HPLA100 - Heavy thermoelastic hollow roll in uniform rotation

Summary

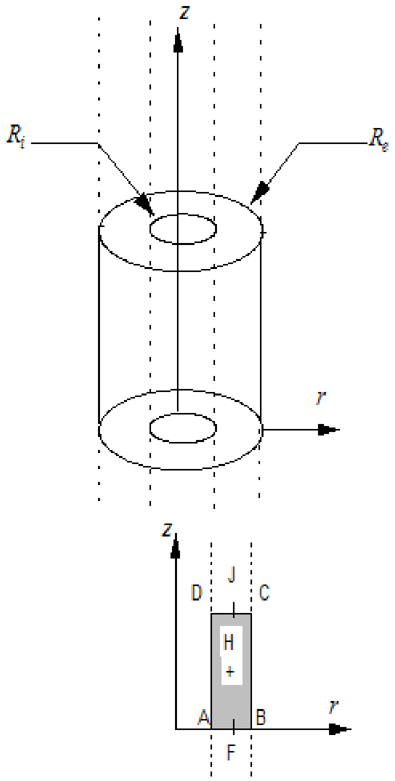
The purpose of this test is to test the second members corresponding to the effects of gravity, a thermal dilation and acceleration due to a uniform rotation. For modelings C and D (hull $3\mathrm{D}$), a chained thermoelastic calculation and a thermoelastoplastic calculation without plastic evolution were carried out.

One has the results for modelings:

- $2\mathrm{D}$ axisymmetric: axisymmetric isoparametric finite elements on meshs `QUAD8`,
- axisymmetric hulls: axisymmetric isoparametric finite elements on meshs `SEG3` (linear grid of the meridian section),
- hulls $3\mathrm{D}$: finite elements `MEC3QU9H`, `MEC3TR7H` on meshs `QUAD9` and `TRIA7`, respectively,
- plates DKT : finite elements plans `DKQ`, `DKT` on meshs `QUAD4` and `TRIA3`, respectively. One tests also the orthotropic hulls of `DEFI_COMPOSITE` for two layers of the same isotropic material and the keyword `ELAS_COQU` of `DEFI_MATERIAU` for the assignment of characteristics of homogenized plates.

1 Problem of reference

1.1 Geometry



Geometrical of the cylinder (m) :

Interior ray	$R_i = 19.5$
External ray	$R_e = 20.5$
Not F	$R = 20.0$
Thickness	$h = 1.0$
Height	$L = 10.0$

1.2 Properties of material

The material is homogeneous isotropic, thermoelastic linear, the initial state is virgin.

Young modulus	$E = 2.0\,E - 5\,N.mm^{-2}$
Poisson's ratio	$\nu = 0.3$
Voluminal density	$\rho = 8.0\,E^{-6}\,kg.mm^{-3}$
Dilation coefficient	$\alpha = 1.0\,E^{-5}\,°C^{-1}$

1.3 Boundary conditions and loadings

- Imposed displacement:
  - $\Omega=1.0\,s^{-1}$ according to the axis OZ
- Imposed loading:
  - gravity, $g=10.0\,m.s^{-2}$ according to the axis OZ
  - force of traction on the higher face: $-160.0\,E^{-4}\,N$ equivalent with a force distributed on CD of $-8.0\,E^{-4}\,N.mm^{-1}$
- Thermal dilation: $T(\rho)-T_{ref}(\rho)=\frac{(T_s+T_i)}{2}+\frac{(T_s-T_i).(r-R)}{h}$ with:
  - case 1: $T_s=0.5\,°C, T_i=-0.5\,°C, T_{ref}=0.0\,°C$
  - case 2: $T_s=0.1\,°C, T_i=0.1\,°C, T_{ref}=0.0\,°C$

These fields of temperature are calculated with THER_LINEAIRE, using a stationary calculation on the same grid, but with a model PLAN in order to have a solution refines in the thickness.

The boundary conditions in displacement (and rotation) being different according to modeling considered, they will be described later on (in the paragraphs relating to modelings).

2 Reference solution

2.1 Method of calculating used for the reference solution

The method of calculating used for the reference solution was determined by F. Voldoire (EDF R & D/AMA) and is presented in the appendix.

The analytical results of reference are:

- displacements and rotations,
- axial stress, generalized efforts (in theory of hulls),

in skins internal and external on the sections AB and CD.

In 2D axisymmetric, the total solutions given in appendix are such as $\varepsilon_{rz}=0$ where r and z are the directions radial and axial cylinder, respectively. For the loadings of uniform rotation and thermal dilation, the boundary conditions are selected so that the solutions do not depend on z (one has in particular $\varepsilon_{zz}=0$).

For the hulls, with boundary conditions equivalent, rotation θ_θ around the axis orthoradial is worthless for the loadings of uniform rotation and thermal dilation, which is not the case of the loading of gravity where rotation is constant (the cylinder is formatted conical then). On the other hand in all the cases, the transverse distortion is worthless; thus the theories of Coils - Kirchhoff and those of Hencky-Mindlin provide the same reference solution.

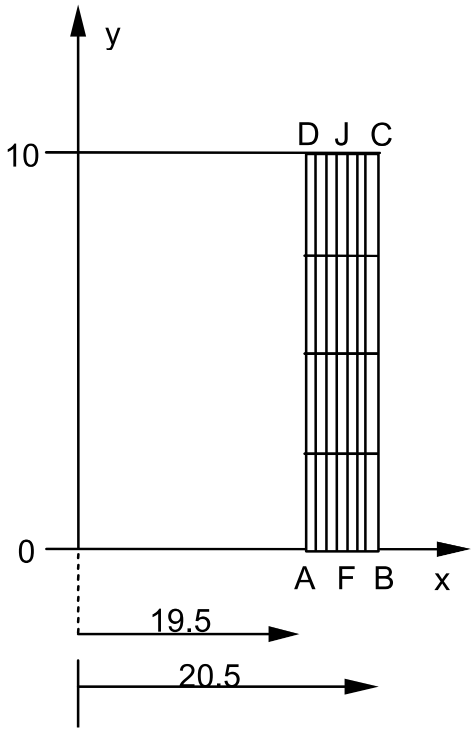
2.2 Reference variable

DX : displacement according to the axis OX,
DY : displacement according to the axis OY,
DZ : displacement according to the axis OZ,
NXX : normal effort according to the axis OX,
NYY : normal effort according to the axis OY,
MXX : moment around the axis OX,
MYY : moment around the axis OY,
$SIXX$: constraint according to the axis OX,
$SIYY$: constraint according to the axis OY.

3 Modeling A

3.1 Characteristics of modeling

Finite elements 2D axisymmetric



The discretized geometry is represented above:

Bord	*group _ no*
BC	*BC*
DA	*DA*
AB	*BAS*
CD	*HAUT*

3.2 Characteristics of the grid

The grid is regular: 4 elements in the height, 8 in the thickness.

