SSNV515 – Tensile test with the law of Rankine

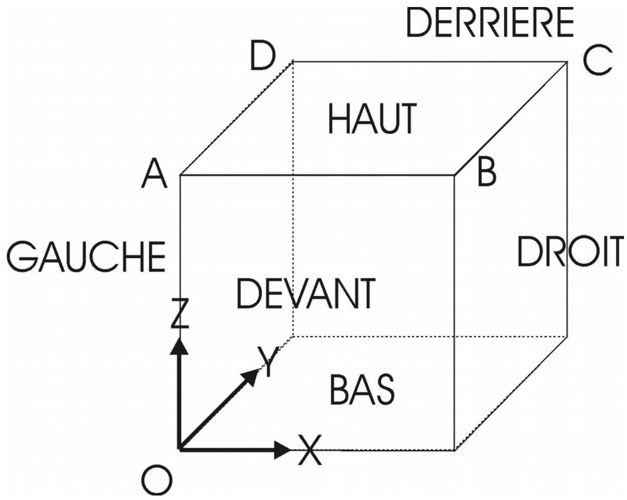
Summary

One is carried out simple tensile test with *the law of Rankine*. The calculated solutions are compared with an analytical solution. Three modelings of which suggested :

- a modeling 0D with SIMU_POINT_MAT ;
- a modeling 3D ;
- Uaxisymmetric modeling 2D ;

1 Problem of reference

1.1 Geometry



The test of simple traction is carried out on only one isoparametric finite element of cubic form $CUB4$. The length of each edge is worth 1. The various facets of this cube are named groups of meshs $HAUT$, BAS, $DEVANT$, $DERRIERE$, $DROIT$ and $GAUCHE$. The group of meshs *SYM* contains the groups of meshs in addition BAS, $DEVANT$ and $GAUCHE$; the group of meshs $COTE$ groups of meshs $DERRIERE$ and $DROIT$.

1.2 Material properties

The elastic properties are:

- module D'Young : $E=1\,MPa$
- Poisson's ratio : $\nu=0,25$

Llimit in traction has is equal to $\sigma_t=1\,kPa$

1.3 Boundary conditions and loadings

The simple tensile test consist in imposing on the test-tube one elongation vertical all while keeping the side pressure constant and equalizes with the initial isotropic constraint $P_0=10\,kPa$
In the model considered, the cubic element represents a eighth of the sample. The limiting conditions are thus the following ones:

- Conditions of symmetry:
  - $u_z=0$ on the group of mesh BAS
  - $u_x=0$ on the group of mesh $GAUCHE$
  - $u_y=0$ on the group of mesh $DEVANT$
- Conditions of side pressure:
  - $P_n=P_0=10\,kPa$ onS groupS of meshS $DROIT$ and $ARRIERE$
- Conditions of loading:
  - $u_z=+1$ on the group of mesh $HAUT$

The loading is carried out in 30 pas de time enter $t=0$ and $t=30$ during which displacement imposed on the group of meshs $HAUT$ vary of $u_z=0$ with $u_z=0.3$ (total vertical deformation of 30%).

1.4 Results

The solutions post-are treated with the point C, in terms of:

- constraint verticalE σ_{zz} ;
- deformation horizontale ϵ_{xx} ;
- normalizes deformation plastic déviatoriquE $e^P = \|\boldsymbol{e}^P\|$

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters $t=0$ and $t=20$.

2 Analytical solution

Let us introduce initially the notations suvantes:

$$\begin{cases} A=K+\frac{4}{3}G \\ B=K-\frac{2}{3}G \\ C=2\left(K+\frac{G}{3}\right) \end{cases} \quad (1)$$

With $K=\frac{E}{3(1-2\nu)}$ and $G=\frac{E}{2(1+\nu)}$ modules of compressibility and shearing, respectively.
That is to say $\boldsymbol{C}$ the tensor of ealsticity of Hooke, one will have with the assumption $\epsilon_{yy}=\epsilon_{xx}$:

$$\boldsymbol{C}.d\boldsymbol{\epsilon}=\begin{cases} B\,d\epsilon_{zz}+C\,d\epsilon_{xx} \\ B\,d\epsilon_{zz}+C\,d\epsilon_{xx} \\ A\,d\epsilon_{zz}+2B\,d\epsilon_{xx} \end{cases} \quad (2)$$

