SSNV510 - Uniaxial pressing of a multi-fissured block

Summary:

This test makes it possible to validate the multi-Heaviside approach for the elementS X-FEM. It is about a case test where one introduces several interfaces compressed laterally. The grid is rather coarse so that the meshs see several cracks. Certain nodes see also several cracks. These nodes contain several Heaviside enrichments then. It is checked that these various enrichments are well taken into account at the kinematic level and that the matrices of rigidity associated with each zone between two interfaces make it possible to obtain the good deformations. One test the approach with and without contact.

1 Problem of reference

1.1 Geometry

The structure is a healthy rectangle into which four horizontal cracks are introduced, in red on the figure 1.1.a. dimensions of the structure as well as the position of the cracks are given on this figure in meters.

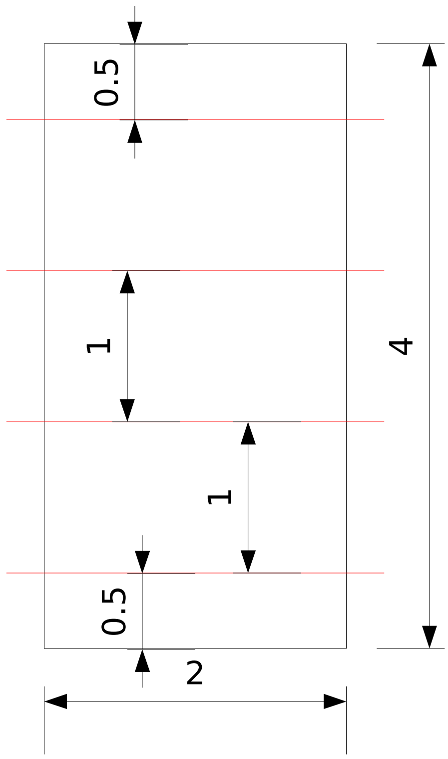

Figure 1.1-a: Geometry of the structure and positioning of the cracks.

1.2 Properties of material

Young modulus: $100\,\mathrm{MPa}$

Poisson's ratio: 0.0

1.3 Boundary conditions and loadings

In the case with contact (modelings A with F), ON blocks the component X displacement on the right part of the structure. The component is blocked Y displacement on its lower part. One applies a loading in pressure according to X constant by piece on the left part, so as to obtain a staircase. This loading is represented figure 1.3-b. The contact is active on the cracks, one applies a constant pressure to the upper part, so as to activate effort of contact on the level as of cracks.
In the case without contact (modelings G with I), the components are blocked X and Y displacement on the right part of the structure and one applies a loading in pressure according to X constant by piece on the left part, so as to obtain a staircase. This loading is represented figure 1.3-b. The contact is not active on the cracks, it does not have thus pressure there to apply according to Y.

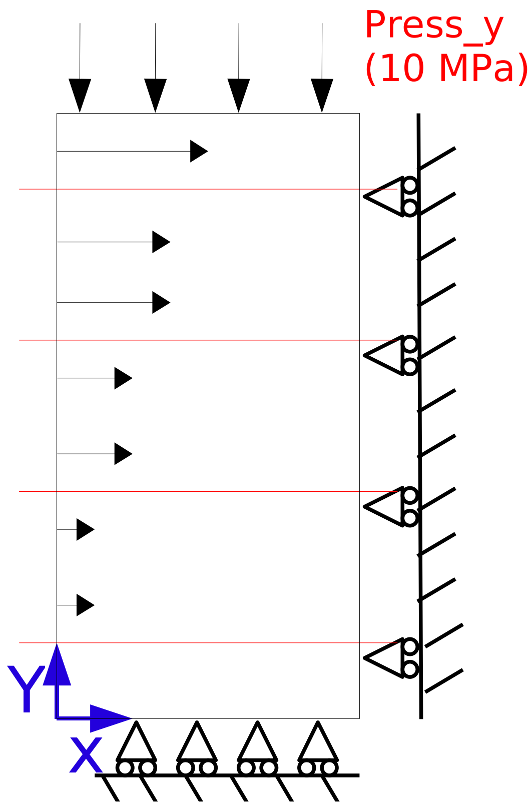

Figure 1.3-a: Illustration of the boundary conditions and the loadings, case with contact.