Many nodes: 121

Number of meshs and type: 32 QUAD8

3.3 Boundary conditions in displacement

3.3.1 Gravity

Displacement DY is blocked at the point F only.

3.3.2 Rotation

Displacement DY is blocked on the sides $[AB]$ (GROUP_NO = 'LOW') and on $[CD]$ (GROUP_NO = 'HIGH').

3.3.3 Thermal case of dilation n°1

Displacement DY is blocked on all the structure.

3.3.4 Thermal case of dilation n°2

Displacement DY is blocked on the sides $[AB]$ (`GROUP_NO = 'LOW'`) and $[CD]$ (`GROUP_NO = 'HIGH'`).

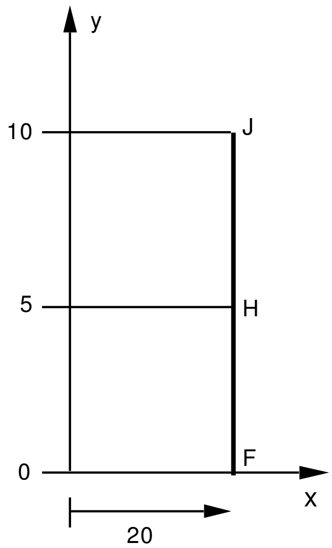
3.4 Results of modeling A

Identification	Node (Mesh)	Value tested	Reference
Gravity	$N78$	DX (mm)	– 2.34000 10^{-8}
	$N120$	DY (mm)	– 1,185 10^{-9}
	$N13$	DY (mm)	1.2150 10^{-9}
	$N78(M13)$	$SIYY$ (MPa)	8.0000 10^{-4}
Uniform rotation - centrifugal force	$N120$	DX (mm)	– 2.94240 10^{-7}
	$N13$	DX (mm)	2.88010 10^{-7}
	$N120(M1)$	$SIYY$ (MPa)	9.94880 10^{-4}
	$N13(M32)$	$SIYY$ (MPa)	9.26310 10^{-4}
Dilation case 1	$N120$	DX (mm)	1.056145 10^{-6}
	$N13$	DX (mm)	1.110317 10^{-6}
	$N120(M1)$	$SIYY$ (MPa)	1.4321427
Dilation case 2	$N120$	DX (mm)	2.53500 10^{-5}
	$N120(M1)$	$SIYY$ (MPa)	– 2.00000 10^{-1}

4 Modeling B

4.1 Characteristics of modeling

Elements of axisymmetric hull



The discretized geometry is represented above. One chooses the theory of hulls of Coils - Kirchhoff (for that one takes a transverse coefficient of shearing of 10^6). One neglects the correction of metric in the thickness. The thickness is of $1\,mm$.

node	GROUP_NO
J	$GRNO13$
H	$GRNO14$
F	$GRNO6$

4.2 Characteristics of the grid

Many nodes: 21

Number of meshs and type: 10 SEG3

4.3 Boundary conditions in displacement and rotation

4.3.1 Gravity

Displacement DY is blocked at the point F only ($GRNO6$).

4.3.2 Rotation

Displacement DY is blocked at the point F ($GRNO6$) at the point J ($GRNO13$).
Rotation around the axis Z is worthless in these two points.

4.3.3 Thermal case of dilation n°1

Displacement DY as well as rotation around the axis Z are blocked on all the structure (GROUP_NO = 'GRNO15').

4.3.4 Thermal case of dilation n°2

Displacement DY is blocked at the point F (`GROUP_NO = 'GRNO06'`) and at the point J (`GROUP_NO = 'GRNO13'`). Rotation around the axis Z is worthless on the same groups of nodes.

4.4 Results of modeling B

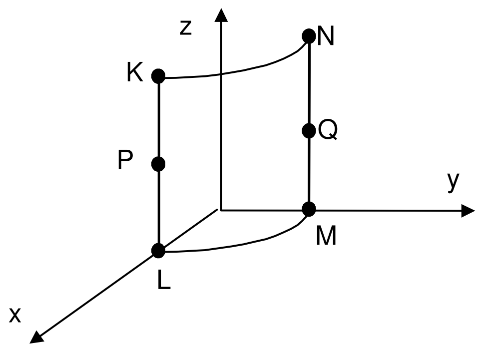
Identification	Node (Mesh)	Value tested	Reference
Gravity	J	$DX(mm)$	$-2.40000\ 10^{-8}$
	H	$DY(mm)$	$5.00000\ 10^{-9}$
	H	DRZ	$2.40000\ 10^{-9}$
	$J(M10)$	$NXX(N)$	$8.00000\ 10^{-4}$
	$J(M10)$ Internal skin	$SIXX(MPa)$	$8.00000\ 10^{-4}$
Rotation - centrifugal force	F	$DX(mm)$	$2.91200\ 10^{-7}$
	$F(M1)$	$NXX(N)$	$9.60000\ 10^{-4}$
	$F(M1)$ Internal skin	$SIXX(MPa)$	$9.60000\ 10^{-4}$
Dilation case 1	$F(M1)$	$MXX(N.mm)$	$-2.38095\ 10^{-1}$
	$F(M1)$ Internal skin	$SIXX(Mpa)$	1.428571
Dilation case 2	F	$DX(mm)$	$26.0000\ 10^{-6}$
	$F(M1)$	$NXX(N)$	$-2.00000\ 10^{-1}$
	$F(M1)$ Internal skin	$SIXX(MPa)$	$-2.00000\ 10^{-1}$

5 Modeling C

5.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of hull 3D: QUAD9



The discretized geometry is represented above.

point	*nœud*
K	*NO72*
L	*NO1*
M	*NO33*
N	*NO39*
P	*NO186*
Q	*NO190*

The factor of correction of shearing A_{CIS} is worth $5/6$ (theory of hulls of Reissner).

5.2 Characteristics of the grid

Many external nodes: 121

Many meshs and types: 32 QUAD9 + 8 SEG3

5.3 Boundary conditions in displacement and rotation

5.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.

