SSNV511 – Block cut out by two interfaces intersected with X-FEM

Summary:

This test makes it possible to validate the approach intersection with X-FEM. It is about a case test where three cracks are introduced. The first crack cuts the field completely. The two other cracks are defined by the same level-set normal. They are connected on both sides of the first via the keyword JUNCTION of the operator DEFI_FISS_XFEM . The double junction forms an intersection then. One tests the approach with and without contact.

1 Problem of reference

1.1 Geometry

The structure is a healthy square into which one introduces two interfaces, in red on the figure 1.1-a. The two interfaces cross, dimensions of the structure as well as the position of the interfaces are given on this figure.

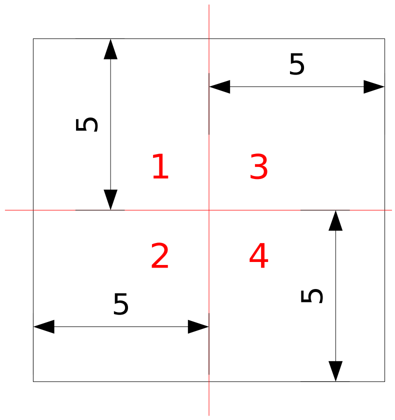

Figure 1.1-a: Geometry of the structure, position of the interfaces and classification of the zones used to test the variation enters calculated displacements and analytical displacements.

1.2 Properties of material

The material has an isotropic elastic behavior whose properties are:
Young modulus: $100\,MPa$
Poisson's ratio: 0.3

1.3 Boundary conditions and loadings

In the case without contact (modelings A with D), one applies conditions in displacement to the edges left and right of the structure, so that each of the 4 zones has a displacement different from the others according to X. This loading is represented figure 1.3-a. One blocks displacements in Y (and in Z for modelings $3D$) on these same edges. One then obtains displacements of rigid modes for the 4 blocks.

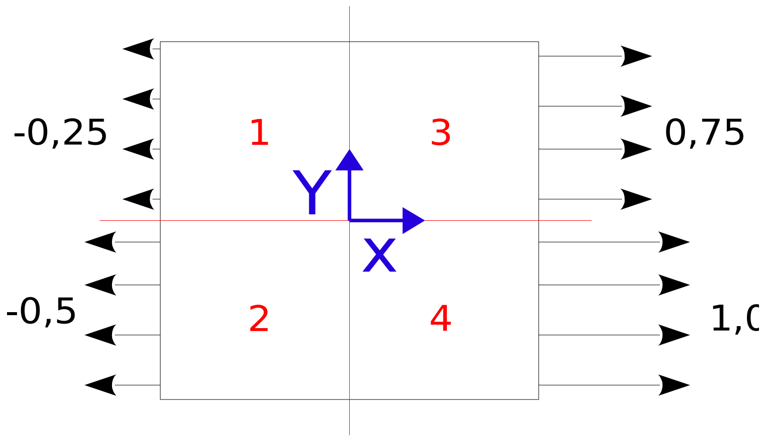

Figure 1.3-a: Illustration of the boundary conditions and the loadings, cases without contact.

In the case of the contact (modelings E with H), one imposes conditions of roller on the edges left and low and one applies the pressure in staircase of the figure 1.3-b to the flat rims and high. This loading is represented figure 1.3-c. Each block is then compressed in a uniform way according to X and Y . For modelings $3D$, a condition of roller is imposed in $Z=0$.

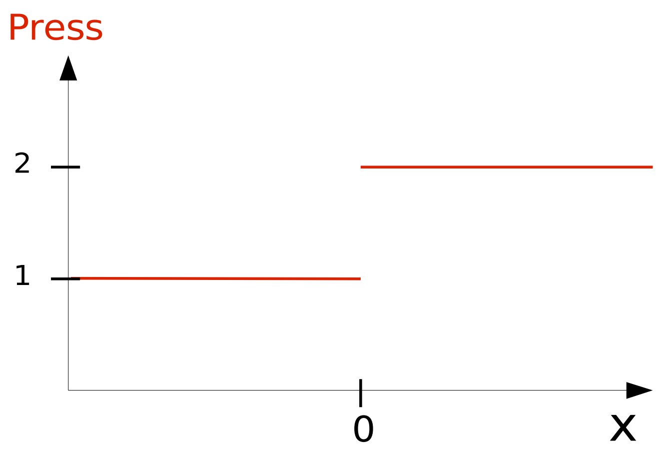

Figure 1.3-b: Pressure imposed according to X on the high edge and according to Y on the flat rim, (in MPa).

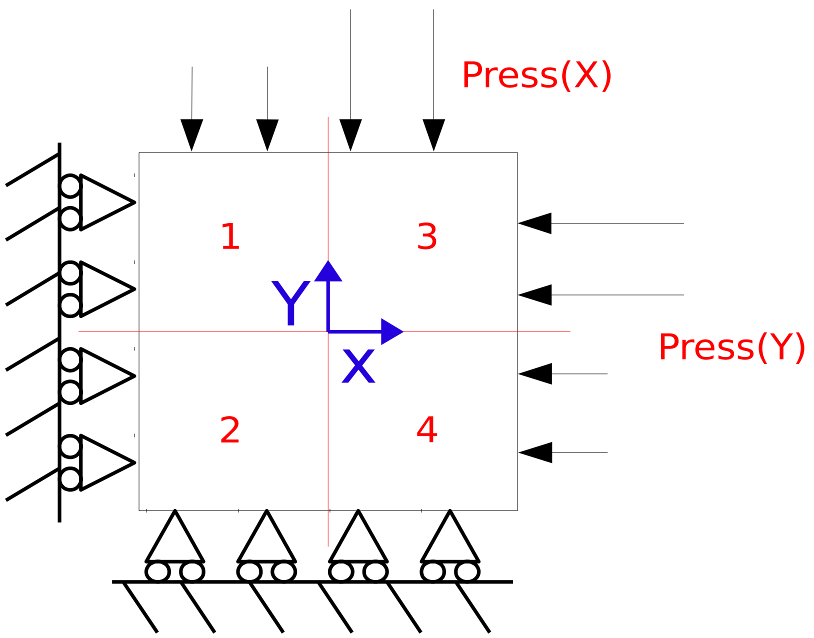

Figure 1.3-c: Illustration of the boundary conditions and the loadings, cases with contact.

2 Reference solution

Shears $\Omega=[-5,+5]\times[-5,+5]$ the field occupied by the solid, in the plan (X,Y) . The field Ω is partitionné in $\Omega=\Omega_1\cup\Omega_2\cup\Omega_3\cup\Omega_{4.}$, where one posed:

$$\Omega_1=[-5,0[\times]0,+5],$$
$$\Omega_2=[-5,0[\times[-5,0[,$$
$$\Omega_3=]0,+5]\times]0,+5],$$
$$\Omega_4=]0,+5]\times[-5,0[.$$

2.1 Case without contact

Without contact, each zone must undergo a rigid movement of body corresponding to the limiting condition imposed on its edge (right or left).

The energy of the structure is thus:

$$E^e=0.$$

The field of analytical solution displacement is:

$$\boldsymbol{u}=u_x(x,y)\,\boldsymbol{e_x},$$

with:

$$u_x(x,y)=\begin{cases}-\frac{1}{4}\ \text{pour}\ (x,y)\in[-5,0[\times]0,+5[,\\ -\frac{1}{2}\ \text{pour}\ (x,y)\in[-5,0[\times[-5,0[,\\ +\frac{3}{4}\ \text{pour}\ (x,y)\in]0,+5]\times]0,+5],\\ +1\ \text{pour}\ (x,y)\in]0,+5]\times[-5,0[,\end{cases}$$

The standard L^2 displacement is defined by:

$$\|\boldsymbol{u}\|_{L^2}^2=\int_\Omega \|\boldsymbol{u}\|^2 d\,\Omega.$$

One thus has:

$$\|\boldsymbol{u}\|_{L^2}^2=25\left(\frac{1}{16}+\frac{1}{4}+\frac{9}{16}+1\right)=25\,\frac{15}{8}.$$

That is to say:

$$\|\boldsymbol{u}\|_{L^2}=\frac{5}{2}\sqrt{\frac{15}{2}}\approx 6{,}84653196881\ \text{m}^2.$$

This result is valid in the case of the plane constraints and of the plane deformations. In the case 3D, the selected thickness is 1 Mr. the expression of the standard L^2 displacement is identical, but the units are modified.