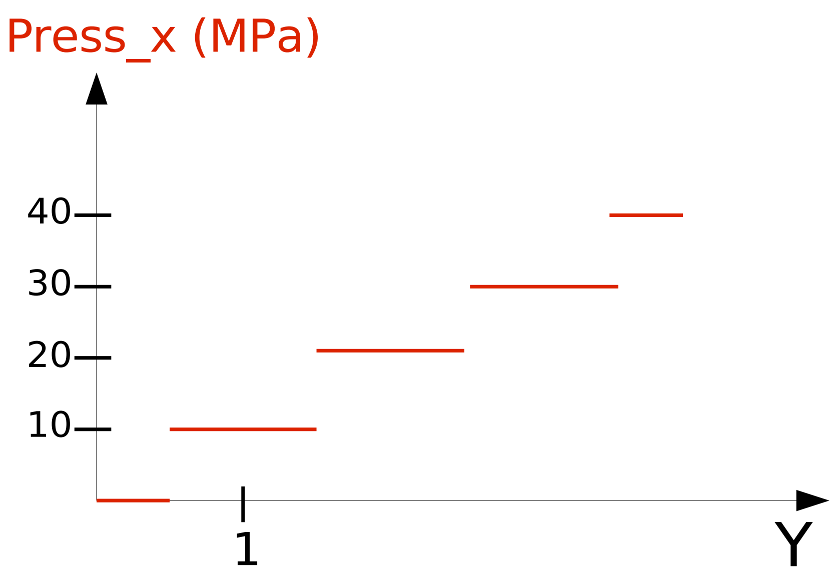

Figure 1.3-b: Pressure imposed according to Y on the left edge (in MPa).

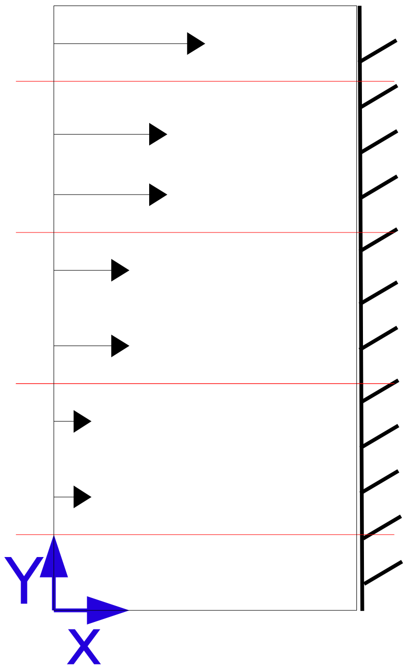

Figure 1.3-C : Illustration of the boundary conditions and the loadings, case without contact.

2 Reference solution

The Poisson's ratio being null, one must find a uniform compression with each of the 4 stages of the structure. In other words for a given stage, the displacement of the structure is proportional to the position according to X and it depends on the Pressure which varies according to Y. Displacement according to Y depends only linearly on Y because imposed pressure $Press_y$ is constant.
There is thus for all the points of the structure the displacement which is worth :

$$Depl_X(X,Y)=(2-X)\frac{Press_x(Y)}{E} \quad \text{éq 2.1-1}$$

$$Depl_Y(Y)=-Y\frac{Press_y}{E} \quad \text{éq 2.1-2}$$

2.1.1 Calculation of the energy of the structure

That is to say $p_x=10\,\mathrm{MPa}$ and $p_y=10\,\mathrm{MPa}$ in the cases with contact, $p_y=0\,\mathrm{MPa}$ in the cases without contact. The tensor of the constraints analytical solution is:

$$\boldsymbol{\sigma}=\sigma_{xx}(y)\boldsymbol{e_x}\otimes\boldsymbol{e_x}-p_y\boldsymbol{e_y}\otimes\boldsymbol{e_y}$$

where one posed:

$$\sigma_{xx}(y)=\begin{cases} 0 \text{ pour } y<\frac{1}{2}, \\ -p_x \text{ pour } y\in\left[\frac{1}{2},\frac{3}{2}\right[, \\ -2p_x \text{ pour } y\in\left[\frac{3}{2},\frac{5}{2}\right[, \\ -3p_x \text{ pour } y\in\left[\frac{5}{2},\frac{7}{2}\right[, \\ -4p_x \text{ pour } y\geq\frac{7}{2}, \end{cases}$$