5.3.2 Rotation

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

5.3.3 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

5.3.4 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

5.4 Results of modeling C

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX\ (mm)$	$-2.40000\ 10^{-8}$
	N	$DY\ (mm)$	$-2.40000\ 10^{-8}$
	P	$DZ\ (mm)$	$5.00000\ 10^{-9}$
	Q	$DZ\ (mm)$	$5.00000\ 10^{-9}$
	P	DRY	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(M4)$	$NYY\ (N)$	$8.00000\ 10^{-4}$
	$N(M32)$	$NYY\ (N)$	$8.00000\ 10^{-4}$
	$K(M4)$ Internal skin	$SIYY\ (MPa)$	$8.00000\ 10^{-4}$
	$N(M32)$ Internal skin	$SIYY\ (MPa)$	$8.00000\ 10^{-4}$
Rotation – centrifugal force	L	$DX\ (mm)$	$2.91200\ 10^{-7}$
	M	$DY\ (mm)$	$2.91200\ 10^{-7}$
	$L(M1)$	$NYY\ (N)$	$9.60000\ 10^{-4}$
	$M(M29)$	$NYY\ (N)$	$9.60000\ 10^{-4}$
	$L(M1)$ Internal skin	$SIYY\ (MPa)$	$9.84600\ 10^{-4}$
	$M(M29)$ Internal skin	$SIYY\ (MPa)$	$9.84600\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY\ (N.mm)$	$-2.38095\ 10^{-1}$
	$M(M29)$	$MYY\ (N.mm)$	$-2.38095\ 10^{-1}$
	$L(M1)$ Internal skin	$SIYY\ (MPa)$	1.428571
	$M(M29)$ Internal skin	$SIYY\ (MPa)$	1.428571
Dilation case 2	L	$DX\ (mm)$	$25.9946\ 10^{-6}$
	M	$DY\ (mm)$	$25.9946\ 10^{-6}$
	$L(M1)$	$NYY\ (N)$	$-2.00000\ 10^{-1}$
	$M(M29)$	$NYY\ (N)$	$-2.00000\ 10^{-1}$
	$L(M1)$ Internal skin	$SIYY\ (MPa)$	$-1.97800\ 10^{-1}$
	$M(M29)$ Internal skin	$SIYY\ (MPa)$	$-2.97800\ 10^{-1}$

5.5 Remarks

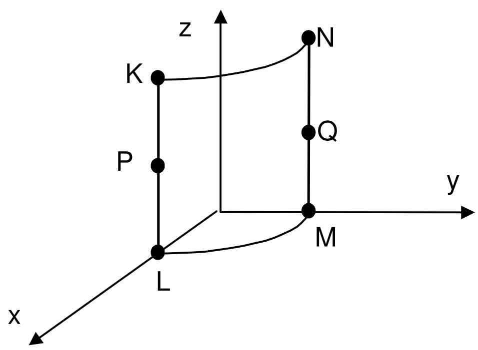
Satisfactory and identical results for calculations with `MECA_STATIQUE` and `STAT_NON_LINE`.

6 Modeling D

6.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of hull 3D: TRIA7



The discretized geometry is represented above.

point	*nœud*
K	$NO72$
L	$NO1$
M	$NO33$
N	$NO39$
P	$NO186$
Q	$NO190$

The factor of correction of shearing A_{CIS} is worth $5/6$ (theory of Reissner hulls).

6.2 Characteristics of the grid

Many external nodes: 153

Many meshs and types: 64 TRIA7 + 8 SEG3

6.3 Boundary conditions in displacement and rotation

6.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.

6.3.2 Rotation

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.

Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

6.3.3 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

6.3.4 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

6.4 Values tested

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10^{-8}
	N	$DY(mm)$	– 2.40000 10^{-8}
	P	$DZ(mm)$	5.00000 10^{-9}
	Q	$DZ(mm)$	5.00000 10^{-9}
	P	$-DRY$	2.40000 10^{-9}
	Q	DRX	2.40000 10^{-9}
	$K(M60)$	$NYY(N)$	8.00000 10^{-4}
	$N(M56)$	$NYY(N)$	8.00000 10^{-4}
	$K(M60)$ Internal skin	$SIYY(MPa)$	8.00000 10^{-4}
	$N(M56)$ Internal skin	$SIYY(MPa)$	8.00000 10^{-4}
Rotation – centrifugal force	L	$DX(mm)$	2.91200 10^{-7}
	M	$DY(mm)$	2.91200 10^{-7}
	$L(M25)$	$NYY(N)$	9.60000 10^{-4}
	$M(M53)$	$NYY(N)$	9.60000 10^{-4}
	$L(M25)$ Internal skin	$SIYY(MPa)$	9.84600 10^{-4}
	$M(M53)$ Internal skin	$SIYY(MPa)$	9.84600 10^{-4}
Dilation case 1	$L(M25)$	$MYY(N.mm)$	– 2.38095 10^{-1}
	$M(M53)$	$MYY(N.mm)$	– 2.38095 10^{-1}
	$L(M25)$ Internal skin	$SIYY(MPa)$	1.428571
	$M(M53)$ Internal skin	$SIYY(MPa)$	1.428571
Dilation case 2	L	$DX(mm)$	26.0000 10^{-6}
	M	$DY(mm)$	26.0000 10^{-6}
	$L(M25)$	$NYY(N)$	– 2.00000 10^{-1}
	$M(M53)$	$NYY(N)$	– 2.00000 10^{-1}
	$L(M25)$ Internal skin	$SIYY(MPa)$	– 1.97800 10^{-1}
	$M(M53)$ Internal skin	$SIYY(MPa)$	– 1.97800 10^{-1}

6.5 Remarks

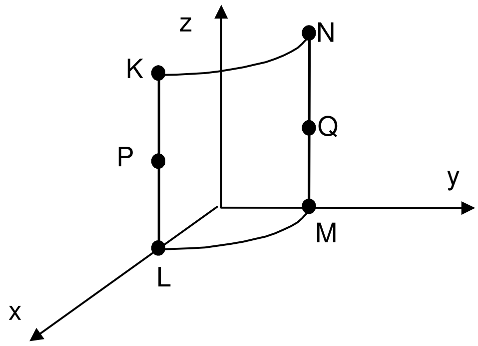
Satisfactory and identical results for calculations with `MECA_STATIQUE` and `STAT_NON_LINE`.

7 Modeling E

7.1 Characteristics of modeling

Modeling of one quarter of cylinder.

Elements of plate DKQ : QUAD4



The discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

7.2 Characteristics of the grid

Many nodes: 231

Many meshs and types: 200 QUAD4 + 80 SEG2

7.3 Boundary conditions in displacement and rotation

7.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .

7.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .

Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

7.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

7.4 Results of modeling E

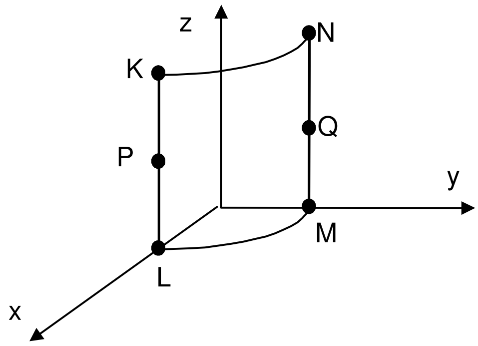
Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	$-2.40000\ 10^{-8}$
	N	$DY(mm)$	$-2.40000\ 10^{-8}$
	P	$DZ(mm)$	$5.00000\ 10^{-9}$
	Q	$DZ(mm)$	$5.00000\ 10^{-9}$
	P	$-DRY$	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(181)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$N(M200)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$K(M181)$ Internal skin	$SIYY(MPa)$	$8.00000\ 10^{-4}$
	$N(M200)$ Internal skin	$SIYY(MPa)$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$M(M20)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$L(M1)$ Internal skin	$SIYY(MPa)$	1.428571
	$M(M20)$ Internal skin	$SIYY(MPa)$	1.428571
Dilation case 2	L	$DX(mm)$	$26.0000\ 10^{-6}$
	M	$DY(mm)$	$26.0000\ 10^{-6}$
	$L(M1)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$M(M20)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$L(M1)$ Internal skin	$SIYY(MPa)$	$-2.00000\ 10^{-1}$
	$M(M20)$ Internal skin	$SIYY(MPa)$	$-2.00000\ 10^{-1}$

8 Modeling F

8.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKT : TRIA3



The discretized geometry is represented above.

not	node
K	$NO1$
L	$NO11$
M	$NO161$
N	$NO227$
P	$NO6$
Q	$NO215$

8.2 Characteristics of the grid

Many nodes: 231

Number of meshs and type: 400 TRIA3 + 80 SEG2

8.3 Boundary conditions in displacement and rotation

8.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.