One has then:

$$\|\boldsymbol{u}\|_{L^2}=\frac{5}{2}\sqrt{\frac{15}{2}}\approx 6{,}84653196881\ \mathrm{m}^{\frac{5}{2}}.$$

2.2 Case with contact

That is to say:

$$p_x(y)=\begin{cases}1\,\mathrm{MPa\ pour}\ \ y\in[-5,0[\\ 2\,\mathrm{MPa\ pour}\ \ y\in]0,+5]\end{cases}\ \text{et}\ \ p_y(x)=\begin{cases}1\,\mathrm{MPa\ pour}\ \ x\in[-5,0[\\ 2\,\mathrm{MPa\ pour}\ \ x\in]0,+5]\end{cases}.$$

One thus has by definition:

$$\begin{aligned}&p_x(y)=2\ \text{ et }\ p_y(x)=1,\ \text{dans}\ \Omega_{1,}\\ &p_x(y)=1\ \text{ et }\ p_y(x)=1,\ \text{dans}\ \Omega_{2,}\\ &p_x(y)=2\ \text{ et }\ p_y(x)=2,\ \text{dans}\ \Omega_{3,}\\ &p_x(y)=1\ \text{ et }\ p_y(x)=2,\ \text{dans}\ \Omega_{4.}\end{aligned} \qquad \text{éq 2.2-1}$$

2.2.1 Case of deformationS plane

The tensor of the constraints analytical solution is:

$$\boldsymbol{\sigma}=-p_x(y)\boldsymbol{e}_x\otimes\boldsymbol{e}_x-p_y(x)\boldsymbol{e}_y\otimes\boldsymbol{e}_y-\nu\left(p_x(y)+p_y(x)\right)\boldsymbol{e}_z\otimes\boldsymbol{e}_z$$

One a:

$$tr(\boldsymbol{\sigma})=-(1+\nu)\left(p_x(y)+p_y(x)\right).$$

The tensor of the deformations is obtained by applying Llaw of Hook hasE :

$$\boldsymbol{\varepsilon}=\frac{1+\nu}{E}\boldsymbol{\sigma}-\frac{\nu}{E}tr(\boldsymbol{\sigma})\boldsymbol{I},$$

where $\boldsymbol{I}$ is the tensor identity.
One thus has:

$$\begin{aligned}\boldsymbol{\varepsilon}=&-\left(\frac{(1+\nu)(1-\nu)\,p_x(y)}{E}-\frac{\nu(1+\nu)\,p_y(x)}{E}\right)\boldsymbol{e}_x\otimes\boldsymbol{e}_x\\ &-\left(\frac{(1+\nu)(1-\nu)\,p_y(x)}{E}-\frac{\nu(1+\nu)\,p_x(y)}{E}\right)\boldsymbol{e}_y\otimes\boldsymbol{e}_y\end{aligned}$$

One thus has:

$$\boldsymbol{\sigma}:\boldsymbol{\varepsilon}=\frac{(1+\nu)(1-\nu)}{E}\left(p_x(y)\right)^2-2\frac{\nu(1+\nu)}{E}p_x(y)\,p_y(x)+\frac{(1+\nu)(1-\nu)}{E}\left(p_y(x)\right)^2.$$

From where:

$$E^e=\frac{1}{2}\frac{1+\nu}{E}\int_\Omega\left[(1-\nu)\left(p_x(y)\right)^2-2\nu\,p_x(y)\,p_y(x)+(1-\nu)\left(p_y(x)\right)^2\right]d\Omega$$

One thus has according to equation 2.2-1 :

$$E^e=\frac{1}{2}\frac{1+\nu}{E}\Big[\int_{\Omega_1}[4(1-\nu)-4\nu+(1-\nu)]d\Omega+\int_{\Omega_2}[(1-\nu)-2\nu+(1-\nu)]d\Omega$$
$$+\int_{\Omega_3}[4(1-\nu)-8\nu+4(1-\nu)]d\Omega+\int_{\Omega_4}[(1-\nu)-4\nu+4(1-\nu)]d\Omega\Big].$$

That is to say:

$$E^e=\frac{1}{2}\frac{1+\nu}{E}\left[(5-9\nu)|\Omega_1|+(2-4\nu)|\Omega_2|+(8-16\nu)|\Omega_3|+(5-9\nu)|\Omega_4|\right].$$

One a:

$$|\Omega_1|=|\Omega_2|=|\Omega_3|=|\Omega_4|=5\times5=25.$$

One thus has:

$$E^e=\frac{1}{2}\frac{1+\nu}{E}25[(5-9\nu)+(2-4\nu)+(8-16\nu)+(5-9\nu).]$$

And finally:

$$E^e=\frac{25(1+\nu)(10-19\nu)}{E}=1{,}3975\ \mathrm{MJ}\times\mathrm{m}^{-1}.$$

The analytical field of displacement $\boldsymbol{u}=u_x\boldsymbol{e_x}+u_y\boldsymbol{e_y}$ is obtained by integrating the deformations:

$$u_x=\int_{-5}^{x}\varepsilon_{xx}dx\,,$$
$$u_y=\int_{-5}^{y}\varepsilon_{yy}dy\,,$$

because the limiting conditions applied are $u_x(x=-5)=0$ and $u_y(y=-5)=0$.
It should be noted that the tensor of the deformations is discontinuous. One has indeed:

$$\boldsymbol{\varepsilon}=\begin{cases}-\frac{(1+\nu)(2-3\nu)}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{(1+\nu)(1-3\nu)}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y,\ \text{dans }[-5,0[\times]0,+5]\\-\frac{(1+\nu)(1-2\nu)}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{(1+\nu)(1-2\nu)}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y,\ \text{dans }[-5,0[\times[-5,0[\\-\frac{2(1+\nu)(1-2\nu)}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{2(1+\nu)(1-2\nu)}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y,\ \text{dans }]0,+5]\times]0,+5]\\-\frac{(1+\nu)(1-3\nu)}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{(1+\nu)(2-3\nu)}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y,\ \text{dans }]0,+5]\times[-5,0[\end{cases}$$

It is noticed that the field of deformations is discontinuous through the right-hand sides of equation $x=0$ and $y=0$. It is thus necessary to distinguish the cases according to the sign from the coordinate on which one integrate to clarify the value of the integrals.

One has thus :

$$u_x=\begin{cases} \int_{-5}^{x}\left[-\frac{(1+\nu)(2-3\nu)}{E}\right]dx\text{, dans } [-5,0[\times]0,+5] \\ \int_{-5}^{x}\left[-\frac{(1+\nu)(1-2\nu)}{E}\right]dx\text{, dans } [-5,0[\times[-5,0[\\ \int_{-5}^{0}\left[-\frac{(1+\nu)(2-3\nu)}{E}\right]dx+\int_{0}^{x}\left[-\frac{2(1+\nu)(1-2\nu)}{E}\right]dx\text{, dans }]0,+5]\times]0,+5] \\ \int_{-5}^{0}\left[-\frac{(1+\nu)(1-2\nu)}{E}\right]dx+\int_{0}^{x}\left[-\frac{(1+\nu)(1-3\nu)}{E}\right]dx\text{, dans }]0,+5]\times[-5,0[\end{cases},$$

and

$$u_y=\begin{cases} \int_{-5}^{0}\left[-\frac{(1+\nu)(1-2\nu)}{E}\right]dy+\int_{0}^{y}\left[-\frac{(1+\nu)(1-3\nu)}{E}\right]dy\text{, dans } [-5,0[\times]0,+5] \\ \int_{-5}^{y}\left[-\frac{(1+\nu)(1-2\nu)}{E}\right]dy\text{, dans } [-5,0[\times[-5,0[\\ \int_{-5}^{0}\left[-\frac{(1+\nu)(2-3\nu)}{E}\right]dy+\int_{0}^{y}\left[-\frac{2(1+\nu)(1-2\nu)}{E}\right]dy\text{, dans }]0,+5]\times]0,+5] \\ \int_{-5}^{y}\left[-\frac{(1+\nu)(2-3\nu)}{E}\right]dy\text{, dans }]0,+5]\times[-5,0[\end{cases}.$$