S hears $\Omega=[0,2]\times[0,4]$ the field occupied by the solid. The energy of the structure is defined by:

$$E^e=\frac{1}{2}\int_\Omega \boldsymbol{\sigma}:\boldsymbol{\varepsilon}\,dS$$

Since $\nu=0$, one has $\boldsymbol{\varepsilon}=\frac{1}{E}\boldsymbol{\sigma}$. From where:

$$\boldsymbol{\sigma}:\boldsymbol{\varepsilon}=\frac{1}{E}\left(\sigma_{xx}(y)^2+p_y^2\right)$$

One thus has:

$$E^e=\frac{1}{2}\frac{1}{E}\int_0^2\left(\int_0^4\left(\sigma_{xx}(y)^2+p_y^2\right)dy\right)dx$$

That is to say:

$$E^e=\frac{1}{E}\left(\int_{\frac{1}{2}}^{\frac{3}{2}} p_x^2\,dy+\int_{\frac{3}{2}}^{\frac{5}{2}} 4\,p_x^2\,dy+\int_{\frac{5}{2}}^{\frac{7}{2}} 9\,p_x^2\,dy+\int_{\frac{7}{2}}^{4} 16\,p_x^2\,dy+\int_0^4 p_y^2\,dy\right)$$

One thus has , for p_x and p_x expressed in MPa :

$$E^e=\frac{1}{E}\left(22\,p_x^2+4\,p_y^2\right)$$

This result is valid in the case of the plane constraints and of the plane deformations. In the case 3D, the selected thickness is 1 Mr. the expression of energy is identical, but the units are modified. One has then, in plane constraints and plane deformations, with contact :

$$E^e=2{,}6\times10^7\ \mathrm{MJ}\times\mathrm{m}^{-1},$$

in 3D with contact:

$$E^e=2{,}6\times10^7\ \mathrm{MJ},$$

in plane constraints and plane deformations, without contact:

$$E^e=2{,}2\times10^7\ \mathrm{MJ}\times\mathrm{m}^{-1},$$

in 3D, without contact:

$$E^e=2{,}2\times10^7\ \mathrm{MJ}\ .$$

2.1.2 Calculation of the standard L ² displacement

The field of analytical solution displacement is

$$\boldsymbol{u}=\frac{\sigma_{xx}(y)}{E}(2-x)\,\boldsymbol{e}_x-\frac{p_y}{E}\,y\,\boldsymbol{e}_y,$$

The standard L^2 displacement is defined by:

$$\|\boldsymbol{u}\|_{L^2}^2=\int_\Omega\|\boldsymbol{u}\|^2\,dS$$

One a:

$$\|\boldsymbol{u}\|^2=\frac{\sigma_{xx}(y)^2}{E^2}(2-x)^2-\frac{p_y^2}{E^2}y^2$$

One thus has:

$$\|\boldsymbol{u}\|_{L^2}^2=\frac{1}{E^2}\int_0^4\left(\int_0^2\left[\sigma_{xx}(y)^2(2-x)^2-p_y^2y^2\right]dx\right)dy$$

That is to say:

$$\|\boldsymbol{u}\|_{L^2}^2=\frac{1}{E^2}\int_0^4\left(\frac{8}{3}\sigma_{xx}(y)^2-2p_y^2y^2\right)dy.$$

From where:

$$\|\boldsymbol{u}\|_{L^2}^2=\frac{1}{E^2}\left[\frac{8}{3}\left(\int_{\frac{1}{2}}^{\frac{3}{2}}p_x^2dy+\int_{\frac{3}{2}}^{\frac{5}{2}}4p_x^2dy+\int_{\frac{5}{2}}^{\frac{7}{2}}9p_x^2dy+\int_{\frac{7}{2}}^{4}16p_x^2dy\right)+2\int_0^4p_y^2y^2dy\right].$$