One will note to simplify the vertical constraint at the moment + $\sigma^{+}=\sigma^{+}_{zz}$, so that the criterion of Rankine is written:

$$\sigma^{+}\leq\sigma_t \quad (3)$$

One has in addition:

$$\begin{cases} \sigma^{\text{préd}}=\sigma^{-}+\boldsymbol{n}.\boldsymbol{C}.d\boldsymbol{\epsilon} \\ \sigma^{+}=\sigma^{-}+\boldsymbol{n}.\boldsymbol{C}.(d\boldsymbol{\epsilon}-d\lambda\,\boldsymbol{n})=\sigma^{\text{préd}}-\underbrace{d\lambda\,\boldsymbol{n}.\boldsymbol{C}.\boldsymbol{n}}_{\Delta\sigma_C} \end{cases} \quad (4)$$

With $\boldsymbol{n}=\begin{pmatrix}0\\0\\1\end{pmatrix}$ and Où:

$$d\lambda=\frac{\langle\sigma^{préd}-\sigma_t\rangle_{+}}{A} \quad (5)$$

According to the law of flow associated, one also has:

$$\begin{cases} d\epsilon^{P}_{zz}=d\lambda=d\epsilon^{P}_{v} \\ e^{P}=\frac{2}{3}d\lambda \end{cases} \quad (6)$$

Like $\boldsymbol{n}=\begin{pmatrix}0\\0\\1\end{pmatrix}$, one obtains:

$$\Delta\sigma_C=d\lambda\,A \quad (7)$$

Combination of the equations (**7**), (**5**) and (**4**) we gives the constraint σ^{+}_{zz} . Lbe equationS (**6**) and (**5**) we givesNT the standard of the deviatoric plastic deformation e^{P} .

Let us try to obtain now the expression of the deformation rubber band horizontal $\epsilon^{\text{élas}}_{xx}$.
Latéralement, there is the condition $\sigma^{+}_{xx}=P_0$, that is to say:

$$\sigma_{xx}^{\text{préd}}-\Delta\sigma_{xx,C}=P_0 \tag{8}$$

With $\Delta\sigma_{xx,C}=d\lambda B$
One obtains then by using the equation (**2**):

$$\sigma_{xx}^{-}+C\,d\epsilon_{xx}^{\text{élas}}+B\,d\epsilon_{zz}-B\,d\lambda=P_0 \tag{9}$$

From where the increment of horizontal elastic strain:

$$d\epsilon_{xx}^{\text{élas}}=\frac{P_0-\sigma_{xx}^{-}+B(d\lambda-d\epsilon_{zz})}{C} \tag{10}$$

3 Modeling A

3.1 Characteristics of modeling

Modeling A is realized on a material point 0D with SIMU_POINT_MAT.

3.2 Sizes tested and results

3.2.1 Values tested

The solutions post-are treated with the point C, in terms of:

- constraint verticalE σ_{zz} ;
- deformation horizontale ϵ_{xx} ;
- normalizes deformation plastic déviatoriquE $e^P = \|\boldsymbol{e}^P\|$

They are compared with an analytical solution (described in the following paragraph) in terms ofmaximum variation enters $t=0$ and $t=20$. The results are recapitulated in the following tables:

$$Q=\sqrt{\frac{1}{2}\underline{\underline{s}}:\underline{\underline{s}}}\quad [Pa]$$

Variable	Absolute deviation | Code_Aster − Analytical |
σ_{zz}	0
ϵ_{xx}	3.10^{-5}
e^P	$1{,}333\ 10^{-5}$

3.2.2 Comments

The variation with the analytical solution is very weak.

4 Modeling B

4.1 Characteristics of modeling

Modeling B is realized in 3D with STAT_NON_LINE.

4.2 Sizes tested and results

4.2.1 Values tested

The solutions post-are treated with the point C, in terms of:

- constraint verticalE σ_{zz} ;
- deformation horizontale ϵ_{xx} ;
- normalizes deformation plastic déviatoriquE $e^P=\|\boldsymbol{e}^P\|$

They are compared with an analytical solution (described in the following paragraph) in terms ofmaximum variation enters $t=0$ and $t=20$. The results are recapitulated in the following tables:

$$Q=\sqrt{\frac{1}{2}\underline{\underline{s}}:\underline{\underline{s}}}\ [Pa]$$

Variable	Absolute deviation | Code_Aster − Analytical |
σ_{zz}	0
ϵ_{xx}	3.10^{-5}
e^P	$1,333\ 10^{-5}$

4.2.2 Comments

The variation with the analytical solution is very weak.

