Coulomb: 2.0

1.3 Boundary conditions and loadings

The lower face is blocked by an embedding and a displacement $u_y=-1.10^{-6}m$ is imposed on the higher face.

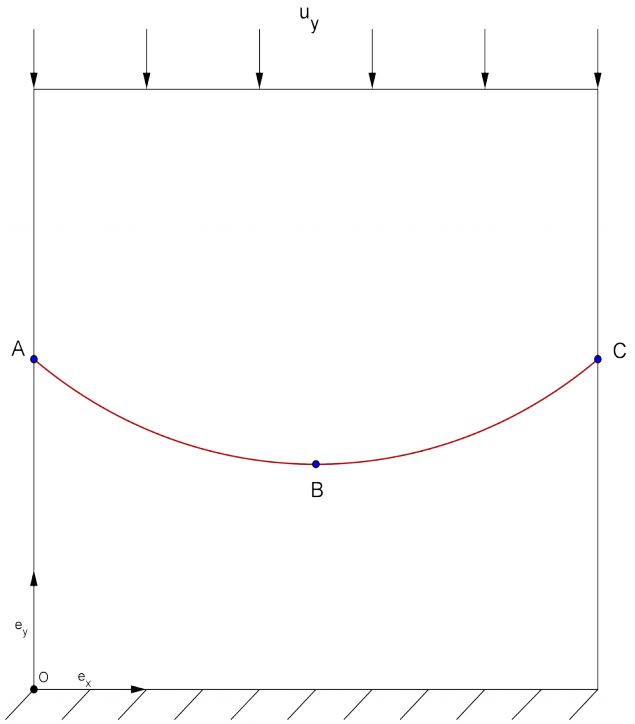

Figure 1.3 -a: Geometry of the structure and positioning of the interface and loadings

2 Reference solution

2.1 Method of calculating

The normal with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix},\qquad \tau=\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix} \qquad \text{éq 2.1-1}$$

$$\text{with } \theta=\arctan\frac{y-11,77}{x-5} \qquad \text{éq 2.1-2}$$

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there differences between plane constraints and deformations).

The method of Lagrangian increased is used for the treatment of the contact/friction.

The interface presents a slope which varies from one end to another of the plate. However at the places where the slope is strongest, close to the side edges, there is likely to be slip. To avoid that, one increases adherence via the coefficient of friction of Coulomb: one takes $\mu=2$.

The value of the contact pressure on the interface is function of the normal n :

$$\lambda=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}=n_y\sigma_{yy}n_y \qquad \text{éq .2.1-3}$$

- where n_y is the component according to y of $\boldsymbol{n}$
- where σ_{yy} is the constraint according to y in the plan of normal $\boldsymbol{e_y}$ in the structure without interface:

$$\sigma_{yy}=E\frac{u_y}{L_y}$$

The semi-multiplier of friction Λ is defined by:

$$\boldsymbol{r}_\tau=\lambda\mu\Lambda \qquad \text{éq 2.1.1-4}$$

With the density of tangential stress being written as follows:

$$\boldsymbol{r}_\tau=(\boldsymbol{\tau}.\boldsymbol{\sigma}.\boldsymbol{n})\boldsymbol{\tau} \qquad \text{éq 2.1-5}$$

From where:

$$\Lambda=\left(\frac{1}{\mu}\frac{\boldsymbol{\tau}.\boldsymbol{\sigma}.\boldsymbol{n}}{\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}}\right)\boldsymbol{\tau}=\left(\frac{1}{\mu}\frac{\tau_y}{n_y}\right)\boldsymbol{\tau} \qquad \text{éq 2.1-6}$$

With the digital values previously introduced:

$$\lambda(x,y)=\frac{-1}{10}\sin^2(\arctan\frac{11,77-y}{5-x})\,Pa \qquad \text{éq 2.1-7}$$

$$\Lambda(x,y)=\boldsymbol{\Lambda}.\boldsymbol{\tau}=\frac{x-5}{2(y-11,77)} \quad \textbf{éq 2.1-8}$$

2.2 Sizes and results of reference

Minimum of the contact pressure LAGS_C on the interface (at points A and C): -0.1
Maximum of the contact pressure LAGS_C on the interface (at the item B): -0.06110

Minimum of the density of tangential stress LAGS_F1 on the interface (at the item A): 0.39894
Maximum of the density of tangential stress LAGS_F1 on the interface (at the item B): 0

2.3 Uncertainties of the solution

No (analytical solution).