And finally:

$$\|\boldsymbol{u}\|_{L^2}^2=\frac{1}{E^2}\left(\frac{176}{3}p_x^2+\frac{128}{3}p_y^2\right).$$

That is to say:

$$\|\boldsymbol{u}\|_{L^2}=\frac{1}{E}\sqrt{\frac{176}{3}p_x^2+\frac{128}{3}p_y^2}.$$

This result is valid in the case of the plane constraints and of the plane deformations. In the case 3D, the selected thickness is 1 Mr. the expression of the standard L^2 displacement is identical, but the units are modified.
One has then , in plane constraints and plane deformations, with contacT :

$$\|\boldsymbol{u}\|_{L^2}\approx 1{,}00664459137\ \text{m}^2$$

in 3D with contact:

$$\|\boldsymbol{u}\|_{L^2}\approx 1{,}00664459137\ \text{m}^{\frac{5}{2}}$$

in plane constraints and plane deformations, without contact:

$$\|\boldsymbol{u}\|_{L^2}\approx 0{,}765941686205\ \text{m}^2$$

in 3D, without contact:

$$\|\boldsymbol{u}\|_{L^2}\approx 0{,}765941686205\ \text{m}^{\frac{5}{2}}$$

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling X-FEM, in plane deformations, where the interfaces are defined by functions of level (level sets noted normals LN).

The equations of the functions of levels for the three horizontal cracks are the following ones:

$$LN\,1=Y-0.5 \quad \text{éq 3.1-1}$$
$$LN\,1=Y-1.5 \quad \text{éq 3.1-2}$$
$$LN2=Y-2.5 \quad \text{éq 3.1-3}$$
$$LN\,3=Y-3.5 \quad \text{éq 3.1-4}$$

3.2 Characteristics of the grid

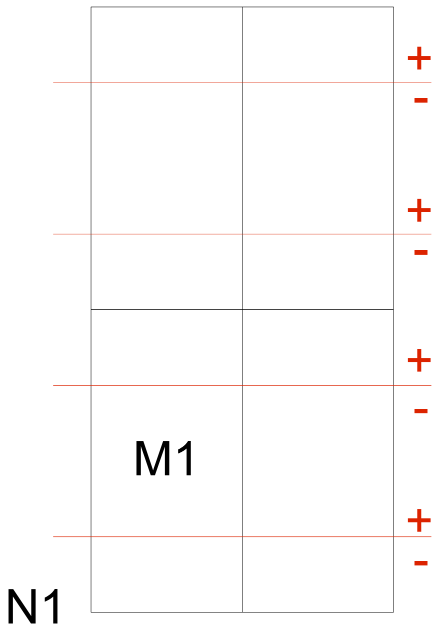

Figure 3.2-a: Grid of modeling A.

The grid comprises 4 meshs of the type QUAD4, represented on the figure 3.2-a.
One notices on this figure for example that the node $N1$ sees 2 cracks. It must thus be enriched 2 times and it has then the degrees of freedom kinematics DX, DY, H1X, H1Y, H2X and H2Y. The contact being active, the node $N1$ have also the degrees of Lagrange LAGS_C and LAG2_C.
In addition it is noticed for example that the mesh $M1$ "sees" 4 cracks. The element associated with this mesh will thus store the fields of the four cracks, independently of the degrees of freedom which are associated with its nodes.

3.3 Sizes tested and results

Displacements are tested on the level of the lips of the crack. Displacement DX must follow the function $Depl\_X$ equation 2.1-1. Displacement DY must follow the function $Depl\_Y$ equation 2.1-2. One obtains the deformation in staircase of the figure 3.4-a.