5 Modeling C

5.1 Characteristics of modeling

Modeling C is realized on a material point 2D axisymmetric with STAT_NON_LINE.

5.2 Sizes tested and results

5.2.1 Values tested

The solutions post-are treated with the point C, in terms of:

- constraint verticalE σ_{zz} ;
- deformation horizontale ϵ_{xx} ;
- normalizes deformation plastic déviatoriquE $e^P=\|\boldsymbol{e}^P\|$

They are compared with an analytical solution (described in the following paragraph) in terms ofmaximum variation enters $t=0$ and $t=20$. The results are recapitulated in the following tables:

$$Q=\sqrt{\frac{1}{2}\underline{\underline{s}}:\underline{\underline{s}}}\quad [Pa]$$

Variable	Absolute deviation | Code_Aster − Analytical |
σ_{zz}	0
ϵ_{xx}	3.10^{-5}
e^P	$1{,}333\ 10^{-5}$

5.2.2 Comments

The variation with the analytical solution is very weak.

6 Modeling D

6.1 Characteristics of modeling

Modeling D is realized on a material point 3D with STAT_NON_LINE. The difference compared to modeling B is the calculation of the initial state by a thermal loading. To bring the sample to the initial isotropic constraint $P_0 = 10\,kPa$, one brings the sample of 20° to 30° celcius. Displacements of the sample are blocked, so that thermal dilation brings the sample in compression. One obtains:

$$\sigma_0 = \frac{E}{9(1-2\nu)} \alpha \Delta T = P_0$$

That is to say the following value of the thermal dilation coefficient: $\alpha = \frac{9(1-2\nu) P_0}{E \Delta T}$

6.2 Sizes tested and results

6.2.1 Values tested

The solutions post-are treated with the point C , in terms of:

- constraint verticalE σ_{zz} ;
- deformation horizontale ϵ_{xx} ;
- normalizes deformation plastic déviatoriquE $e^P = \| \boldsymbol{e}^P \|$

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters $t=0$ and $t=20$. The results are recapitulated in the following tables:

$$Q = \sqrt{\frac{1}{2} \underline{\underline{s}} : \underline{\underline{s}}} \quad [Pa]$$

Variable	Absolute deviation | Code_Aster − Analytical |
σ_{zz}	0
ϵ_{xx}	3.10^{-5}
e^P	$1,333\ 10^{-5}$

6.2.2 Comments

The variation with the analytical solution is very weak.

7 Summary of the results

One represents in following figures evolution of the various sizes during the tensile test with the law of Rankine.

