HPLP101 - Plate fissured in thermoelasticity (plane constraints)

Summary:

This test is resulting from the validation independent of *Code_Aster* in breaking process (reference resulting from Murakami: Mura11-17). It makes it possible to validate the operators of breaking process for a two-dimensional problem (assumption of the plane constraints) in isotropic linear thermoelasticity.

This test understands a modeling in plane constraints in which are calculated:

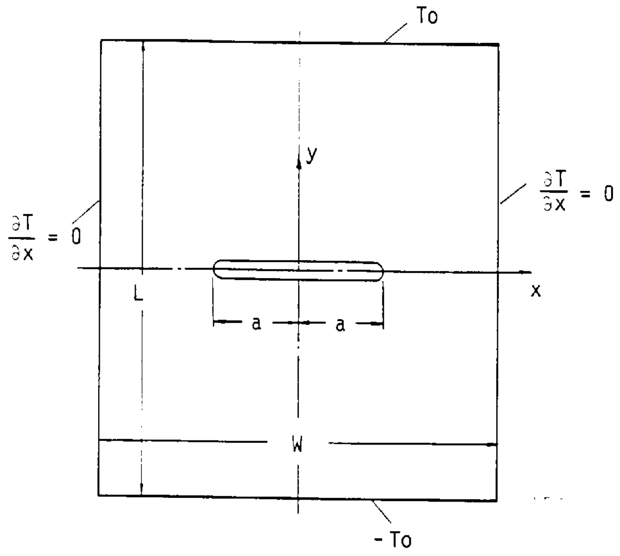
- the rate of refund of energy G (classical calculation by the method theta),
- coefficients of intensity of constraints K_I and K_{II}.

These two calculations are carried out on 6 different crowns of integration.

The interest of the test is to compare the values of G and K_{II} compared to the reference solution and to test the invariance of calculations compared to the various crowns of integration.

1 Problem of reference

1.1 Geometry



Width of the plate: $W=0.6\,m$
Length of the plate: $L=0.3\,m$
Length of the crack: $2a=0.3\,m$

1.2 Properties of material

Notation for thermoelastic properties:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}+\begin{Bmatrix}\alpha_{11}\\ \alpha_{22}\\ 0\end{Bmatrix}\cdot(T-T_{ref})$$

$S_{11}=1/E_x$

$S_{22}=1/E_y$

$S_{12}=-\nu_x/E_x=-\nu_y/E_y$

$S_{66}=1/G_{xy}$

$\alpha_{11}=\alpha_x$

$\alpha_{22}=\alpha_y$

One limits oneself to isotropic material, as well from the thermal point of view as mechanical:

$E_x=E_y=2.10^5\,MPa$

$\nu_x=\nu_y=0.3$

$\alpha_x=\alpha_y=1.2\,10^{-5}\,°C^{-1}$

$\lambda_x=\lambda_y=54.\,W/m\,°C$

1.3 Boundary conditions and loading

Two models are considered:

- the half-model $x=0$
- the complete model

Boundary conditions mechanical:

- half-model

  $UX=0$ along the axis of symmetry $X=0$
  $UY=0$ at the point ($W/2.$)
- complete model

  $UX=0$ at the point $(0, L/2.)$
  $UY=0$ at the points $(-L/2.)$ and $(L/2.)$

Boundary conditions thermal:

- half-model

  $T=100°C$ on the higher edge $Y=L/2$.
  $T=-100°C$ on the lower edge $Y=-L/2$.
  null flow on the axis of symmetry, the free edge $X=W/2.$ and on the edge of the crack
- complete model

  $T=100°C$ on the higher edge $Y=L/2$.
  $T=-100°C$ on the lower edge $Y=-L/2$.
  null flow on the free edges $X=\pm W/2.$ and on the edge of the crack

2 Reference solution

2.1 Method of calculating used for the reference solution

Complex potential [bib1].