8.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.

Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

8.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

8.4 Results of modeling F

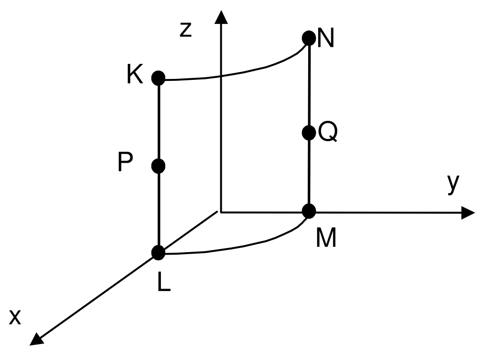
Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	$-2.40000\ 10^{-8}$
	N	$DY(mm)$	$-2.40000\ 10^{-8}$
	P	$DZ(mm)$	$5.00000\ 10^{-9}$
	Q	$DZ(mm)$	$5.00000\ 10^{-9}$
	$-DRX$	$-DRY$	$2.40000\ 10^{-9}$
	DRX	DRX	$2.40000\ 10^{-9}$
	$K(M362)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$N(M400)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$L(M362)$ Internal skin	$SIYY(MPa)$	$8.00000\ 10^{-4}$
	$N(M400)$ Internal skin	$SIYY(MPa)$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$M(M39)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$L(M1)$ Internal skin	$SIXX(MPa)$	1.428571
	$M(M39)$ Internal skin	$SIXX(MPa)$	1.428571
Dilation case 2	L	$DX(mm)$	$26.0000\ 10^{-6}$
	M	$DY(mm)$	$26.0000\ 10^{-6}$
	$L(M1)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$M(M39)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$L(M1)$ Internal skin	$SIYY(MPa)$	$-2.00000\ 10^{-1}$
	$M(M39)$ Internal skin	$SIYY(MPa)$	$-2.00000\ 10^{-1}$

9 Modeling G

9.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKT : TRIA3. Modeling uses a double-layered plate whose characteristics of orthotropism are those of material defined in $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO1$
L	$NO11$
M	$NO161$
N	$NO227$
P	$NO6$
Q	$NO215$

9.2 Characteristics of the grid

Many nodes: 231

Many meshs and types: 400 TRIA3 + 80 SEG2

9.3 Boundary conditions in displacement and rotation

9.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .

9.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .

Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

9.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

9.4 Results of modeling G

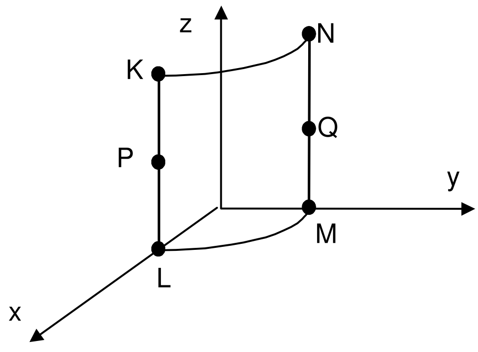
Identification	Node (mesh)	Value tested	Reference
Gravity	K	DX	$-2.40000\ 10^{-8}$
	N	DY	$-2.40000\ 10^{-8}$
	P	DZ	$5.0\ 10^{-9}$
	Q	DZ	$5.0\ 10^{-9}$
	P	$-DRY$	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(M362)$	NYY	$8.00000\ 10^{-4}$
	$N(M400)$	NYY	$8.00000\ 10^{-4}$
	$K(M362)$	$SIYY$	$8.00000\ 10^{-4}$
	$N(M400)$	$SIYY$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	MYY	$-2.38095\ 10^{-1}$
	$M(M39)$	MYY	$-2.38095\ 10^{-1}$
	$L(M1)$, internal skin	$SIXX$	1.428571
	$L(M1)$, external skin	$SIXX$	– 1.428571
	$M(M39)$, internal skin	$SIXX$	1.428571
	$M(M39)$, external skin	$SIXX$	– 1.428571
Dilation case 2	L	DX	$26.0\ 10^{-6}$
	M	DY	$26.0\ 10^{-6}$
	$L(M1)$	NYY	$-2.00000\ 10^{-1}$
	$M(M39)$	NYY	$-2.00000\ 10^{-1}$
	$L(M1)$	$SIYY$	$-2.00000\ 10^{-1}$
	$M(M39)$	$SIYY$	$-2.00000\ 10^{-1}$

10 Modeling H

10.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKQ : QUAD4. Modeling uses a double-layered plate whose characteristics of orthotropism are those of material defined in $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

10.2 Characteristics of the grid

Many external nodes: 231

Many meshs and types: 200 QUAD4 + 80 SEG2

10.3 Boundary conditions in displacement and rotation

10.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .

10.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

10.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

10.4 Results of modeling H

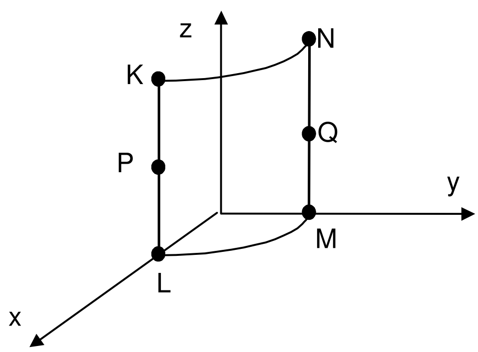
Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	$-2.40000\ 10^{-8}$
	N	$DY(mm)$	$-2.40000\ 10^{-8}$
	P	$DZ(mm)$	$5.00000\ 10^{-9}$
	Q	$DZ(mm)$	$5.00000\ 10^{-9}$
	P	$-DRX$	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(M181)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$N(M200)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$K(M181)$	$SIYY(Pa)$	$8.00000\ 10^{-4}$
	$N(M200)$	$SIYY(Pa)$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$M(M20)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$L(M1)$, internal skin	$SIYY(Pa)$	1.428571
	$L(M1)$, external skin	$SIYY(Pa)$	– 1.428571
	$M(M20)$, internal skin	$SIYY(Pa)$	1.428571
	$M(M20)$, external skin	$SIYY(Pa)$	– 1.428571
Dilation case 2	L	$DX(mm)$	$26.0000\ 10^{-6}$
	M	$DY(mm)$	$26.0000\ 10^{-6}$
	$L(M1)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$M(M20)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$L(M1)$	$SIYY(Pa)$	$-2.00000\ 10^{-1}$
	$M(M20)$	$SIYY(Pa)$	$-2.00000\ 10^{-1}$

11 Modeling I

11.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKQ : QUAD4. Assignment of characteristics of homogenized plates corresponding to material of $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

11.2 Characteristics of the grid

Many external nodes: 231

Many meshs and types: 200 QUAD4 + 80 SEG2

11.3 Boundary conditions in displacement and rotation

11.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .

11.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .

Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

11.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

11.4 Results of modeling I

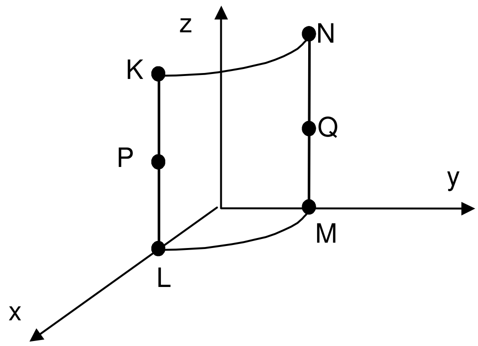
Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	$-2.40000\ 10^{-8}$
	N	$DY(mm)$	$-2.40000\ 10^{-8}$
	P	$DZ(mm)$	$5.00000\ 10^{-9}$
	Q	$DZ(mm)$	$5.00000\ 10^{-9}$
	P	$-DRX$	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(M181)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$N(M200)$	$NYY(N)$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$M(M20)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$L(M1)$, internal skin	$SIYY(Pa)$	1.428571
	$M(M20)$, internal skin	$SIYY(Pa)$	1.428571
Dilation case 2	L	$DX(mm)$	$26.0000\ 10^{-6}$
	M	$DY(mm)$	$26.0000\ 10^{-6}$
	$L(M1)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$M(M20)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$L(M1)$	$SIYY(Pa)$	$-2.00000\ 10^{-1}$
	$M(M20)$	$SIYY(Pa)$	$-2.00000\ 10^{-1}$

12 Modeling J

12.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKT : TRIA3. Assignment of characteristics of homogenized plates corresponding to material of $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO1$
L	$NO11$
M	$NO161$
N	$NO227$
P	$NO6$
Q	$NO215$

12.2 Characteristics of the grid

Many nodes: 231

Many meshs and types: 400 TRIA3 + 80 SEG2

12.3 Boundary conditions in displacement and rotation

12.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN .

12.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL .

Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

12.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

12.4 Results of modeling J

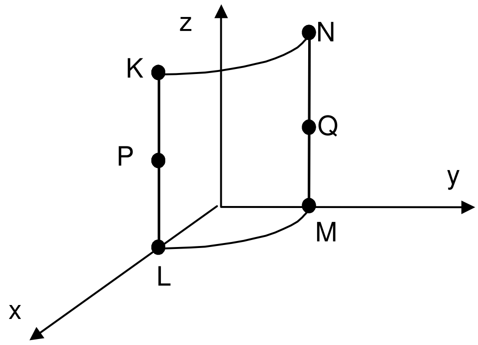
Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	$-2.40000\ 10^{-8}$
	N	$DY(mm)$	$-2.40000\ 10^{-8}$
	P	$DZ(mm)$	$5.0\ 10^{-9}$
	Q	$DZ(mm)$	$5.0\ 10^{-9}$
	P	$-DRY$	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(M362)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$N(M400)$	$NYY(N)$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$M(M39)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$L(M1)$, internal skin	$SIYY(Pa)$	1.428571
	$M(M39)$, internal skin	$SIYY(Pa)$	1.428571
Dilation case 2	L	$DX(mm)$	$26.0\ 10^{-6}$
	M	$DY(mm)$	$26.0\ 10^{-6}$
	$L(M1)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$M(M39)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$L(M1)$	$SIYY(Pa)$	$-2.00000\ 10^{-1}$
	$M(M39)$	$SIYY(Pa)$	$-2.00000\ 10^{-1}$

13 Modeling K

13.1 Characteristics of modeling

Identical to modeling E with elements DKQG (modeling DKTG).
The purpose of this modeling is to test the taking into account of the temperature for the elements DKQG.

Elements of plate DKQG : QUAD4



The discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

13.2 Characteristics of the grid

Many nodes: 231

Many meshs and types: 200 QUAD4 + 80 SEG2

13.3 Boundary conditions in displacement and rotation

13.3.1 Gravity

Displacement DZ is blocked on the group of nodes LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.

13.3.2 Thermal case of dilation n°1

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.

Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

13.3.3 Thermal case of dilation n°2

Displacement DZ as well as rotations around the axes X and Y are blocked on the groups of nodes $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the group of nodes KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the group of nodes MN.
Displacement DZ as well as rotation around the axis Y are blocked on the group of nodes $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the group of nodes $METN$.

13.4 Results of modeling K

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	$-2.40000\ 10^{-8}$
	N	$DY(mm)$	$-2.40000\ 10^{-8}$
	P	$DZ(mm)$	$5.00000\ 10^{-9}$
	Q	$DZ(mm)$	$5.00000\ 10^{-9}$
	P	$-DRY$	$2.40000\ 10^{-9}$
	Q	DRX	$2.40000\ 10^{-9}$
	$K(181)$	$NYY(N)$	$8.00000\ 10^{-4}$
	$N(M200)$	$NYY(N)$	$8.00000\ 10^{-4}$
Dilation case 1	$L(M1)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
	$M(M20)$	$MYY(N.mm)$	$-2.38095\ 10^{-1}$
Dilation case 2	L	$DX(mm)$	$26.0000\ 10^{-6}$
	M	$DY(mm)$	$26.0000\ 10^{-6}$
	$L(M1)$	$NYY(N)$	$-2.00000\ 10^{-1}$
	$M(M20)$	$NYY(N)$	$-2.00000\ 10^{-1}$

14 Summary of the results

Very good performances obtained for modelings A and B are explained by the fact that the reference solutions belong to the space generated by the selected finite elements. Only remain the digital rounding errors.

Satisfactory results were got for modelings of plates and hulls in space C, D, E, F, G, H, I, J and K. For these last, a chained thermoelastic calculation was carried out. Results with modeling DKT (E, F, G, H, I, J) show that the elements quadrangle have a better behavior than the elements triangle. It is necessary to have a sufficiently fine discretization with these elements plans in order to be able to correctly model the circular geometry of the cylindrical hull. Indeed, to discretize the geometry of the cylinder by plane or parabolic facets is not in conformity and induces a parasitic inflection which decreases with the smoothness of grid. Thus a multiplication amongst elements by two on the height of the structure makes fall the maximum error relative of 5.48% (case presented here) to 2.8%. Results with modeling COQUE_3D (C and D) are very good except for gravity with the element triangle.
Calculations with modeling DKTG (K) the same results as with modeling give DKT.