That is to say:

$$u_x=\begin{cases} -\frac{(1+\nu)(2-3\nu)}{E}x-5\frac{(1+\nu)(2-3\nu)}{E}\text{, dans } [-5,0[\times]0,+5] \\ -\frac{(1+\nu)(1-2\nu)}{E}x-5\frac{(1+\nu)(1-2\nu)}{E}\text{, dans } [-5,0[\times[-5,0[\\ -\frac{2(1+\nu)(1-2\nu)}{E}x-5\frac{(1+\nu)(2-3\nu)}{E}\text{, dans }]0,+5]\times]0,+5] \\ -\frac{(1+\nu)(1-3\nu)}{E}x-5\frac{(1+\nu)(1-2\nu)}{E}\text{, dans }]0,+5]\times[-5,0[\end{cases},\qquad \text{éq 2.2-2}$$

and:

$$u_y=\begin{cases} -\frac{(1+\nu)(1-3\nu)}{E}y-5\frac{(1+\nu)(1-2\nu)}{E}\text{, dans } [-5,0[\times]0,+5] \\ -\frac{(1+\nu)(1-2\nu)}{E}y-5\frac{(1+\nu)(1-2\nu)}{E}\text{, dans } [-5,0[\times[-5,0[\\ -\frac{2(1+\nu)(1-2\nu)}{E}y-5\frac{(1+\nu)(2-3\nu)}{E}\text{, dans }]0,+5]\times]0,+5] \\ -\frac{(1+\nu)(2-3\nu)}{E}y-5\frac{(1+\nu)(2-3\nu)}{E}\text{, dans }]0,+5]\times[-5,0[\end{cases}.\qquad \text{éq 2.2-3}$$

It should be noted that the field of displacement is not continuous. The field being solution of a problem of contact, the normal part with the interfaces of displacement is continuous and one a:

$$u_x(x=0^-)=u_x(x=0^+)=\begin{cases}-5\dfrac{(1+\nu)(1-2\nu)}{E}, \text{ pour } y\in[-5,0[\\ -5\dfrac{(1+\nu)(2-3\nu)}{E}, \text{ pour } y\in]0,+5]\end{cases},$$

and

$$u_y(y=0^-)=u_y(y=0^+)=\begin{cases}-5\dfrac{(1+\nu)(1-2\nu)}{E}, \text{ pour } x\in[-5,0[\\ -5\dfrac{(1+\nu)(2-3\nu)}{E}, \text{ pour } x\in]0,+5]\end{cases}.$$

On the other hand, the tangential part of displacement is discontinuous and one a:

$$u_y(x=0^+)-u_y(x=0^-)=-\frac{(y+5)(1+\nu)(1-\nu)}{E},$$

and

$$u_x(y=0^+)-u_x(y=0^-)=-\frac{(x+5)(1+\nu)(1-\nu)}{E}.$$

The calculation of the integral of the square of the standard of displacement thus must still once to use a partition of the field conforms to the interfaces of equation $x=0$ and $y=0$.

One thus has:

$$\int_\Omega \|\boldsymbol{u}\|^2 d\Omega=\int_{\Omega_1}\left(u_x^2+u_y^2\right)d\Omega+\int_{\Omega_2}\left(u_x^2+u_y^2\right)d\Omega+\int_{\Omega_3}\left(u_x^2+u_y^2\right)d\Omega+\int_{\Omega_4}\left(u_x^2+u_y^2\right)d\Omega.$$

One has finally:

$$\int_\Omega \|\boldsymbol{u}\|^2 d\Omega=\frac{1250\left(131\nu^4+119\nu^3-115\nu^2-3685\nu+40\right)}{3E^2}.$$

From where:

$$\|\boldsymbol{u}\|_{L^2}=\frac{25}{E}\sqrt{\frac{2\left(131\nu^4+119\nu^3-115\nu^2-63\nu+40\right)}{3}}\approx 0{,}791204250915\ \text{m}^2.$$

2.2.2 Case of the plane constraints

The tensor of the constraints analytical solution is:

$$\sigma=-p_x(y)\boldsymbol{e}_x\otimes\boldsymbol{e}_x-p_y(x)\boldsymbol{e}_y\otimes\boldsymbol{e}_y \quad \text{éq 2.2-4}$$

One a:

$$tr(\sigma)=-(p_x(y)+p_y(x)).$$

The tensor of the deformations is obtained by applying the law of Hooke and one a:

$$\boldsymbol{\varepsilon}=\left[-\frac{1+\nu}{E}p_x(y)+\frac{\nu}{E}(p_x(y)+p_y(x))\right]\boldsymbol{e_x}\otimes\boldsymbol{e_x}+\left[-\frac{1+\nu}{E}p_y(x)+\frac{\nu}{E}(p_x(y)+p_y(x))\right]\boldsymbol{e_y}\otimes\boldsymbol{e_y}$$
$$+\frac{\nu}{E}(p_x(y)+p_y(x))\boldsymbol{e_z}\otimes\boldsymbol{e_z}$$

And finally:

$$\boldsymbol{\varepsilon}=-\left(\frac{p_x(y)}{E}-\nu\frac{p_y(x)}{E}\right)\boldsymbol{e_x}\otimes\boldsymbol{e_x}-\left(\frac{p_y(x)}{E}-\nu\frac{p_x(y)}{E}\right)\boldsymbol{e_y}\otimes\boldsymbol{e_y}+\frac{\nu}{E}\left(p_x(y)+p_y(x)\right)\boldsymbol{e_z}\otimes\boldsymbol{e_z}$$

The energy of the structure is :

$$E^e=\frac{1}{2}\int_\Omega\boldsymbol{\sigma}:\boldsymbol{\varepsilon}\,d\Omega$$

One a:

$$\boldsymbol{\sigma}:\boldsymbol{\varepsilon}=p_x(y)\left(\frac{p_x(y)}{E}-\nu\frac{p_y(x)}{E}\right)+p_y(x)\left(\frac{p_y(x)}{E}-\nu\frac{p_x(y)}{E}\right)$$

That is to say:

$$\boldsymbol{\sigma}:\boldsymbol{\varepsilon}=\frac{(p_x(y))^2}{E}-2\nu\frac{p_x(y)p_y(x)}{E}+\frac{(p_y(x))^2}{E}$$

One thus has:

$$E^e=\frac{1}{2}\frac{1}{E}\int_\Omega\left[(p_x(y))^2-2\nu\,p_x(y)\,p_y(x)+(p_y(x))^2\right]d\Omega$$

From where according to equation 2.2-1 :

$$E^e=\frac{1}{2}\frac{1}{E}\left[\int_{\Omega_1}(4-4\nu+1)\,d\Omega+\int_{\Omega_2}(1-2\nu+1)\,d\Omega+\int_{\Omega_3}(4-8\nu+4)\,d\Omega+\int_{\Omega_4}(1-4\nu+4)\,d\Omega\right].$$

That is to say:

$$E^e=\frac{1}{2}\frac{1}{E}\left[(5-4\nu)|\Omega_1|+(2-2\nu)|\Omega_2|+(8-8\nu)|\Omega_3|+(5-4\nu)|\Omega_4|\right].$$

One thus has:

$$E^e=\frac{1}{2}\frac{1}{E}25\left[(5-4\nu)+(2-2\nu)+(8-8\nu)+(5-4\nu).\right]$$

And finally:

$$E^e=\frac{1}{E}25(10-9\nu)=1{,}825\ \mathrm{MJ}\times\mathrm{m}^{-1}. \quad \text{éq 2.2-5}$$