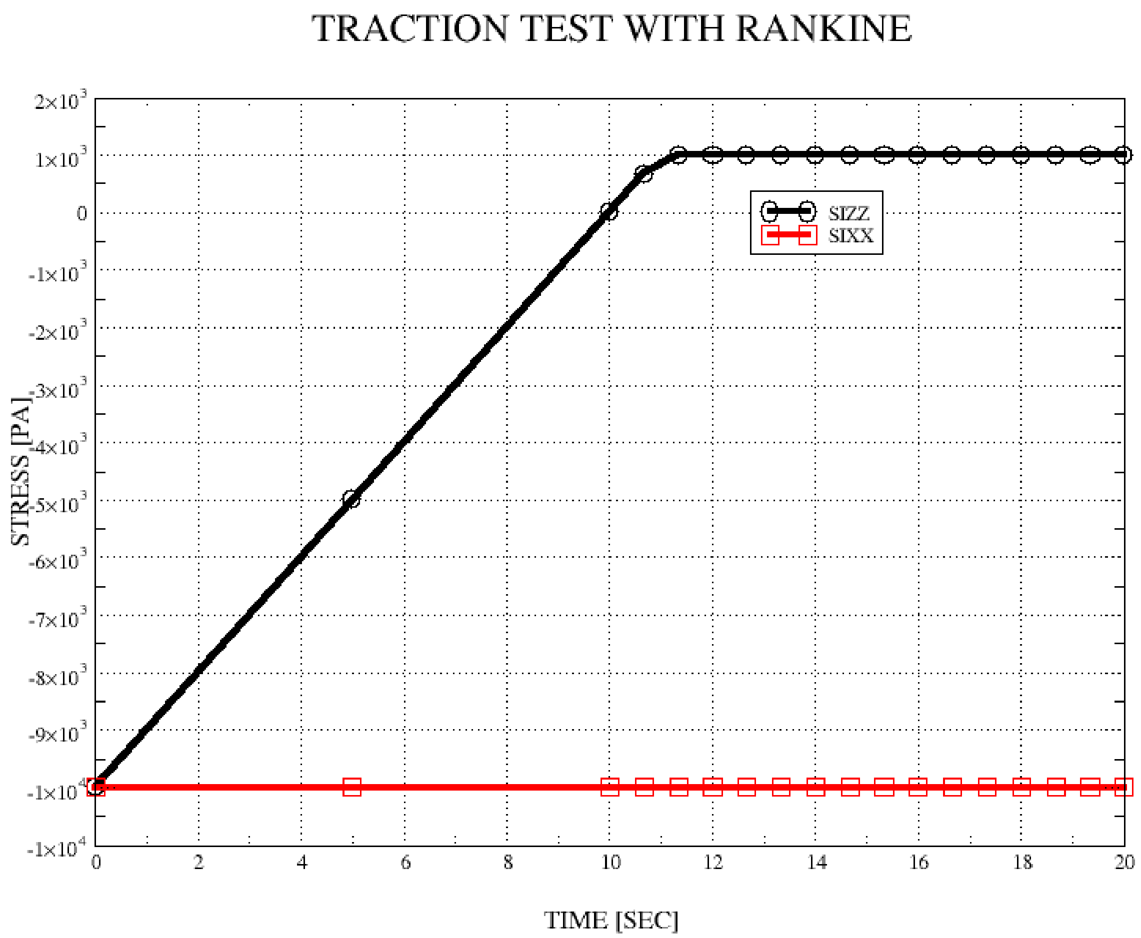


Figure **1** : *Evolution of the constraints during the tensile test*

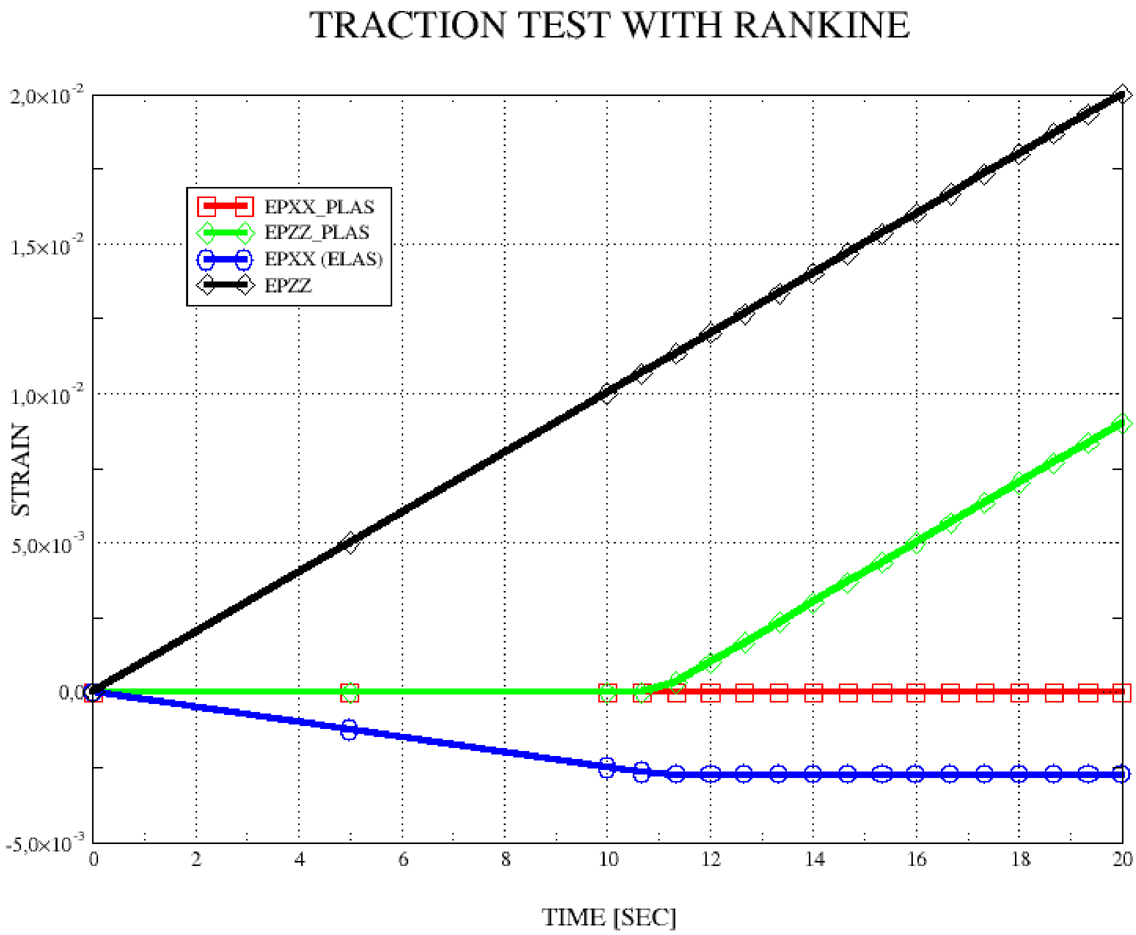