15 Appendix

15.1 Uniform loading of rotation around OZ

15.1.1 Axisymmetric model 2D

The density of centrifugal force is: $\rho \Omega^2 r e_r$.

One considers the boundary conditions following:

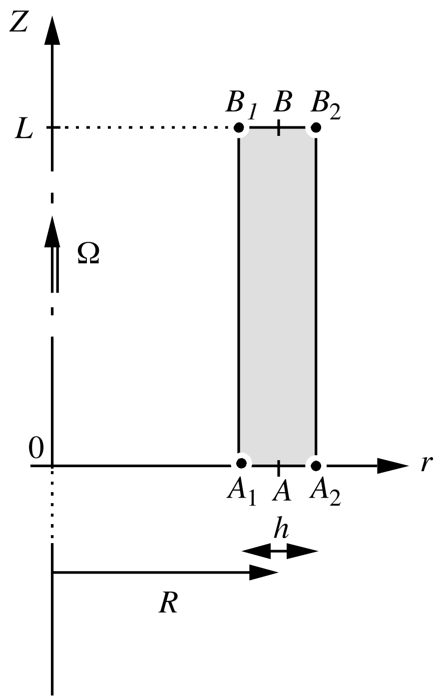
$u_z(r, z)=0$ in $z=0$ and $z=L$

One applies displacement in the form:

$u_r=u(r)$
$u_z=u_\theta=0$

As follows:

$\varepsilon_{rr}=u'$; $\varepsilon_{\theta\theta}=\frac{u}{r}$; $\varepsilon_{zz}=\varepsilon_{rz}=\varepsilon_{\theta z}=\varepsilon_{r\theta}=0$



Geometry of the hollow roll:

$R=20\,mm$
$h=1\,mm$

The elastic constraints are expressed:

$$\sigma_{rr}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)u'+\nu\frac{u}{r}]$$
$$\sigma_{\theta\theta}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{u}{r}+\nu u']$$
$$\sigma_{zz}=\frac{\nu E}{(1+\nu)(1-2\nu)}[\frac{u}{r}+u']$$

The radial equilibrium equation is written:

$$(r\sigma_{rr})_{,r}-\sigma_{\theta\theta}=-\rho\Omega^2 r^2$$

As follows:

$$(\frac{(ru)'}{r})'=\frac{-(1+\nu)(1-2\nu)}{(1-\nu)E}\rho\Omega^2 r \qquad éq\,1.1-1$$

Note:

$$\frac{u}{r}+u'=\frac{(ru)'}{r}$$

From where the general solution:

$$u(r)=\frac{-(1+\nu)(1-2\nu)}{(1-\nu)E}\rho\Omega^2\frac{r^3}{8}+Ar+\frac{B}{r} \qquad éq\,1.1-2$$

The constraints are then:

$$\sigma_{rr}(r)=\frac{-3-2\nu}{1-\nu}\rho\Omega^2\frac{r^2}{8}+\frac{E}{(1+\nu)(1-2\nu)}(A-(1-2\nu)\frac{B}{r^2})$$
$$\sigma_{\theta\theta}(r)=\frac{-1+2\nu}{1-\nu}\rho\Omega^2\frac{r^2}{8}+\frac{E}{(1+\nu)(1-2\nu)}(A-(1-2\nu)\frac{B}{r^2}) \qquad éq\,1.1-3$$
$$\sigma_{zz}(r)=\frac{-\nu}{1-\nu}\rho\Omega^2\frac{r^2}{2}+\frac{2\nu E}{(1+\nu)(1-2\nu)}A$$

The boundary conditions in constraints are:

$$\sigma_{rr}=0 \text{ in } r=R\pm\frac{h}{2}$$

One notes:

$$x=\frac{h}{2R}$$

One obtains thanks to $[éq\,1.1-3]$:

$$B=\frac{(3-2\nu)(1+\nu)}{8(1-\nu)E}\rho\,\Omega^2 R^4(1-x^2)^2$$

then:

$$A=\frac{(3-2\nu)(1+\nu)(1-2\nu)}{4(1-\nu)E}\rho\,\Omega^2 R^2(1-x^2)$$

Digital application:

$$\rho=8.10-6\ \ kg/mm^3$$
$$\Omega=1\ \ s^{-1}$$
$$E=2.105\,N/mm^2$$
$$\nu=0.3$$

From where: $A=7.13588.10^{-9}mm^2$
$B=3.561258.10^{-6}\ mm^2$

Note:

$$\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}\rho\frac{\Omega^2}{8}=3.714286\ \ E-12\ \ mm^2$$
$$\frac{\nu}{1-\nu}\rho\frac{\Omega^2}{2}=1.714286\ \ E-6\ \ MPa.mm^2$$

As follows:

- in internal skin: $\begin{cases} u_r=2.9424\,E^{-7}\,mm \\ \sigma_{zz}=0.99488\,E^{-3}\,Mpa \end{cases}$
- in external skin: $\begin{cases} u_r=2.8801\,E^{-7}\,mm \\ \sigma_{zz}=0.92631\,E^{-3}\,Mpa \end{cases}$

15.1.2 Axisymmetric model hull

The centrifugal force is equivalent to one **pressure distributed** :

$$p=\rho\,\Omega^2 hR(1+\frac{h^2}{12R^2})$$

The solution is membranous, normal balance is written:

$$N_{\theta\theta}=pR$$

The membrane deformation is: $E_{\theta\theta}=\frac{w}{R}$, whereas $E_{\theta\theta}=0=K_{\theta\theta}=K_{zz}$.

In elasticity:

$$N_{\theta\theta}=\frac{Eh}{1-\nu^2}E_{\theta\theta}\ ;\ N_{zz}=\nu\, N_{\theta\theta}\ ;\ M_{\alpha\beta}=0$$

From where the solution (marks with arrows and circumferential normal effort):

$$w=\frac{(1-\nu^2)\rho\,\Omega^2}{E}R^3(1+\frac{h^2}{12\mathrm{R}^2})\ ;\ N_{\theta\theta}=\rho\,\Omega^2 R^2 h(1+\frac{h^2}{12\mathrm{R}^2})$$

Axial stress is worth:

$$\sigma_{zz}=\nu\,\rho\,\Omega^2 R^2(1+\frac{h^2}{12\mathrm{R}^2})$$ (constant in the thickness)

If one does not take account of the correction of metric, the term should be removed $(1+\frac{h^2}{12\mathrm{R}^2})$ in the preceding expressions.

Digital application (without correction of metric) **:**

$$p=1{,}600000.10-4\,MPa$$
$$w=2{,}912000.10-7\,mm$$
$$N_{zz}=0{,}96000.10-3\,N/mm$$
$$\sigma_{zz}=0{,}96000.10-3\,MPa$$

15.2 Loading of gravity

15.2.1 Axisymmetric model 2D

The density of force is: $-\rho\, g\, e_z$ (vertical gravity).