The analytical field of displacement $\boldsymbol{u}=u_x\boldsymbol{e}_x+u_y\boldsymbol{e}_y$ is obtained by integrating the deformations:

$$u_x=\int_{-5}^{x}\varepsilon_{xx}\,dx,$$
$$u_y=\int_{-5}^{y}\varepsilon_{yy}\,dy,$$

because the limiting conditions applied are $u_x(x=-5)=0$ and $u_y(y=-5)=0$.
It should be noted that the tensor of the deformations is discontinuous. One has indeed:

$$\boldsymbol{\varepsilon}=\begin{cases}-\frac{2-\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{1-2\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+3\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z,\text{ dans }[-5,0[\times]0,+5]\\ -\frac{1-\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{1-\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+2\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z,\text{ dans }[-5,0[\times[-5,0[\\ -\frac{2-2\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{2-2\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+4\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z,\text{ dans }]0,+5]\times]0,+5]\\ -\frac{1-2\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{2-\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+3\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z,\text{ dans }]0,+5]\times[-5,0[\end{cases}. \quad \text{éq 2.2-6}$$

It is noticed that the field of deformations is discontinuous through the right-hand sides of equation $x=0$ and $y=0$. It is thus necessary to distinguish the cases according to the sign from the coordinate on which one integrate to clarify the value of the integrals. One has thus :

$$u_x=\begin{cases}\int_{-5}^{x}\left(-\frac{2-\nu}{E}\right)dx,\text{ dans }[-5,0[\times]0,+5]\\ \int_{-5}^{x}\left(-\frac{1-\nu}{E}\right)dx,\text{ dans }[-5,0[\times[-5,0[\\ \int_{-5}^{0}\left(-\frac{2-\nu}{E}\right)dx+\int_{0}^{x}\left(-\frac{2-2\nu}{E}\right)dx,\text{ dans }]0,+5]\times]0,+5]\\ \int_{-5}^{0}\left(-\frac{1-\nu}{E}\right)dx+\int_{0}^{x}\left(-\frac{1-2\nu}{E}\right)dx,\text{ dans }]0,+5]\times[-5,0[\end{cases},$$

and

$$u_y=\begin{cases}\int_{-5}^{0}\left(-\frac{1-\nu}{E}\right)dy+\int_{0}^{y}\left(-\frac{1-2\nu}{E}\right)dy,\ \text{dans } [-5,0[\times]0,+5]\\ \int_{-5}^{y}\left(-\frac{1-\nu}{E}\right)dy,\ \text{dans } [-5,0[\times[-5,0[\\ \int_{-5}^{0}\left(-\frac{2-\nu}{E}\right)dy+\int_{0}^{y}\left(-\frac{2-2\nu}{E}\right)dy,\ \text{dans }]0,+5]\times]0,+5]\\ \int_{-5}^{y}\left(-\frac{2-\nu}{E}\right)dy,\ \text{dans }]0,+5]\times[-5,0[\end{cases}.$$

That is to say:

$$u_x=\begin{cases}-\frac{2-\nu}{E}x-5\frac{2-\nu}{E},\ \text{dans } [-5,0[\times]0,+5]\\ -\frac{1-\nu}{E}x-5\frac{1-\nu}{E},\ \text{dans } [-5,0[\times[-5,0[\\ -\frac{2-2\nu}{E}x-5\frac{2-\nu}{E},\ \text{dans }]0,+5]\times]0,+5]\\ -\frac{1-2\nu}{E}x-5\frac{1-\nu}{E},\ \text{dans }]0,+5]\times[-5,0[\end{cases}, \qquad \text{éq 2.2-7}$$

and:

$$u_y=\begin{cases}-\frac{1-2\nu}{E}y-5\frac{1-\nu}{E},\ \text{dans } [-5,0[\times]0,+5]\\ -\frac{1-\nu}{E}y-5\frac{1-\nu}{E},\ \text{dans } [-5,0[\times[-5,0[\\ -\frac{2-2\nu}{E}y-5\frac{2-\nu}{E},\ \text{dans }]0,+5]\times]0,+5]\\ -\frac{2-\nu}{E}y-5\frac{2-\nu}{E},\ \text{dans }]0,+5]\times[-5,0[\end{cases}. \qquad \text{éq 2.2-8}$$

It should be noted that the field of displacement is not continuous. The field being solution of a problem of contact, the normal part with the interfaces of displacement is continuous and one a:

$$u_x(x=0^-)=u_x(x=0^+)=\begin{cases}-5\frac{1-\nu}{E},\ \text{pour } y\in[-5,0[\\ -5\frac{2-\nu}{E},\ \text{pour } y\in]0,+5]\end{cases},$$

and

$$u_y(y=0^-)=u_y(y=0^+)=\begin{cases}-5\frac{2-\nu}{E}, \text{ pour } x\in[-5,0[\\ -5\frac{1-\nu}{E}, \text{ pour } x\in]0,+5]\end{cases}.$$

On the other hand, the tangential part of displacement is discontinuous and one a:

$$u_y(x=0^+)-u_y(x=0^-)=-\frac{y+5}{E},$$

and

$$u_x(y=0^+)-u_x(y=0^-)=-\frac{x+5}{E}.$$

The calculation of the integral of the square of the standard of displacement thus must still once to use a partition of the field conforms to the interfaces of equation $x=0$ and $y=0$.

One thus has:

$$\int_\Omega \|\boldsymbol{u}\|^2 d\Omega=\int_{\Omega_1}(u_x^2+u_y^2)d\Omega+\int_{\Omega_2}(u_x^2+u_y^2)d\Omega+\int_{\Omega_3}(u_x^2+u_y^2)d\Omega+\int_{\Omega_4}(u_x^2+u_y^2)d\Omega.$$

One has finally:

$$\int_\Omega \|\boldsymbol{u}\|^2 d\Omega=\frac{1250(28\nu^2-63\nu+40)}{3E^2}.$$

From where:

$$\|\boldsymbol{u}\|_{L^2}=\frac{25}{E}\sqrt{\frac{2(28\nu^2-63\nu+40)}{3}}\approx 0{,}992051745962 \text{ m}^2.$$

2.2.3 Case 3D

The structure occupies the field $\Omega_{3D}=\Omega\times[0,1]$. The boundary conditions of the case 3D are the same ones as those of the cases 2D in the plan (X,Y), a condition of roller is imposed in $Z=0$ and the edge $Z=1$ is free constraints. The tensor of the constraints analytical solution is thus identical to the case of the plane constraints (*cf.* éq 2.2-4):

$$\boldsymbol{\sigma}=-p_x(y)\boldsymbol{e_x}\otimes\boldsymbol{e_x}-p_y(x)\boldsymbol{e_y}\otimes\boldsymbol{e_y}$$

The density of energy elastic is thus identical to that of the case 3D. The solid is thickness unit in the direction Z . The expression of the energy of the structure is thus identical to the case of the plane constraints, but the units are modified. One has then (*cf.* éq 2.2-5) :

$$E^e=\frac{1}{E}25(10-9\nu)=1{,}825 \text{ MJ}.$$

The analytical field of displacement $\boldsymbol{u}=u_x\boldsymbol{e_x}+u_y\boldsymbol{e_y}+u_z\boldsymbol{e_z}$ is obtained by integrating the deformations:

$$u_x=\int_{-5}^{x}\varepsilon_{xx}\,dx\,,$$
$$u_y=\int_{-5}^{y}\varepsilon_{yy}\,dy\,,$$
$$u_z=\int_{0}^{z}\varepsilon_{zz}\,dz\,,$$

because the limiting conditions applied are $u_x(x=-5)=0$, $u_y(y=-5)=0$ and $u_z(z=0)=0$.
It should be noted that the tensor of the deformations is discontinuous. One has indeed (*cf.* éq 2.2- 6) :

$$\boldsymbol{\varepsilon}=\begin{cases}-\frac{2-\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{1-2\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+3\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z\text{, dans }[-5,0[\times]0,+5]\times[0,1]\\ -\frac{1-\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{1-\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+2\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z\text{, dans }[-5,0[\times[-5,0[\times[0,1]\\ -\frac{2-2\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{2-2\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+4\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z\text{, dans }]0,+5]\times]0,+5]\times[0,1]\\ -\frac{1-2\nu}{E}\boldsymbol{e}_x\otimes\boldsymbol{e}_x-\frac{2-\nu}{E}\boldsymbol{e}_y\otimes\boldsymbol{e}_y+3\frac{\nu}{E}\boldsymbol{e}_z\otimes\boldsymbol{e}_z\text{, dans }]0,+5]\times[-5,0[\times[0,1]\end{cases}.$$