Identification	*Reference*
SOMM_ABS for DX- $\mathrm{Depl\_X}$ (with dimensions Master)	0
SOMM_ABS for DY- $\mathrm{Depl\_Y}$ (with dimensions Master)	0
SOMM_ABS for DX- $\mathrm{Depl\_X}$ (with dimensions slave)	0
SOMM_ABS for DY- $\mathrm{Depl\_Y}$ (with dimensions slave)	0

Table 3.3-1

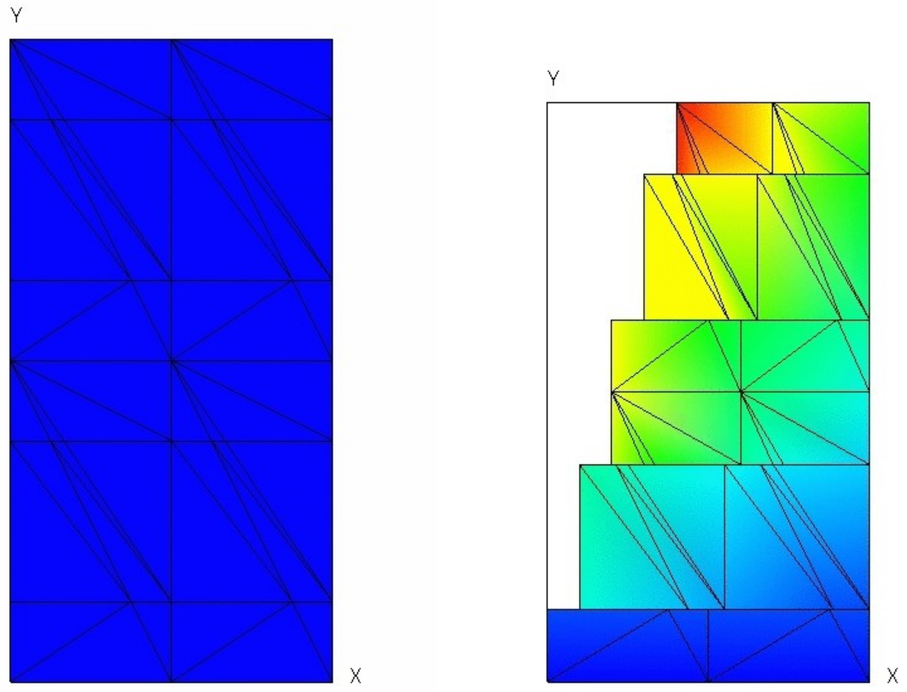

Figure 3.4-a: Deformation of the structure.

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E ^{E}	'ANALYTICAL'	2.6 10^7	0.1%

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	1.00664459137	0.1%

4 Modeling B

4.1 Characteristics of modeling

It is acted of the same modeling as modeling A, but as plane constraints.

4.2 Characteristics of the grid

The grid is the same one as that of modeling A.

4.3 Sizes tested and results

The sizes tested are identical to those described in modeling A. One obtains the deformation in staircase of the figure 3.4-a.

Identification	*Reference*
SOMM_ABS for DX- $\mathrm{Depl\_X}$ (with dimensions Master)	0
SOMM_ABS for DY- $\mathrm{Depl\_Y}$ (with dimensions Master)	0
SOMM_ABS for DX- $\mathrm{Depl\_X}$ (with dimensions slave)	0
SOMM_ABS for DY- $\mathrm{Depl\_Y}$ (with dimensions slave)	0

Table 4.3-1

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E E	'ANALYTICAL'	$2.6\ 10^7$	0.1%

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes 2	'ANALYTICAL'	1.00664459137	0.1%

5 Modeling C

5.1 Characteristics of modeling

It is the same modeling as modeling A, but in 3D.

5.2 Characteristics of the grid

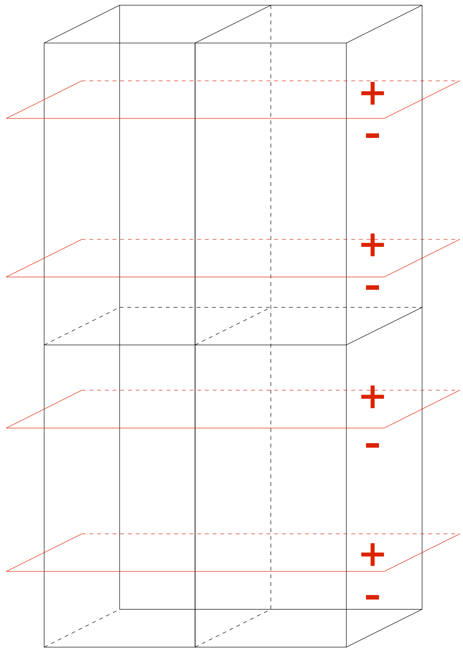

Figure 5.2-a: Grid of modeling C.

