SSND105 - Law of behavior visco-élasto-plastic with effect of memory

Summary:

The problem is quasi-static non-linear in mechanics of the structures. Laws tested, VMIS_CIN2_MEMO and VISC_CIN2_MEMO, are laws with non-linear kinematic work hardening, isotropic work hardening, and memory of maximum work hardening. One analyzes the answer in a material point, with a pre-work hardening, then a cyclic loading.

Modeling A makes it possible to validate the effect of memory with VMIS_CIN2_MEMO in a case where work hardening is purely isotropic, for a simple traction. The reference solution for this modeling is analytical.

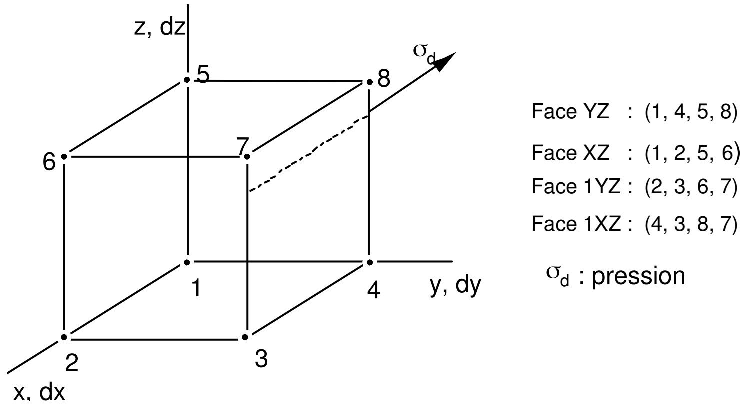
Modeling B compares the results got with effect of memory, and without effect of memory between the laws VISC_CIN2_MEMO and VISCOCHAB, for a cyclic loading with pre-work hardening.

Modeling C is similar to modeling B, and makes it possible to validate the two models into axisymmetric.

Modeling D is similar to modeling C, and makes it possible to check that the models being able to take into account the effect of nonproportionality give in this case results identical to the preceding models.

1 Problem of reference

1.1 Geometry



1.2 Properties of materials

Isotropic elasticity $E=145\,000\,MPa$ $\nu=0.3$

Elastoplasticity with effect of memory (modeling A): model `VISC_CIN2_MEMO`

Isotropic work hardening

R_0	$35\,MPa$	B	12

Memory

DRIVEN	19	Q_0	140MPa
ETA	0.5	Q_M	460MPa

Kinematic work hardening (modeling A)

C1	0	G1_0	0
C2	0	G2_0	0

Viscoplasticity with effect of memory (modelings B and C): model `VISC_CIN2_MEMO`
Parameters identical to the preceding values, except:

LEMAITRE

UN_SUR_K	$1/70(MPa\,S^{1/N})^{-1}$ =0.0142857	NR	24

Kinematic work hardening (modeling B)

C1	1950 MPa	G1_0	50
C2	65000 MPa	G2_0	1300

Model viscoplasticity `VISCOCHAB` (modelings B and C)

k	$35\,MPa$	B	12	ETA	0.5	C2	$65000\,MPa$
A_K	0	M_R	1	C1	$1950\,MPa$	M_2	1
A_R	1	G_R	0	M_1	1	D2	1
K_0	$70\,MPa\,S^{1/N}$	DRIVEN	19	D1	1	G_X2	0
NR	24	Q_M	460	G_X1	0	G2_0	$1300\,MPa$
ALP	0 MPa	Q_0	40 MPa	G1_0	$50\,MPa$	A_I	1

	QR_0	200 MPa	

1.3 Boundary conditions and loadings

$N6 \quad dy=dz=0$
$N2 \quad dy=0$
$FACE1YZ \quad dx=0$

Traction (modeling A): $FACEYZ \quad Fx=-0.25\times coef \qquad Coef=120$ for $t=8s$.

Pre-work hardening (modeling B) $FACEYZ \quad Sxx=250\,MPa\times coef2 \qquad Sxx=250\,Mpa\times coef2$

$coef\ 2=1$ for $t=10s$, then discharge ($coef2=0$) for $t=11s$.