It is noticed that the field of deformations is discontinuous through the right-hand sides of equation $x=0$ and $y=0$. It is thus necessary to distinguish the cases according to the sign from the coordinate on which one integrate to clarify the value of the integrals. Expressions of the components u_x and u_y are identical to the case of the plane constraints (*cf.* éq 2.2- 7 and 2.2- 8) and one has for u_z :

$$u_z=\begin{cases}\int_0^1\frac{3\nu}{E}dz\text{, dans }[-5,0[\times]0,+5]\times[0,1]\\ \int_0^1\frac{2\nu}{E}dz\text{, dans }[-5,0[\times[-5,0[\times[0,1]\\ \int_0^1\frac{4\nu}{E}dz\text{, dans }]0,+5]\times]0,+5]\times[0,1]\\ \int_0^1\frac{3\nu}{E}dz\text{, dans }]0,+5]\times[-5,0[\times[0,1]\end{cases}.$$

That is to say:

$$u_z=\begin{cases}\frac{3\nu}{E}z\text{, dans }[-5,0[\times]0,+5]\times[0,1]\\ \frac{2\nu}{E}z\text{, dans }[-5,0[\times[-5,0[\times[0,1]\\ \frac{4\nu}{E}z\text{, dans }]0,+5]\times]0,+5]\times[0,1]\\ \frac{3\nu}{E}z\text{, dans }]0,+5]\times[-5,0[\times[0,1]\end{cases}. \qquad \text{éq 2.2-9}$$

The calculation of the integral of the square of the standard of displacement thus must still once to use a partition of the field conforms to the interfaces of equation $x=0$ and $y=0$. One thus has:

$$\int_{\Omega_{3D}} \|\boldsymbol{u}\|^2 d\Omega = \int_{\Omega_1 \times [0,1]} \left(u_x^2 + u_y^2 + u_z^2\right) d\Omega + \int_{\Omega_2 \times [0,1]} \left(u_x^2 + u_y^2 + u_z^2\right) d\Omega$$
$$+ \int_{\Omega_3 \times [0,1]} \left(u_x^2 + u_y^2 + u_z^2\right) d\Omega + \int_{\Omega_4 \times [0,1]} \left(u_x^2 + u_y^2 + u_z^2\right) d\Omega.$$

One has finally:

$$\int_{\Omega_{3D}} \|\boldsymbol{u}\|^2 d\Omega = \frac{50\left(719\nu^2 - 1575\nu + 1000\right)}{3E^2}.$$

From where:

$$\|\boldsymbol{u}\|_{L^2} = \frac{5}{E}\sqrt{\frac{2\left(719\nu^2 - 1575\nu + 1000\right)}{3}} \approx 0{,}99348712456\ \mathrm{m}^{\frac{5}{2}}.$$

3 Modeling A

3.1 Characteristics of modeling

It acts of a modeling X-FEM, into cubesplane formations. The interfaces are defined by functions of levels (level sets noted normals LN).

The equations of the functions of levels for the interfaces horizontal and vertical are the following ones:

$$\mathrm{LN}1 = Y \quad \text{éq 3.1-1}$$
$$\mathrm{LN}2 = X \quad \text{éq 3.1-2}$$

The horizontal interface is defined in a classical way by using the operator `DEFI_FISS_XFEM` with the level set normal $\mathrm{LN1}$.
To define the vertical interface, one proceeds in two stages. The operator for the first time is called `DEFI_FISS_XFEM` with the level set normal $\mathrm{LN2}$, by defining a point “in the top” of the horizontal crack for the keyword `JUNCTION` (the point is not obligatorily on the level set). This stage makes it possible to define the upper part of the vertical interface (see figure 3.1-a in the center). The operator for the second time is called `DEFI_FISS_XFEM` same manner, but by defining a point “in lower part” of the crack (see figure 3.1-a on the right).
One thus called on the whole 3 times `DEFI_FISS_XFEM` (creation of 3 cracks objects) to define the two interfaces which are intersected. From a theoretical point of view, each object fissures addition an enrichment of the Heaviside type. What makes a total of three degrees of Heaviside freedom besides the classical degrees of freedom. There are thus well 4 degree of freedom on the level of the intersection, which makes it possible to move in manner independent the 4 zones generated by the 2 interfaces.

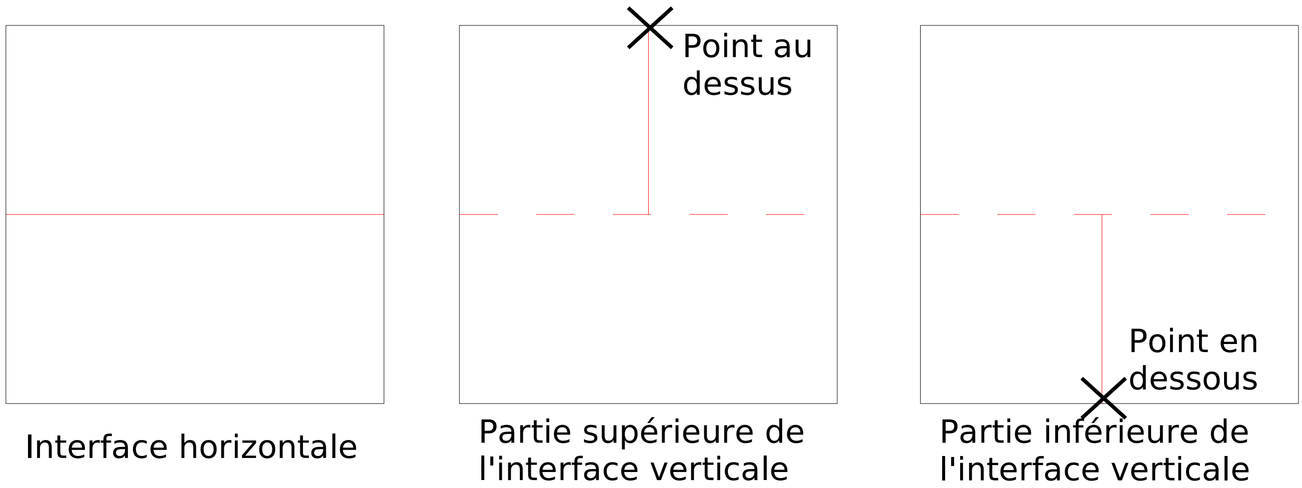

Figure 3.1-a: Stages of construction of the intersection.

3.2 Characteristics of the grid

The grid which comprises 25 meshs of the type `QUAD4`, is represented on the figure 3.2-a.
One notices on this figure that the central mesh is cut by the two interfaces. This test thus makes it possible to validate multiple cutting. Let us note that the nodes of this mesh are nouveau riches 3 times, they thus have the degrees of freedom DX , DY , $H1X$, $H1Y$, $H2X$, $H2Y$, $H3X$ and $H3Y$.

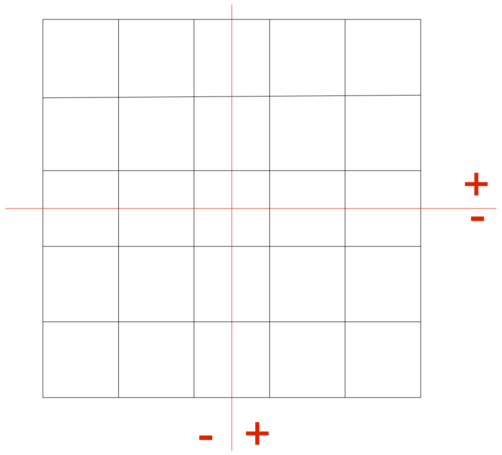

Figure 3.2-a: Grid of modeling A.