One considers the boundary conditions following:

$U_z(r,z)=0$ in $r=R$ and $z=0$ (circle of support)

with uniform traction: $\sigma_{zz}(r,z)=\rho\, g\, L$ in $z=L$, balancing the weight.

One applies the elastic solution of the type:

$$\sigma=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

so that:

$$\varepsilon_{rr}=\varepsilon_{\theta\theta}=-\nu\,\varepsilon_{zz}=-\nu\, u_{z,z}=-\nu\frac{\sigma_{zz}}{E}\ ;\ \varepsilon_{rz}=0=\varepsilon_{r\theta}=\varepsilon_{\theta z}$$

One observes as follows:

$$u_{r,r}=\frac{u_r}{r}\ \Leftrightarrow\ u_r(r,z)=-\nu A'(z)\, r$$

Then:

$$\begin{aligned} -\nu A'(z)=\varepsilon_{rr}=-\nu\,\varepsilon_{zz}\ &\Leftrightarrow\ u_{z,z}(r,z)=A'(z)\\ &\Leftrightarrow\ u_r(r,z)=A(z)+B \end{aligned}$$

Of $\varepsilon_{rz}=0$, one draws:

$$B'(r)-\nu\, rA''(z)=0$$

that is to say:

$$A''(z)=cste=\alpha \quad ; \quad B'(r)=\alpha\,\nu\, r$$

Boundary conditions in effort, one obtains:

$$A(z)=\frac{\rho g z^2}{2E}+\beta \quad ; \quad B(r)=\nu\frac{\rho g r^2}{2E}$$

Lastly, β check: $\beta=-\nu\,\rho\,g\dfrac{R^2}{2E}$

As follows:

$$u_r(r,z)=\frac{-\nu\,\rho\,g\,z\,r}{E}\quad;\quad u_z(r,z)=\frac{\rho\,g}{2E}(z^2+\nu(r^2-R^2))\qquad éq\,2.1-1$$
$$\sigma_{zz}(r,z)=\rho\,g\,z$$

Digital application

$$g=10\;N/kg$$
$$\rho=8.10^{-6}\;kg/mm^3$$
$$R=20\;mm$$
$$L=10\;mm$$
$$E=2.10^5\;N/mm^2$$
$$\nu=0.3$$
$$h=1\;mm$$

- in internal skin: $\begin{cases}u_r(L)=-2.34000\,E^{-8}\,mm\\ \sigma_{zz}(L)=8.0000\,E^{-4}\,Mpa\\ u_z(O)=-1.185000\,E^{-9}\,mm\end{cases}$
- in external skin: $\begin{cases}u_r(L)=-2.46000\,E^{-8}\,mm\\ \sigma_{zz}(L)=8.0000\,E^{-4}\,Mpa\\ u_z(O)=1.215000\,E^{-9}\,mm\end{cases}$

15.2.2 Axisymmetric model hull

A vertical traction is exerted in $z=L$:

$$F=\rho\,g\,h\,L$$

Gravity leads to a vertical force:

$$f=-\rho\,g\,h\,e_z$$

The boundary condition on the circle of support is: $u_z(z)=0$ in $z=0$

The solution is membranous, vertical balance is written:

$$N_{zz,z}=\rho\,g\,h$$

Moreover: $N_{\theta\theta}=0$. In elasticity, one deduces then:

$$E_{\theta\theta}=\frac{w}{R}=\frac{-\nu\,N_{zz}}{Eh}=\frac{-\nu\,\rho\,g\,z}{E}$$
$$E_{zz}=u_{z,z}=\frac{N_{zz}}{Eh}\Rightarrow u_z(z)=\frac{\rho\,g}{2E}z^2$$

Axial stress is:

$\sigma_{zz}=\rho\,g\,z$ (constant in the thickness)

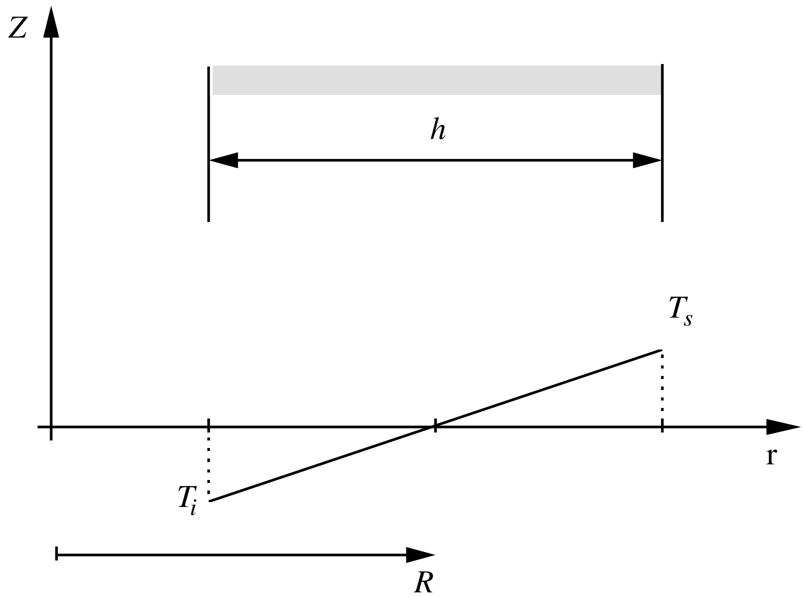
Digital application:

$$F=8.10^{-4}\,N.mm^{-1}$$
$$w(L)=-2.4000.10^{-8}\,mm$$
$$N_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$
$$\sigma_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$

15.3 Thermomechanical loading

15.3.1 Axisymmetric model 2D

$$T(r)-T_{réf}(r)=\frac{T_s+T_i}{2}+\frac{(T_s-T_i)}{h}(r-R) \qquad éq\,3.1.-1$$



One applies displacement in the form:

$u_r=u(r)$; $u_z=u_\theta=0$

with the boundary conditions suitable. Thus, the elastic constraints are expressed:

$$\sigma_{rr}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)u'+\nu\frac{u}{r}]-\frac{\alpha E}{1-2\nu}(T-T_{réf})$$

$$\sigma_{\theta\theta}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{u}{r}+\nu u']-\frac{\alpha E}{1-2\nu}(T-T_{réf}) \qquad éq\,1.1-3$$

$$\sigma_{zz}=\frac{\nu E}{(1+\nu)(1-2\nu)}[\frac{u}{r}+u']-\frac{\alpha E}{1-2\nu}(T-T_{réf})$$

The radial equilibrium equation $(r,\sigma_{rr})_{,r}-\sigma_{\theta\theta}=0$ give:

$$\left(\frac{(ru)'}{r}\right)'=\frac{\alpha(1+\nu')}{(1-\nu)}(T-T_{réf})' \qquad éq\,3.1-2$$

From where the general solution:

$$u(r)=\frac{\alpha(1+\nu)}{(1-\nu)}\frac{(T_s-T_i)}{h}\frac{r^2}{3}+Ar+\frac{B}{r} \qquad éq\,3.1-3$$