Figure **2** : *Evolution of the deformations during the tensile test*

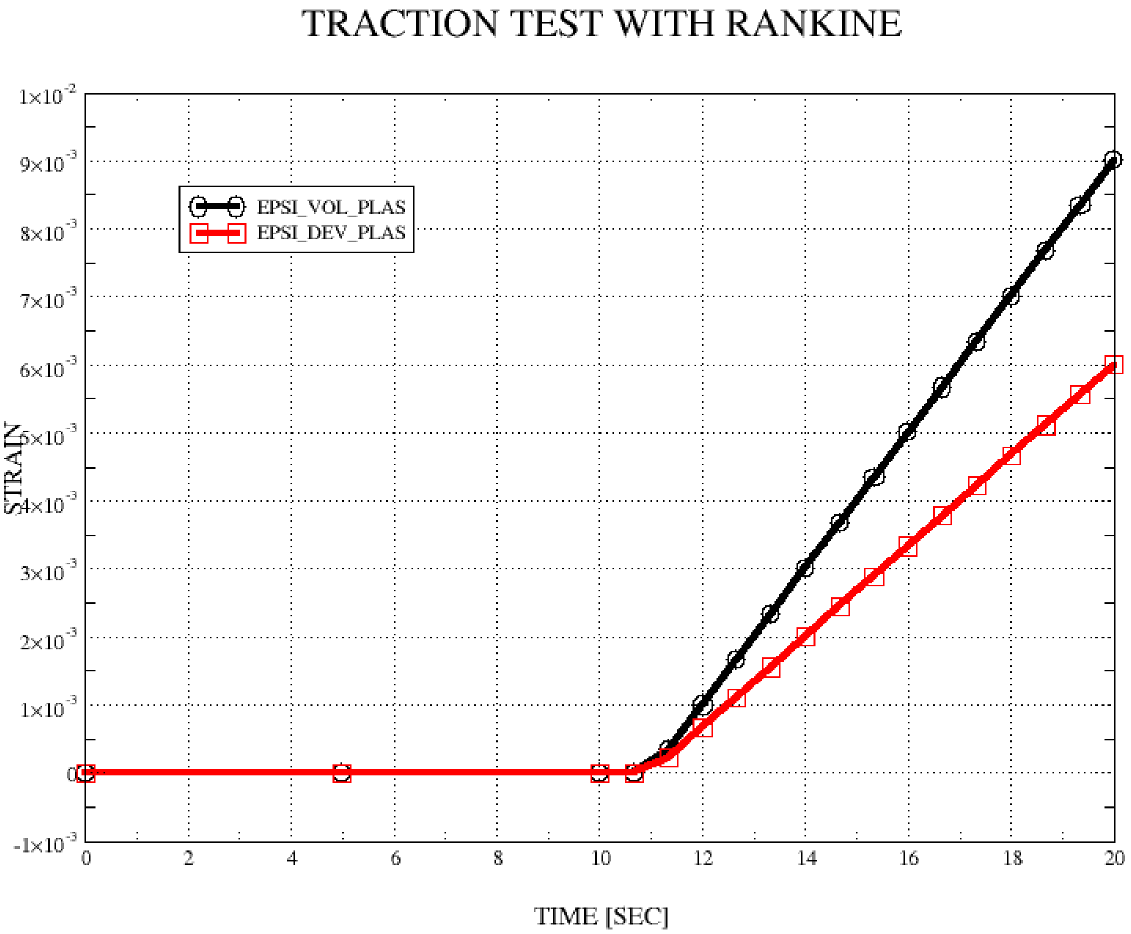


Figure **3** : *Evolution of the internal variables during the tensile test*