3.3 Sizes tested and results

Displacements are tested on the level as of lips of the cracks after having carried out the operations of postprocessings relative to $X\text{-FEM}$ (POST_MAIL_XFEM and POST_CHAM_XFEM). Displacement DX must correspond to the loading imposed of the figure 1.3-a on each zone and DY must be null. One tests the min and the max on the lips of each zone.

Identification			Reference	Type of reference	Precision
DEPZON_1	DX	MIN	-0.25	'ANALYTICAL'	10^-12%
		MAX	-0.25	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_2	DX	MIN	-0.5	'ANALYTICAL'	10^-12%
		MAX	-0.5	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_3	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_4	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%

Table 3.3-1

The deformation is represented on the figure 3.4-a. The code color represents the field of displacement.

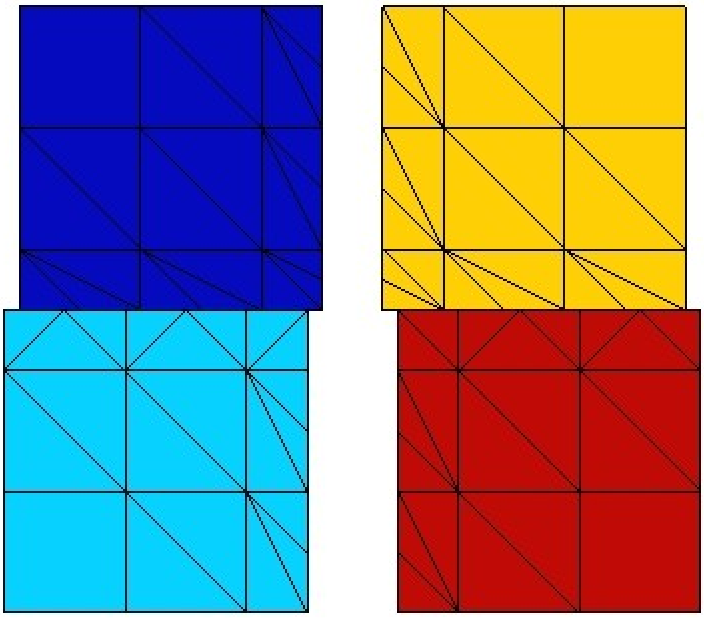

Figure 3.4-a: Deformation of the structure.

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference
E E	'ANALYTICAL'	0

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes 2	'ANALYTICAL'	6.84653196881	0.1%

4 Modeling B

4.1 Characteristics of modeling

It is acted of the same modeling as modeling A, but as plane constraints. The intersection is built same manner.

4.2 Characteristics of the grid

The grid which comprises 54 meshs of the type `TRIA3` is represented on the figure 4.2-a.

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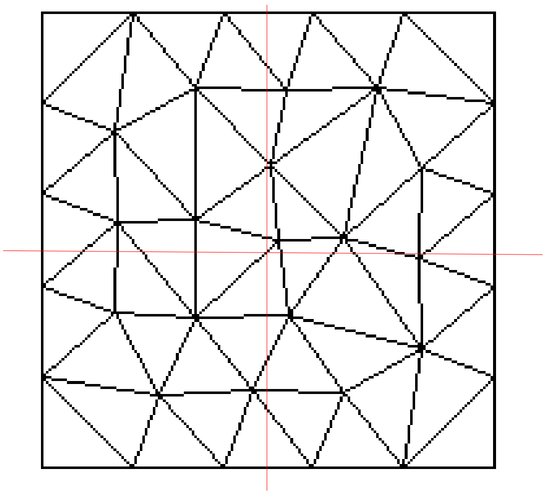

Figure 4.2-a: Grid of modeling B.

4.3 Sizes tested and results

The sizes tested are identical to those presented for modeling A.

Identification			Reference	Type of reference	Precision
DEPZON_1	DX	MIN	-0.25	'ANALYTICAL'	10^-12%
		MAX	-0.25	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_2	DX	MIN	-0.5	'ANALYTICAL'	10^-12%
		MAX	-0.5	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_3	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_4	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%

Table 4.3-1

The deformation is represented on the figure 4.4-a.

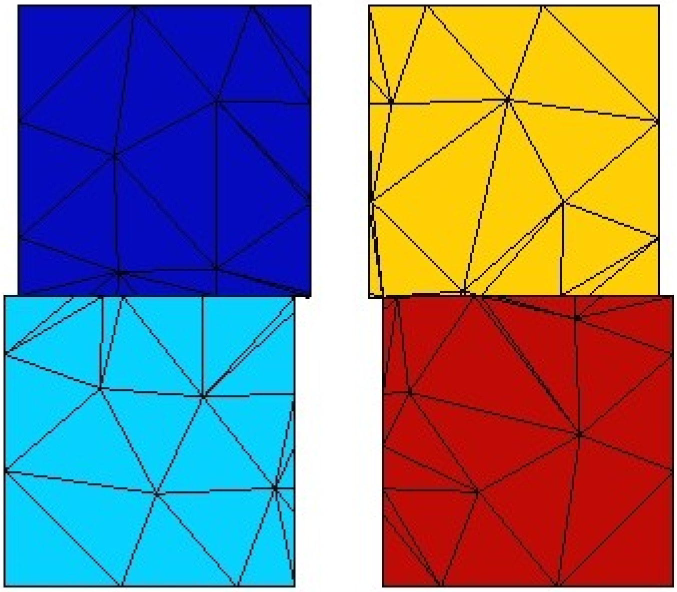

Figure 4.4-a: Deformation of the structure.

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference
E^E	'ANALYTICAL'	0

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	6.84653196881	0.1%

5 Modeling C

5.1 Characteristics of modeling

It is the same modeling as modeling A, but in $3D$. The intersection is built same manner.

5.2 Characteristics of the grid

The grid which comprises 25 meshs of the type HEXA8 is represented on the figure 5.2-a.

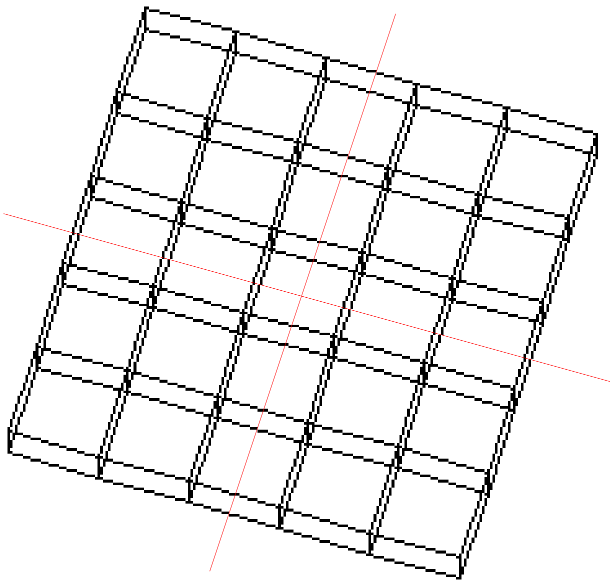

Figure 5.2-a: Grid of modeling C.

5.3 Sizes tested and results

The sizes tested are identical to those presented for modeling A. One adds tests on DZ .

Identification			Reference	Type of reference	Precision
DEPZON_1	DX	MIN	-0.25	'ANALYTICAL'	10^-12%
		MAX	-0.25	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_2	DX	MIN	-0.5	'ANALYTICAL'	10^-12%
		MAX	-0.5	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_3	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_4	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%

Table 5.3-1

The deformation is represented on the figure 5.4-a.