2.2 Results of reference

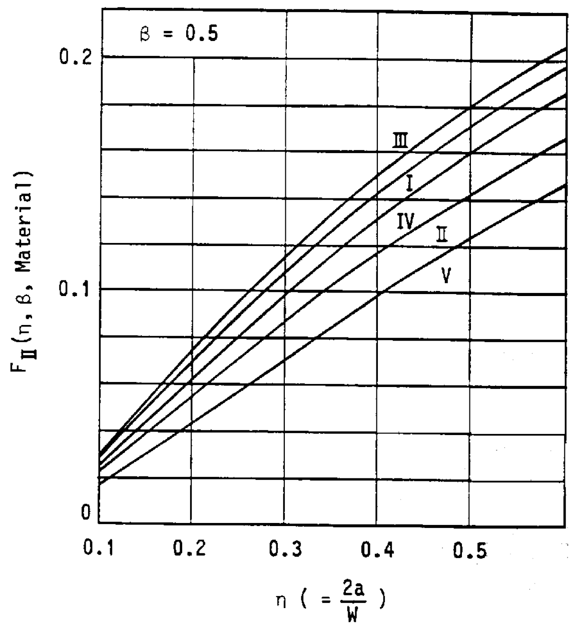
$$\eta=\frac{2a}{W}$$

$$\beta=\frac{L}{W}$$

$$K_{II}=\frac{\alpha_{11} T_0}{S_{11}}\cdot\sqrt{\frac{W}{2}}\cdot F_{II}$$

where the geometrical factor of correction F_{II} is given according to η for each material, in the typical case $\beta=0.5$ on the curves below.

The isotropic material being represented by the curve I



2.3 Uncertainty on the solution

Nondefinite precision.

2.4 Bibliographical references

1) Y. MURAKAMI: Stress Intensity Factors Handbook, box 11.17, pages 1045-1047. The Society of Materials Science, Japan, Pergamon Near, 1987.

3 Modeling A

3.1 Characteristics of modeling

For this modeling, the 3 topological parameters of the block crack are:

- NS : many sectors on $90°$
- NC : many crowns
- rt : the ray of the largest crown (with half a: length of the crack)

$NS=8$
$NC=4$
$rt=0{,}001\times a$

Values of the higher and lower rays, to specify in the order `CALC_G` are:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	3,75E-5	7,500E-5	1,125E-4	1,500E-4	1,875E-4	2,250E-4
Rsup	7,50E-5	1,125E-4	1,500E-4	1,875E-4	2,250E-4	3,000E-4

3.2 Characteristics of the grid

Half-grid; grid radiating at the right end of the crack.

3831 nodes,
1516 elements,
884 TRI6,
632 QUA8.

3.3 Sizes tested and results of modeling A

Identification	**Reference**	***Aster***	**% difference**
K_{II} , crown n°1	2,2347E+7	2,2814E+7	2.09
K_{II} , crown n°2	2,2347E+7	2,2813E+7	2.08
K_{II} , crown n°3	2,2347E+7	2,2814E+7	2.09
K_{II} , crown n°4	2,2347E+7	2,2814E+7	2.09
K_{II} , crown n°5	2,2347E+7	2,2817E+7	2.10
K_{II} , crown n°6	2,2347E+7	2,2818E+7	2.11
G , crown n°1	2,4969E+3	2,5984E+3	4.07
G , crown n°2	2,4969E+3	2,5990E+3	4.09
G , crown n°3	2,4969E+3	2,5992E+3	4.10
G , crown n°4	2,4969E+3	2,5993E+3	4.10
G , crown n°5	2,4969E+3	2,6013E+3	4.18
G , crown n°6	2,4969E+3	2,5985E+3	4.07

3.4 Remarks

In the reference, the author supposes that $K_I=0$, but it does not check it a posteriori. With the sights of the deformations resulting from Code_Aster, the coefficient K_I is different from zero, but there remains very weak compared to K_{II} (the crack slips more than it does not open).

With regard to the rate of refund of energy G , if we suppose that $K_I=0$, we draw the value of reference starting from the formula from IRWIN in plane constraints:

$$G_{ref}=(1/E)\times K_{II}^2$$

4 Summary of the results

Differences between the reference solution and the results of *Code_Aster* do not exceed 2% on the coefficients of intensity of constraints and 4% for the rate of refund of energy. One checks the invariance of the results compared to the various crowns of integration.