From $11s$, 20 cycles in imposed deformation (+ 0.5%)

2 Reference solution

2.1 Method of calculating used for the reference solution

One can calculate the analytical solution corresponding to pre-work hardening (traction monotonous, modeling A):

The system of equations of the problem with effect of memory is written (20 equations) [R5.03.04]:

Elasticity : $\tilde{\sigma}=2\mu(\tilde{\varepsilon}-\varepsilon^p)$

Criterion of plasticity $\left(\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha}_1-\frac{2}{3}C_2\boldsymbol{\alpha}_2\right)_{eq}=R_0+R(p)$

Plastic flow: $\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}=\dot{p}\,\mathbf{n}$ with $\mathbf{n}=\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha}_1-\frac{2}{3}C_2\boldsymbol{\alpha}_2}{\left(\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha}_1-\frac{2}{3}C_2\boldsymbol{\alpha}_2\right)_{eq}}$

Isotropic work hardening : $\dot{R}=b(Q-R)\dot{p}$

Memory of maximum work hardening: $Q=Q_0+(Q_m-Q_0)(1-e^{-2\mu q})$

where Q is determined by:

•a field $F(\varepsilon^p,\xi,q)=\frac{2}{3}J_2(\varepsilon^p-\xi)-q\leq 0$ characterizing the maximum plastic deformations, whose Q measures the ray and ξ the center

• ξ is calculated according to a law of normality i.e.: $\dot{\xi}=\dot{q}\mathbf{n}^*$, with $\mathbf{n}^*=\frac{3}{2}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\xi}{J_2(\boldsymbol{\varepsilon}^{\mathbf{p}}-\xi)}$

On the surface of the field of maximum work hardening, one has $F=0$. By applying the condition $dF=0$, one obtains the expression of speed: $\dot{q}=\eta\langle\mathbf{n}:\mathbf{n}^*\rangle\dot{p}$

For a material point in uniaxial load, the fields (uniform) have as components:

$$\boldsymbol{\sigma}=\sigma\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\qquad \boldsymbol{\varepsilon}^{\mathbf{p}}=p\begin{bmatrix}1 & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & -\frac{1}{2}\end{bmatrix}$$

In this case, at the time of the first uniaxial load in direction X:

$$\boldsymbol{\xi}^{-}=0$$
$$q^{-}=0$$
$$\Delta q=\eta\varepsilon_x{}^p$$

In this case, $q=\frac{1}{2}\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}{}_{\max}$, implies that $\eta=\frac{1}{2}$. In this case, $\Delta\boldsymbol{\xi}=\frac{1}{2}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}\right)$

Moreover, in the case of a cycle of symmetrical traction compression (in plastic deformation), one obtains, during the first symmetrical discharge (with $\eta=\frac{1}{2}$):

$$\boldsymbol{\xi}^{-}=\frac{1}{2}\varepsilon^{p}{}_{\max}$$
$$q^{-}=\frac{1}{2}\varepsilon^{p}_{xx\max}$$
$$\Delta q=\eta\left(\frac{2}{3}J_2\left(\varepsilon^p\right)-q^{-}\right)=\eta\left(\left|\varepsilon^p_{xx\min}-\xi^{-}\right|-\frac{1}{2}\varepsilon^p_{xx\max}\right)=\frac{1}{2}\left|\varepsilon^p_{xx\min}\right|$$

$$q=q^{-}+\Delta q=\varepsilon^p_{xx\max}=\frac{1}{2}\Delta\varepsilon^p_{xx}$$
$$\Delta\xi=\frac{(1-\eta)\Delta q\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}^{-}\right)}{\eta q^{-}+\Delta q}=-\frac{1}{2}\Delta\varepsilon^p_{xx\max}$$

$\xi=\xi^0+\Delta\xi=0$ what corresponds well to the expected result (cf [bib2]): field $F=0$ centered on the origin, and of ray the half-amplitude of plastic deformation.

In the case of an increasing traction, and if kinematic work hardening is neglected, the equations to be solved become:

$$\sigma\leq R_0+R(p)$$

The function thus should be calculated $R(p)$, such as:

$$dR=b(Q-R)dp \qquad \text{with } Q=Q_0+(Q_m-Q_0)\left(1-e^{-2\mu q}\right).$$

Moreover, it is considered that one is in load, therefore $F\left(\varepsilon^p,\xi,q\right)=0$

$$dq=\eta dp$$

The differential equation thus should be integrated:

$$dR = b\left(Q_0 + (Q_m - Q_0)\left(1 - e^{-2\mu q}\right) - R\right)dp$$

what is integrated in the following way:

$$dR + bR = 0 \Rightarrow R = \lambda e^{-bp}$$

Method of variation of the constant: $R = \lambda(p)e^{-bp}$

$$d\lambda e^{-bp} = b\left(Q_m - (Q_m - Q_0)e^{-2\mu\eta p}\right)dp$$

$$d\lambda = bQ_m e^{bp}dp + b(Q_0 - Q_m)e^{(b-2\mu\eta)p}dp$$

while integrating:

$$\lambda = Q_m e^{bp} + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)}e^{(b-2\mu\eta)p} + K$$

from where

$$R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)}e^{-2\mu\eta p} + Ke^{-bp}$$