3 Modeling A

3.1 Characteristics of modeling

Modeling: C_PLAN. It is used to validate the keyword 'VERSION3' used for ALGO_LAGR in DEFI_CONTACT. The structure is a healthy square, in which an interface in form ofarc of a circle is introduced.

The contact/friction is treated with elements X-FEM quadratic $P2$ (displacement) and $P1$ (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and Lagrange of contact/friction on the nodes tops.

3.2 Characteristics of the grid

Many nodes: 4961
Many meshs and types: 1600 QUAD8 for the plate and 160 SEG3 for the edges.

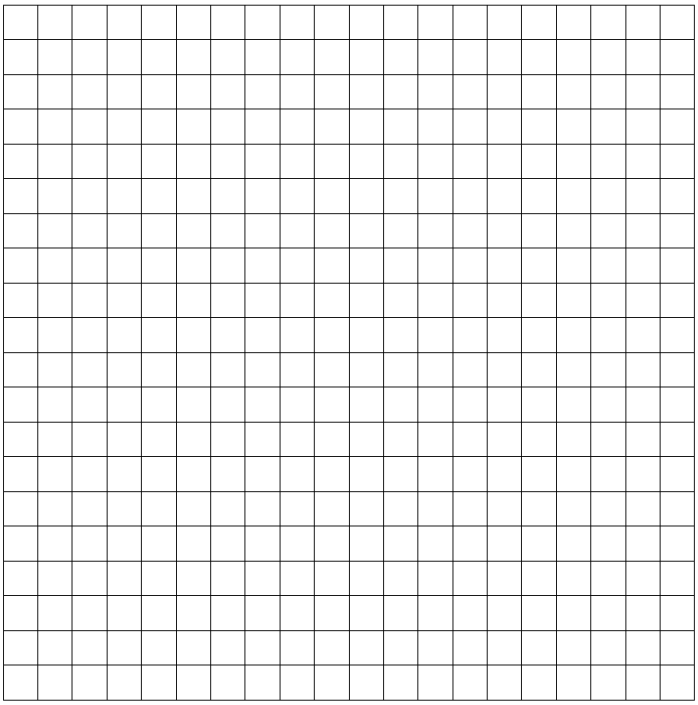

Figure 3.2 -a: Grid 2D quadrangle

3.3 Sizes tested and results

The values appear only with the nodes of the interface resulting from the new grid.

Identification	Type of reference	Value of Référence	Precision
LAGS_C at the points A and C (MIN)	'ANALYTICAL'	-0.1	0.1 %
LAGS_C at the point B (MAX)	'ANALYTICAL'	-0.06110	0.1 %
LAGS_F1 at the point B (MIN)	'ANALYTICAL'	0	0.1 %
LAGS_F1 at the point A (MAX)	'ANALYTICAL'	0.39894	0,5%

3.4 Comments

This valid test:
- the calculation of the matrix of rigidity (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom),
- the calculation of the matrices of contact (integration on one SE3 at the points of Gauss),
- under cutting (configurations in curved interface and elements on right board),
- postprocessing X-FEM elements P2P1 ,
- the algorithm version 3 of the relations of equality on the edges cut for the quadratic elements in small slips, in order to satisfy the LBB.

The solvor MUMPS detect a singularity in the matrix. This problem already arose on many other cases tests X-FEM in contact. The readjustment of the level set normal makes it possible to improve conditioning of the matrix but to the detriment of a too great error of discretization of the level set. One thus keeps the option of desactivation of the detection of singularity of the solvor before suggesting a more satisfactory solution.