The grid comprises 4 meshs of the type HEXA8, represented on the figure 5.2-a.
Even notices that for the grid of modeling A. Certaines meshs "see" 2 cracks, they have the degrees of kinematic freedom DX, DY, DZ, H1X, H1Y, H1Z, H2X and H2Y, H2Z as well as the degrees of freedom of Lagrange LAGS_C and LAG2_C. others "see" 4 cracks, they have also the degrees of freedom kinematics H3X, H3Y, H3Z, H4X and H4Y, H4Z as well as the degrees of freedom of Lagrange LAG3_C and LAG4_C.

5.3 Sizes tested and results

The sizes tested are identical to those described in modeling A. One obtains the deformation in staircase of the figure 5.4-a.

Identification	*Reference*
SOMM_ABS for DX- $Depl\_X$ (with dimensions Master)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions Master)	0
SOMM_ABS for DX- $Depl\_X$ (with dimensions slave)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions slave)	0

Table 5.3-1

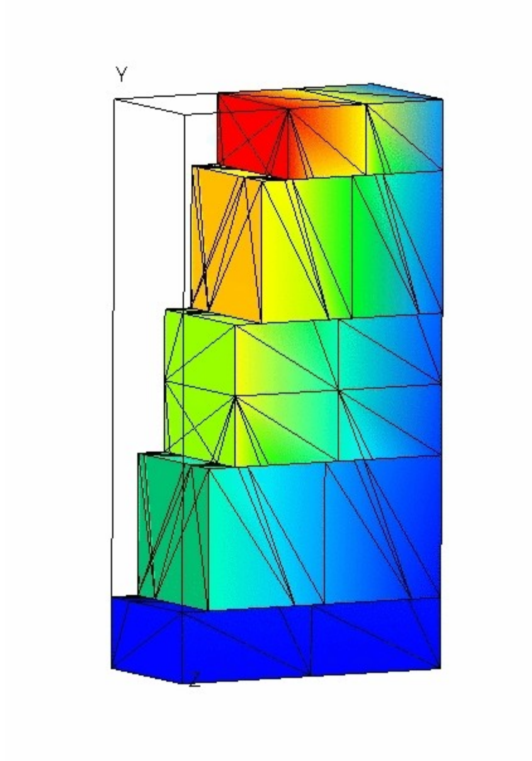

Figure 5.4-a: Deformation of the structure.

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E E	'ANALYTICAL'	2.6 10^7	0.1%

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes 2	'ANALYTICAL'	1.00664459137	0.1%

6 Modeling D

6.1 Characteristics of modeling

It is the same modeling as modeling A, but the contact in great slip is activated.

6.2 Characteristics of the grid

The grid is the same one as that of modeling A.

6.3 Features tested

Idem modeling A. One tests this time the contact for the multi-Heaviside approach in great slip, using the option REAC_GEOM of the operator DEFI_CONTACT. One thus passes by a phase of pairing of elements before calculating the contribution of contact.

6.4 Sizes tested and results

The sizes tested are identical to those described in modeling A. One obtains the deformation in staircase of the figure 3.4-a.

Identification	***Reference***
SOMM_ABS for DX- $Depl\_X$ (with dimensions Master)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions Master)	0
SOMM_ABS for DX- $Depl\_X$ (with dimensions slave)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions slave)	0

Table 6.4-1

7 Modeling E

7.1 Characteristics of modeling

It is acted of the same modeling as modeling D, but as plane constraints.

7.2 Characteristics of the grid

The grid is the same one as that of modeling A.

7.3 Sizes tested and results

The sizes tested are identical to those described in modeling A. One obtains the deformation in staircase of the figure 3.4-a.