The constant K is defined by the initial conditions: for $p=0$, $R=0$

$$0 = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)} + K \text{ that is to say } K = \frac{b(Q_m - Q_0)}{(b - 2\mu\eta)} - Q_m = \frac{-bQ_0 + 2\mu\eta Q_m}{(b - 2\mu\eta)}$$

Finally:

$$R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)}e^{-2\mu\eta p} + \frac{2\mu\eta Q_m - bQ_0}{(b - 2\mu\eta)}e^{-bp}$$

One thus has in load: $\sigma = R_0 + R(p)$

2.2 Results of reference

Modeling a:

Value of $SIXX$ at the final moment: $\sigma = R_0 + R(p)$

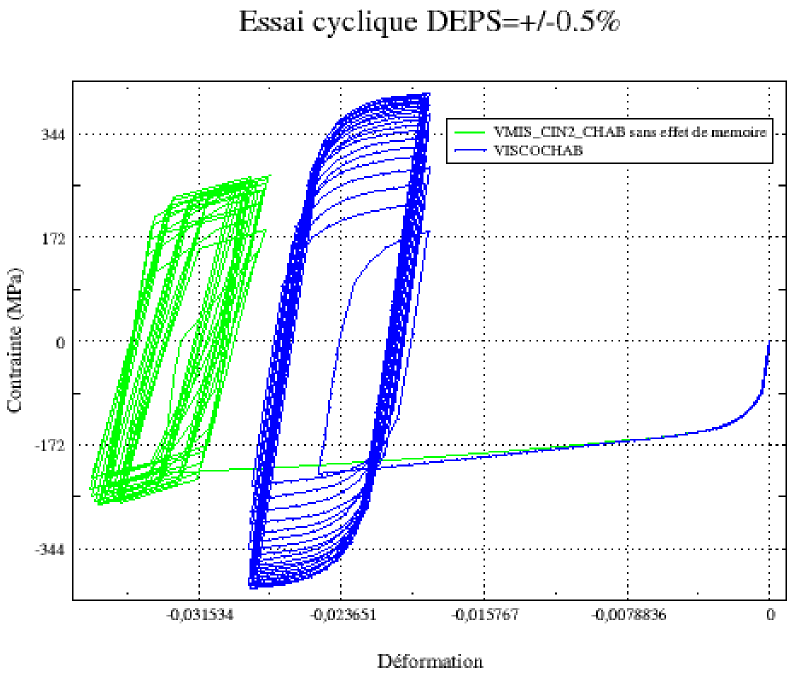
with $R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)}e^{-2\mu\eta p} + \frac{2\mu\eta Q_m - bQ_0}{(b - 2\mu\eta)}e^{-bp}$

$t=8\text{s}$, one must find $SIXX = 120\,Mpa$.

For that one calculation $R(p)$ starting from the value of p at the moment $t=8\text{s}$.

Modeling b:

One will compare the results obtained with VISC_CIN2_MEMO with those obtained with VISCOCHAB, at the end of pre-work hardening and at the end of 10 cycles. The curves below highlight of the effect of memory (per comparison with VISC_CIN2_CHAB who does not model it): after a pre-work hardening, the cycles with imposed deformation are stabilized with an amplitude of constraints higher than that obtained without effect of memory:



2.3 Uncertainty on the solution

- Analytical modeling a:
- Modeling b: intercomparison enters VISCOCHAB and VISC_CIN2_MEMO : precision of the digital integration, estimated at less 1% .
- Modeling C: validation ddes behaviors in 2D AXIS ; the results must be identical to those of modeling B.

2.4 Bibliographical references

[1] R5.03.04 “Behaviors élasto-visco-plastics of J.L.Chaboche”.

[2] J.M.PROIX “ Comportement viscoplastic fascinating in account not proportionality of the loading” EDF R & D - CR-AMA12-284, 12/12/12

3 Modeling A

3.1 Characteristics of modeling

Modeling 3D, 1 hexa8. Simple traction.