The constraints are then:

$$\sigma_{rr}(r)=\frac{\alpha E(T_s-T_i)}{h}\left(\frac{R}{1-2\nu}-\frac{r}{3(1-\nu)}\right)-\frac{\alpha E}{1-2\nu}(T_s-T_i)+\frac{E}{(1+\nu)(1-2\nu)}\left(A-(1-2\nu)\frac{B}{r^2}\right)$$
$$\sigma_{\theta\theta}(r)=\frac{\alpha E(T_s-T_i)}{h}\left(\frac{R}{1-2\nu}-\frac{2r}{3(1-\nu)}\right)-\frac{\alpha E}{1-2\nu}(T_s-T_i)+\frac{E}{(1+\nu)(1-2\nu)}\left(A-(1-2\nu)\frac{B}{r^2}\right)$$
$$\sigma_{zz}(r)=\frac{\alpha E(T_s-T_i)}{h}\left(\frac{R}{1-2\nu}-\frac{r}{3(1-\nu)}\right)-\frac{\alpha E}{1-2\nu}(T_s-T_i)+\frac{2\nu E}{(1+\nu)(1-2\nu)}A \qquad éq\,3.1-4$$

The boundary conditions in efforts are: in $r=R\pm\frac{h}{2}$, $\sigma_{rr}=0$. One notes: $x=\frac{h}{2R}$. One obtains thanks to $[éq\,3.1-4]$:

$$B=\frac{\alpha(T_s-T_i)}{6h(1-\nu)}(1+\nu)R^3(1-x^2)^2$$

then:

$$A=\alpha(1+\nu)\left[\frac{-(T_s-T_i)R}{6h(1-\nu)}(3-(1-2\nu)x^2)+\frac{(T_s+T_i)}{2}\right]$$

Digital application:

$$F=8.10^{-4}\,N.mm^{-1}$$
$$w(L)=-2.4000.10^{-8}\,mm$$
$$N_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$
$$\sigma_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$

From where: $A=-0.18569881.10^{-3}\,mm^2$
$B=0.02473096\,mm^2$

Note:

$$\frac{\alpha(1+\nu)}{1-\nu}\frac{T_s-T_i}{3h}=0.61904762\,E-5$$

- in internal skin: $\begin{cases}u_r=1.056145\,E^{-6}\,mm\\ \sigma_{zz}=1.4321427\,Mpa\end{cases}$
- in external skin: $\begin{cases}u_r=1.110317\,E^{-6}\,mm\\ \sigma_{zz}=-1.4250001\,Mpa\end{cases}$

$$\sigma_{zz}=1.449319\,Mpa$$
$$ou$$
$$\sigma_{zz}=1.428671\,Mpa(\text{ sans correction métrique })$$

If one takes $T_s = T_i = 0.1°C$:

$$A = 0{,}00130000 . 10^{-3}$$
$$B = 0{,}0 \ mm^2$$

As follows:

- in internal skin: $\begin{cases} u_r = 25.350000\, E^{-6} mm \\ \sigma_{zz} = -0.200000\, Mpa \end{cases}$
- in external skin: $\begin{cases} u_r = 26.650000\, E^{-6} mm \\ \sigma_{zz} = -0.200000\, Mpa \end{cases}$

15.3.2 Axisymmetric model hull

For the field of temperature in the thickness given by $[éq\,3.1-1]$, one obtains the following expression of the law of behavior:

$$N_{\theta\theta} = \frac{Eh}{1-\nu^2}(E_{\theta\theta} + \nu E_{zz}) - \frac{\alpha E h}{1-\nu}\left[\frac{T_s+T_i}{2} + \frac{T_s-T_i}{12}\frac{h}{R}\right]$$
$$N_{zz} = \frac{Eh}{1-\nu^2}(\nu E_{\theta\theta} + E_{zz}) - \frac{\alpha E h}{1-\nu}\left[\frac{T_s+T_i}{2} + \frac{T_s-T_i}{12}\frac{h}{R}\right] \qquad éq\,3.2-1$$

and:

$$M_{\theta\theta} = \frac{Eh^3}{12(1-\nu^2)}(K_{\theta\theta} + \nu K_{zz}) - \frac{\alpha E h^2}{12(1-\nu)}\left[\frac{T_s+T_i}{2}\frac{h}{R} + T_s - T_i\right]$$
$$M_{zz} = \frac{Eh^3}{12(1-\nu^2)}(\nu K_{\theta\theta} + K_{zz}) - \frac{\alpha E h^2}{12(1-\nu)}\left[\frac{T_s+T_i}{2}\frac{h}{R} + T_s - T_i\right] \qquad éq\,3.2-2$$

According to these expressions, thermal terms in $\frac{h}{R}$ are to be in the case of neglected if one does not consider the correction of metric in the thickness, i.e. the usual models.

In our situation:

$$E_{\theta\theta} = \frac{w}{R}$$
$$E_{zz} = 0$$
$$K_{\theta\theta} = K_{zz} = 0$$

Normal balance with the hull is written:

$$N_{\theta\theta}=0$$

from where the arrow:

$$w=\alpha(1+\nu)[\frac{T_s+T_i}{2}+\frac{T_s+T_i}{12}\frac{h}{R}]R$$

and:

$$N_{zz}=\alpha Eh[\frac{T_s+T_i}{2}+\frac{T_s+T_i}{12}\frac{h}{R}]$$

$$M_{zz}=\frac{-\alpha Eh^2}{12(1-\nu)}[(T_s-T_i)+\frac{T_s+T_i}{2}\frac{h}{R}]$$

As the second member of dilation does not take account of the correction of metric, terms in h/R above are neglected.

Digital application

$$R=20\ mm$$
$$h=1\ mm$$
$$\alpha=10^{-5}\,°C^{-1}$$
$$T_s=T_i=0.5°C$$
$$\nu=0.3$$
$$E=2.10^5\,N/mm^2$$

From where:

$$M_{zz}=-0.2380952\,N$$

• in internal skin:

$$\sigma_{zz}=1.449319\,Mpa$$
$$ou$$
$$\sigma_{zz}=1.428671\,Mpa(\text{ sans correction métrique })$$

If one takes $T_s=T_i=0{,}1°C$:

$$w=26.00000.10^{-6}\,mm$$
$$N_{zz}=-0.2\,N.mm^{-1}$$
$$M_{zz}=-0.001190476\,N$$

• in internal skin:

$$\sigma_{zz}=-0.2122466\,Mpa$$
$$ou$$
$$\sigma_{zz}=-0.200000\,Mpa(\text{ sans correction métrique })$$

** the constraints in the thickness with correction of metric are given by:*

$$\sigma_{zz}(x_3)=\frac{N_{zz}-\frac{M_{zz}}{R}}{h(1-\frac{h^2}{12R^2})}+(M_{zz}-N_{zz}\frac{h^2}{12R})\frac{12x_3}{h^3(1-\frac{h^2}{12R^2})}$$