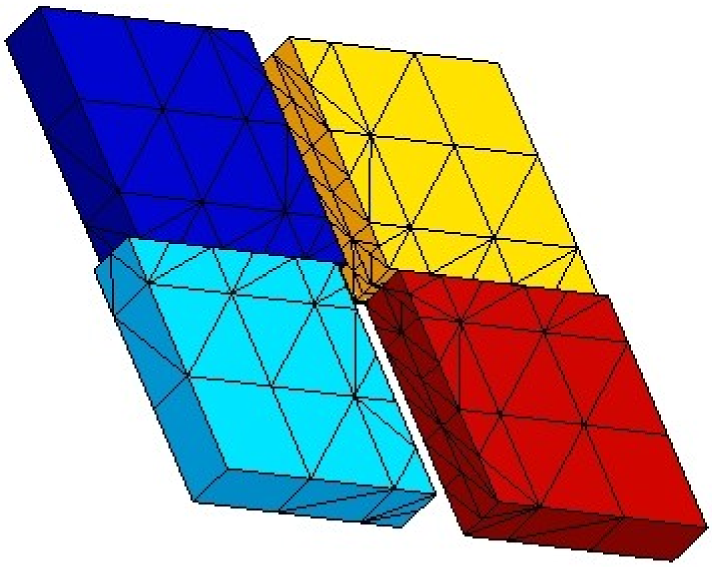

Figure 5.4-a: Deformation of the structure.

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference
E ^{E}	'ANALYTICAL'	0

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	6.84653196881	0.1%

6 Modeling D

6.1 Characteristics of modeling

It is the same modeling as modeling C.

6.2 Characteristics of the grid

The grid which comprises 162 meshs of the type TETRA4 is represented on the figure 6.2-a.

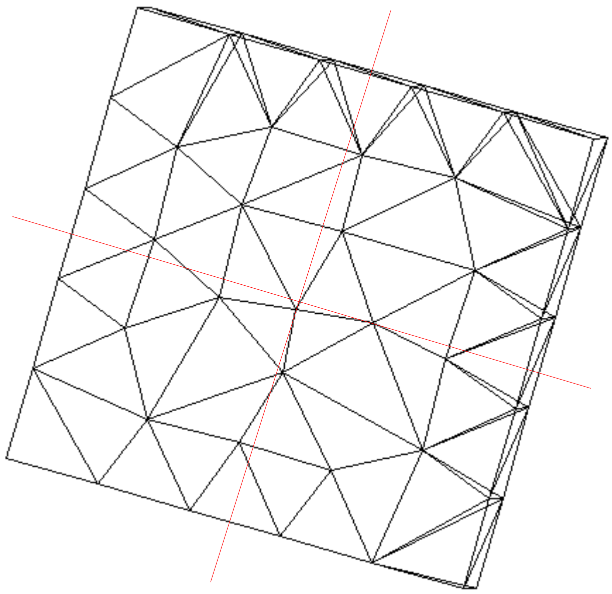

Figure 6.2-a: Grid of modeling D.

6.3 Sizes tested and results

The sizes tested are identical to those presented for modeling C.

Identification			Reference	Type of reference	Precision
DEPZON_1	DX	MIN	-0.25	'ANALYTICAL'	10^-12%
		MAX	-0.25	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_2	DX	MIN	-0.5	'ANALYTICAL'	10^-12%
		MAX	-0.5	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_3	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%
DEPZON_4	DX	MIN	0.75	'ANALYTICAL'	10^-12%
		MAX	0.75	'ANALYTICAL'	10^-12%
	DY	MIN	0	'ANALYTICAL'	10^-12%
		MAX	0	'ANALYTICAL'	10^-12%

Table 6.3-1

The deformation is represented on the figure 6.4-a.

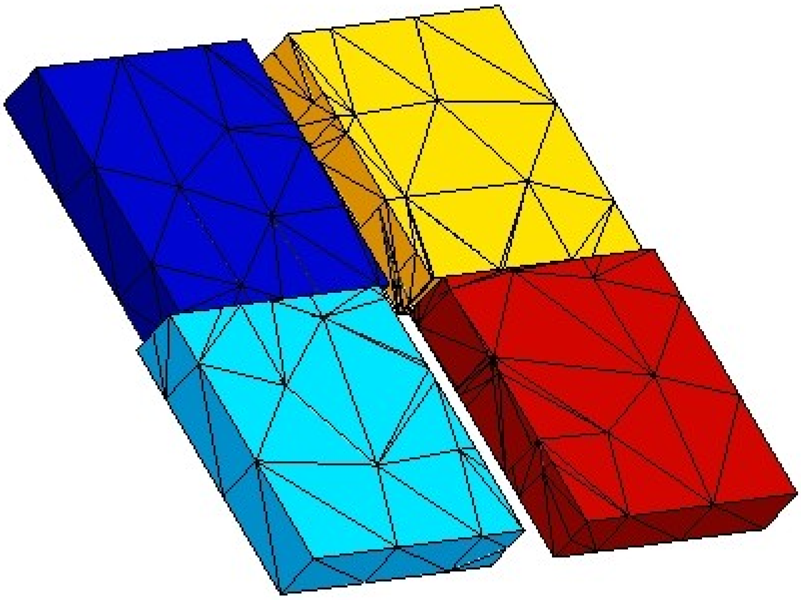

Figure 6.4-a: Deformation of the structure.

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference
E E	'ANALYTICAL'	0

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes 2	'ANALYTICAL'	6.84653196881	0.1%

7 Modeling E

7.1 Characteristics of modeling

It is the same modeling that modeling A, but the conditions of loading in contact are applied. The intersection is built with X-FEM and the functions of levels in the same way as for modeling A.

7.2 Characteristics of the grid

The grid identical to that of modeling A, is represented figure 3.2-a. Let us note that the nodes of the intersected mesh are nouveau riches 3 times, they thus have the degrees of freedom of contact $LAGS\_C$, $LAG2\_C$ and $LAG3\_C$ besides degrees of freedom kinematics.

7.3 Sizes tested and results

Displacements are tested on the level as of lips of the cracks after having carried out the operations of postprocessings relative to X-FEM (POST_MAIL_XFEM and POST_CHAM_XFEM). Displacement DX must follow the function u_x equation 2.2-2. Displacement DY must follow the function u_y equation 2.2-3. One obtains the deformation of the figure 7.4-a.

Identification			Reference
DEPZON_1	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
DEPZON_2	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
DEPZON_3	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
DEPZON_4	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0

Table 7.3-1

The deformation is represented on the figure 7.4-a. The code color represents the field of displacement.

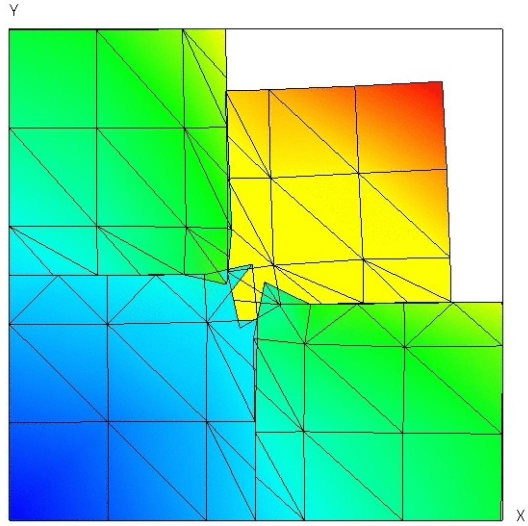

Figure 7.4-a: Deformation of the structure (exaggeration 10).

One tests the value of E^e product by the operator POST_ERREUR (expressedE in $\mathrm{J} \times \mathrm{m}^{-1}$).

Identification	Type of reference	Value of reference
E^E	'ANALYTICAL'	$1.3975\ 10^{6}$

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ²	'ANALYTICAL'	0.791204250915	0.1%

7.4 Remarks

A high error is obtained. Indeed the implementation of the recutting of the facets of contact was not implemented. The efforts of contact on these facets are not taken into account in calculation. The zone affected relates to in particular the point of intersection of the cracks (which one does not test) as well as the element the container. Let us note that the results are clearly to improve when the grid is refined.

8 Modeling F

8.1 Characteristics of modeling

It is acted of the same modeling as modeling E, but as plane constraints. The intersection is built same manner.

8.2 Characteristics of the grid

The grid identical to that of modeling B, is represented on the figure 4.2-a.