3.2 Sizes tested and results

Identification	Reference	tolerance
σ_{xx}	120	0,20%
p	$3.70925\,E-2$	0,10%

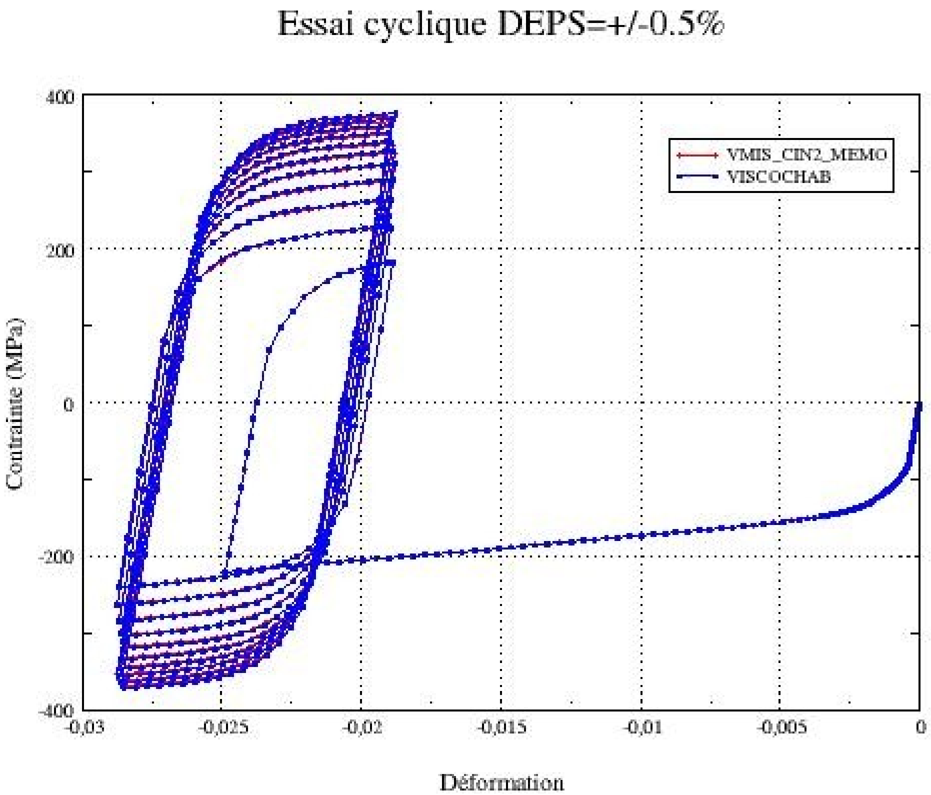
4 Modeling B

4.1 Characteristics of modeling

Pre-work hardening in traction then cycles with imposed deformation, comparison VISCOCHAB and VISC_CIN2_MEMO. 250 pas de time for 10 cycles.

4.2 Sizes tested and results

Identification	Moment	VISCOCHAB	VISC_CIN2_MEMO	$\%$ difference
σ_{xx}	10	220	220	0
σ_{xx}	11	0	0	0
σ_{xx}	113.5	$3.75459E+02$	$3.72353E+02$	-0.8
ε_{xx}	113.5	$-1.87638E-02$	$-1.87638E-02$	0



4.3 Remarks

The difference of 0.8% on the constraints at the final moment grows blurred if the step of time is refined: with a step of time 2 times finer, the variation become 0.4% .

5 Modeling C

5.1 Characteristics of modeling

Pre-work hardening in traction then cycles with imposed deformation, comparison VISCOCHAB and VISC_CIN2_MEMO. 250 pas de time for 10 cycles. Modeling 2D AXIS.

5.2 Sizes tested and results

Identification	Moment	VISCOCHAB	VISC_CIN2_MEMO	% difference
σ_{xx}	113.5	3.75459E+02	3.72353E+02	−0.8
ε_{xx}	113.5	−1.87638E−02	−1.87638E−02	

6 Modeling D

6.1 Characteristics of modeling

This modeling is identical to modeling C, with models of the type NRAD (not radiality). Results of the models VISC_MEMO_NRAD and VISC_ CIN2 _NRAD can be compared with those of modeling C, since the effect of nonradiality must be inoperative here. Tests of VMIS_MEMO_NRAD, VMIS_CIN2_NRAD (without viscosity) are of nonregression.

6.2 Sizes tested and results

Behavior VISC_MEMO_NRAD

Identification	Moment	Reference VISC_CIN2_MEMO
σ_{xx}	113.5	369.679
ε_{xx}	113.5	−1.8773E−02

Behavior VISC_CIN2_NRAD

Identification	Moment	Reference VISC_CIN2_CHAB
σ_{xx}	113.5	269.6
σ_{xx}	10	220

Behavior VMIS_MEMO_NRAD

Identification	Moment	Reference
σ_{xx}	113.5	372.2 (not regression)
σ_{xx}	10	220 (analytical)

Behavior VMIS_CIN2_NRAD

Identification	Moment	Reference VISC_CIN2_MEMO
σ_{xx}	113.5	225.254 (not regression)

σ_{xx}	10	220 (analytical)

7 Summary of the results

Four modelings make it possible to validate, on a material point, the behaviors of the kinematic type nonlinear for purpose of memory, in plasticity and viscoplasticity.