Identification	*Reference*
SOMM_ABS for DX- Depl_X (with dimensions Master)	0
SOMM_ABS for DY- Depl_Y (with dimensions Master)	0
SOMM_ABS for DX- Depl_X (with dimensions slave)	0
SOMM_ABS for DY- Depl_Y (with dimensions slave)	0

Table 7.3-1

8 Modeling F

8.1 Characteristics of modeling

It is the same modeling as modeling D, but in 3D.

8.2 Characteristics of the grid

The grid is the same one as that of modeling C.

8.3 Sizes tested and results

The sizes tested are identical to those described in modeling A. One obtains the deformation in staircase of the figure 5.4-a.

Identification	***Reference***
SOMM_ABS for DX- $Depl\_X$ (with dimensions Master)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions Master)	0
SOMM_ABS for DX- $Depl\_X$ (with dimensions slave)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions slave)	0

Table 8.3-1

9 Modeling G

9.1 Characteristics of modeling

It is the same modeling as modeling A, without contact.

9.2 Characteristics of the grid

The grid is the same one as that of modeling A.

9.3 Features tested

Idem modeling A, Without contact, one has $\mathrm{Depl\_Y}=0$.

9.4 Sizes tested and results

The sizes tested are identical to those described in modeling A.

Identification	*Reference*
`SOMM_ABS` for `DX`- $\mathrm{Depl\_X}$ (with dimensions Master)	0
`SOMM_ABS` for `DY`- $\mathrm{Depl\_Y}$ (with dimensions Master)	0
`SOMM_ABS` for `DX`- $\mathrm{Depl\_X}$ (with dimensions slave)	0
`SOMM_ABS` for `DY`- $\mathrm{Depl\_Y}$ (with dimensions slave)	0

Table 9.4-1

One tests the value of E^e product by the operator `POST_ERREUR`.

Identification	Type of reference	Value of reference	Tolerance
`E` ^{E}	`'ANALYTICAL'`	2,2 10^7	0.1%

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator `POST_ERREUR`.

Identification	Type of reference	Value of reference	Tolerance
`L normalizes` ^{2}	`'ANALYTICAL'`	0.765941686205	0.1%

10 Modeling H

10.1 Characteristics of modeling

It is the same modeling as modeling G, but in plane constraints.

10.2 Characteristics of the grid

The grid is the same one as that of modeling A.

10.3 Sizes tested and results

The sizes tested are identical to those described in modeling A.

Identification	*Reference*
SOMM_ABS for DX- $\mathrm{Depl\_X}$ (with dimensions Master)	0
SOMM_ABS for DY- $\mathrm{Depl\_Y}$ (with dimensions Master)	0
SOMM_ABS for DX- $\mathrm{Depl\_X}$ (with dimensions slave)	0
SOMM_ABS for DY- $\mathrm{Depl\_Y}$ (with dimensions slave)	0

Table 10.3-1

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E^E	'ANALYTICAL'	2,2 10^7	0.1%

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	0.765941686205	0.1%

11 Modeling I

11.1 Characteristics of modeling

It is the same modeling as modeling G, but in 3D.

11.2 Characteristics of the grid

The grid is the same one as that of modeling C.

11.3 Sizes tested and results

The sizes tested are identical to those described in modeling A.

Identification	*Reference*
SOMM_ABS for DX- $Depl\_X$ (with dimensions Master)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions Master)	0
SOMM_ABS for DX- $Depl\_X$ (with dimensions slave)	0
SOMM_ABS for DY- $Depl\_Y$ (with dimensions slave)	0

Table 11.3-1

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E^E	'ANALYTICAL'	$2{,}2\ 10^7$	0.1%

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	0.765941686205	0.1%

12 Summary of the results

This test makes it possible to activate multi-Heaviside for Lbe elements X-FEM. One montre that one is able to differentiate the number of cracks "seen" by mesh, of that "seen" by node. In the example of this case test, certain nodes see to the maximum 2 cracks whereas the meshs see 4 of them. One also shows that it is possible to take into account conditions of contact on the interfaces, the nodes then are also enriched by multiple degrees of freedom by Lagrange.

The approach was validated in `D_PLAN`, in `C_PLAN` and in `3D`, for the small ones and the great slips.