8.3 Sizes tested and results

One tests displacements on the level of the lips of the cracks after having carried out the operations of postprocessings relative to X-FEM (POST_MAIL_XFEM and POST_CHAM_XFEM). Displacement DX must follow the function u_x equation 2.2-7. Displacement DY must follow the function u_y equation 2.2-8. One obtains the deformation of the figure 7.4-a.

Identification			***Reference***
DEPZON_1	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
DEPZON_2	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
DEPZON_3	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
DEPZON_4	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0

Table 8.3-1

The deformation is represented on the figure 8.4-a.

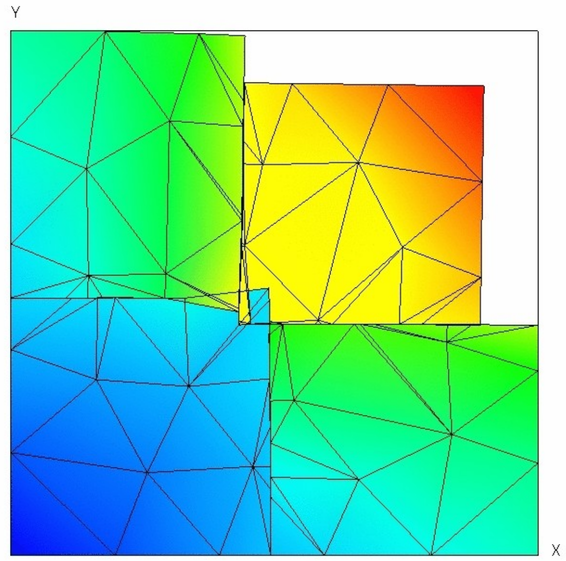

Figure 8.4-a: Deformation of the structure (exaggeration 10).

One tests the value of E^e product by the operator POST_ERREUR (expressedE in $\mathrm{J} \times \mathrm{m}^{-1}$).

Identification	Type of reference	Value of reference
E^E	'ANALYTICAL'	1.825 10^6

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	0.992051745962	0.1%

8.4 Remarks

The remarks are identical to those formulated for modeling E.

9 Modeling G

9.1 Characteristics of modeling

It is the same modeling as modeling E, but in $3D$. The intersection is built same manner.

9.2 Characteristics of the grid

The grid identical to that of modeling C, is represented on the figure 5.2-a.

9.3 Sizes tested and results

One tests displacements on the level of the lips of the cracks after having carried out the operations of postprocessings relative to X-FEM (POST_MAIL_XFEM and POST_CHAM_XFEM). Displacement DX must follow the function u_x equation 2.2-7. Displacement DY must follow the function u_y equation 2.2-8. Displacement DZ must follow the function u_z equation 2.2-9. One obtains the deformation of the figure 7.4-a.

Identification			***Reference***
DEPZON_1	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0
DEPZON_2	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0
DEPZON_3	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0
DEPZON_4	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0

Table 9.3-1

The deformation is represented on the figure 9.4-a.

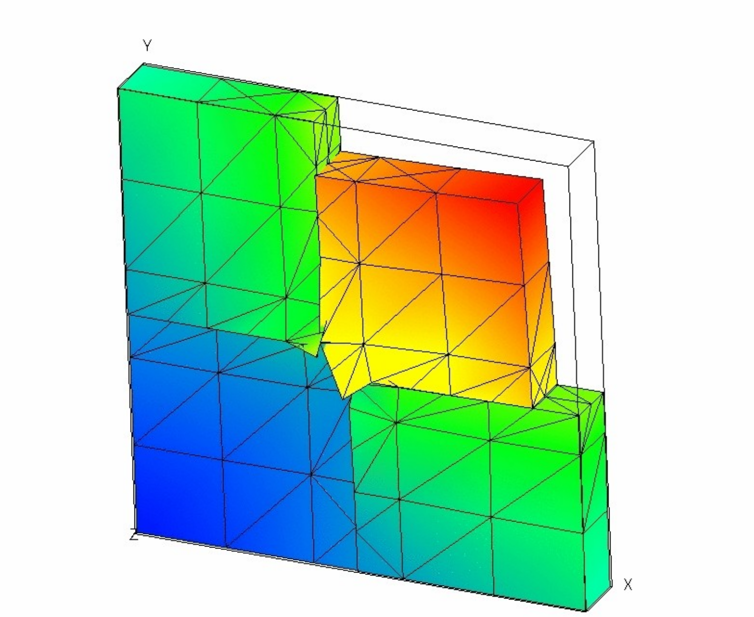

Figure 9.4-a: Deformation of the structure (exaggeration 10).

One tests the value of E^e product by the operator `POST_ERREUR` (expressedE in J).

Identification	Type of reference	Value of reference
E^E	'ANALYTICAL'	1.825 10 ^{6}

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator `POST_ERREUR`.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ^{2}	'ANALYTICAL'	0.99348712456	0.1%

9.4 Remarks

The remarks are identical to those formulated for modeling E.

10 Modeling H

10.1 Characteristics of modeling

It is the same modeling as modeling G.

10.2 Characteristics of the grid

The grid identical to that of modeling D, is represented on the figure 6.2-a.

10.3 Sizes tested and results

The sizes tested are identical to those presented for modeling G.

Identification			Reference
DEPZON_1	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0
DEPZON_2	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0
DEPZON_3	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0
DEPZON_4	DX- u_x	MIN	0
		MAX	0
	DY u_y	MIN	0
		MAX	0
	DZ- u_z	MIN	0
		MAX	0

Table 10.3-1

The deformation is represented on the figure 10.4-a.

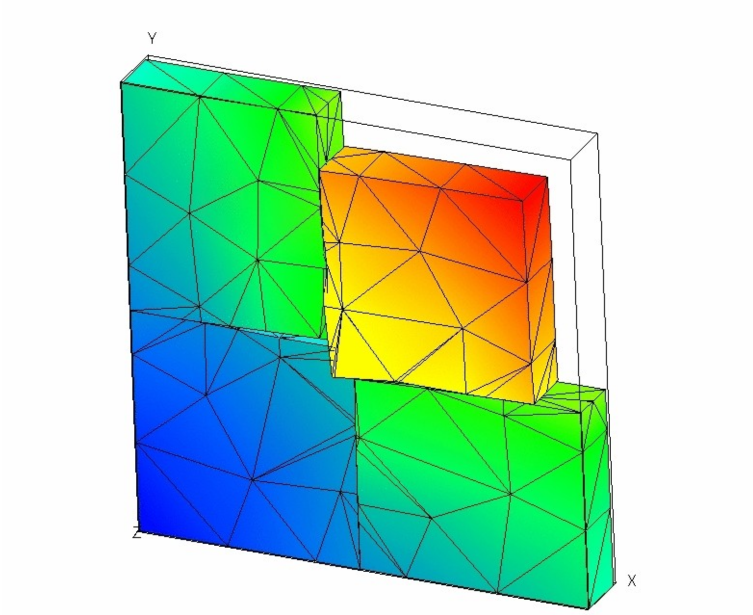

Figure 10.4-a: Deformation of the structure (exaggeration 10).

One tests the value of E^e product by the operator POST_ERREUR (expressedE in J).

Identification	Type of reference	Value of reference
E E	'ANALYTICAL'	1.825 10 6

One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes 2	'ANALYTICAL'	0.99348712456	0.1%

10.4 Remarks

The remarks are identical to those formulated for modeling E.

11 Summary of the results

The representation of junctions with X-FEM allows to model the kinematics of opening of the intersection of two interfaces. It is also possible to make the same thing with funds of cracks on both sides of the junction, but it is necessary to take care to move away the bottom from the intersection (approximately 2 meshs for a topological enrichment because one cannot manage yet the presence of the additional Heaviside for elements Ace-tip. An element Ace-tip cannot currently "see" more than one crack at the same time).

The approach was validated in 2D for modelings C_PLAN and D_PLAN and for the elements of the type QUAD4 and TRIA3. One also validated the approach in 3D for the elements HEXA8 and TETRA4, with and without contact.