4 Modeling B

4.1 Characteristics of modeling

Modeling: C_PLAN. It is used to validate the keyword 'VERSION3' used for ALGO_LAGR in DEFI_CONTACT. The structure is a healthy square, in which an interface in form ofarc of a circle is introduced.

The contact/friction is treated with elements X-FEM quadratic $P2$ (displacement) and $P1$ (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and Lagrange of contact/friction on the nodes tops.

4.2 Characteristics of the grid

Many nodes: 6561
Many meshs and types: 3200 TRIA6 for the plate and 160 SEG3 for the edges.

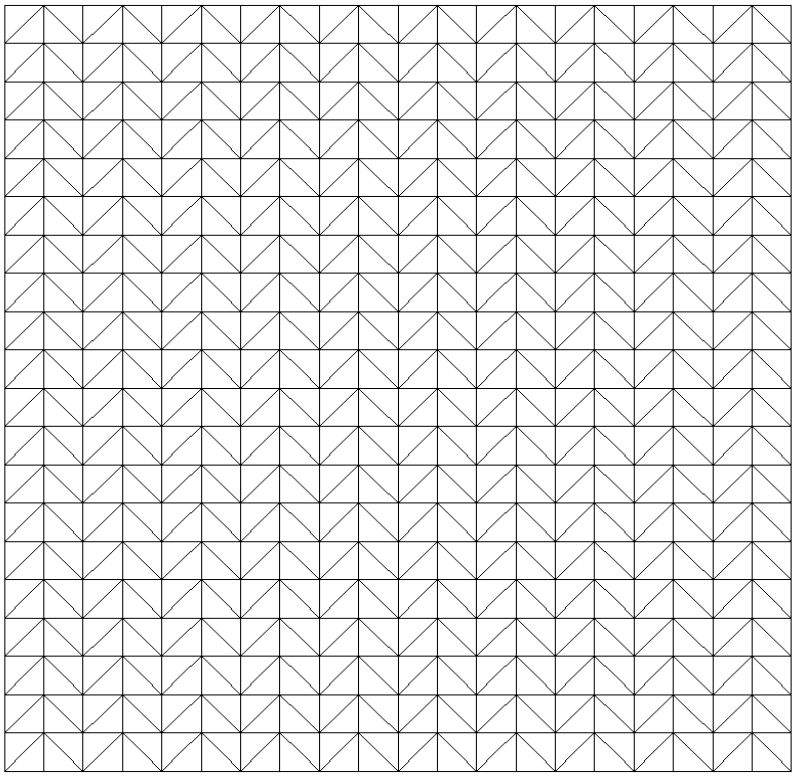

Figure 4.2 -a: Grid 2D triangle

4.3 Sizes tested and results

The values appear only with the nodes of the interface resulting from the new grid.

Identification	Type of reference	Value of Référence	Precision
LAGS_C at the points A and C (MIN)	'ANALYTICAL'	-0.1	0.1 %
LAGS_C at the point B (MAX)	'ANALYTICAL'	-0.06110	0.1 %
LAGS_F1 at the point B (MIN)	'ANALYTICAL'	0	0.1 %
LAGS_F1 at the point A (MAX)	'ANALYTICAL'	0.39894	0,5%

4.4 Comments

This valid test:

- the calculation of the matrix of rigidity (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom),
- the calculation of the matrices of contact (integration on one SE3 at the points of Gauss),
- under cutting (configurations in curved interface and elements on right board),
- postprocessing X-FEM elements P2P1 ,
- the refinement of the grid, makes it possible to get more precise results (compared to modeling A),
- the algorithm version 3 of the relations of equality on the cut edges, for the quadratic elements in small slips, in order to satisfy the LBB.

5 Summaries of the results

The goals of this test are achieved.

- It was a question of showing the feasibility of the taking into account of the contact rubbing on the lips of the chain with the method continues adapted to the framework X-FEM. Only the case of an interface crossing the structure completely was considered (interface).
- The method is validated in 2D P2 (displacement) P1 (pressure) on a grid quadrangle and triangle.
- The method is validated with the method of Lagrangian increased for the treatment of the contact/friction.