HSLS01 - Thin square plate subjected to a heat gradient in the thickness

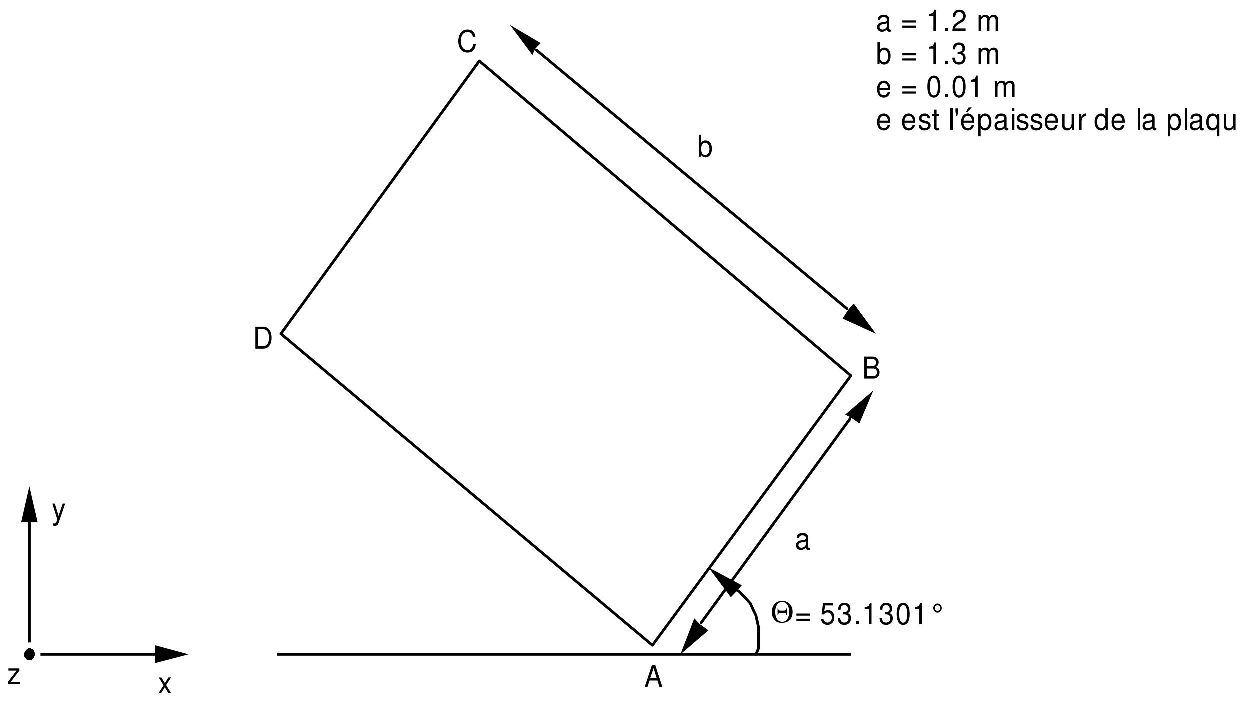
Summary

The purpose of this test is to validate thermal dilation in the elements of plate, where the temperature is variable in the thickness.

Two modelings make it possible to test modelings DKT, DST, Q4G on meshs TRIA3 and QUAD4 and COQUE_3D on the meshs TRIA7 and QUAD9.

1 Problem of reference

1.1 Geometry



1.2 Properties of materials

Young modulus: $E=2.10^{11} Pa$
Poisson's ratio: $\nu=0.3$
Dilation coefficient: $\alpha=1.10^{-5} \circ C^{-1}$

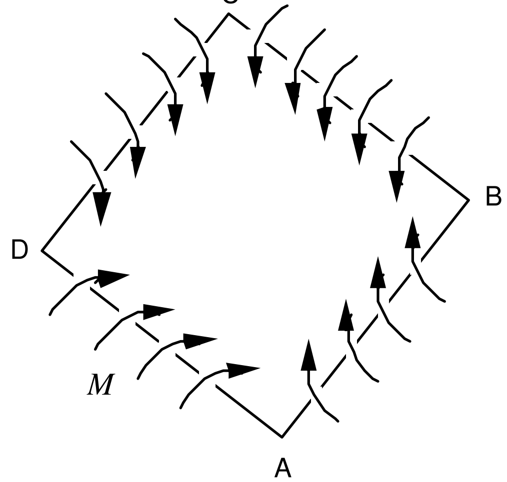
1.3 Boundary conditions and loadings

Sides AB , BC , CD and DA are embedded. The temperature is constant on the higher face and is equal to $T_s=100 \circ C$.
The temperature is constant on the lower face and is equal to $T_i=0 \circ C$; the variation in temperature is supposed to be linear in the thickness.

2 Reference solution

2.1 Method of calculating used for the reference solution

The solution is analytical.



The thermal loading is equivalent to a loading defined by a uniform distribution of moments on the edges such as it appears on the figure.

The value of these moments per unit of length is equal to: $M = \alpha \frac{T_s - T_i}{e} \times \frac{E \times e^3 \times (1+\nu)}{12(1-\nu^2)}$.

That is to say: $M = \alpha (T_s - T_i) \times \frac{E \times e^2}{12(1-\nu)}$. This led to a uniform distribution of M in the plate.

2.2 Results of reference

One thus has $M = 2380.95238\,N$; the plate being turned of an angle $\theta = 53°.1301$, there are components whose absolute value is: $M \times \cos\theta = 1428.5715\,N$ and $M \times \sin\theta = 1904.76184\,N$

The reactions are defined by a distribution of moments equal to the preceding one in absolute value and of contrary sign.

The meshs are squares of which the length is equal to $0.05\,m$, therefore the moments in each node must be equal to $M_1 = M \times \cos\theta \times 0.05 = 71.42857\,N.m$

and $M_2 = M \times \sin\theta \times 0.05 = 95.2381\,N.m$

that is to say $M = \sqrt{M_1^2 + M_2^2} = 119.0476$ N.m

2.3 Uncertainty on the solution

Uncertainty is worthless.

2.4 Bibliographical references

1) TIMOSHENKO: Theory of punts and shells chapter 2, article 14.

3 Modeling A

3.1 Characteristics of modeling

The model consists of:

- 1008 elements,
- 675 nodes,

of which:

- 72 éléme nts `Q4G`,
- 144 elements `T3G`,
- 84 elements `DSQ`,
- 84 elements `DKQ`,
- 312 elements `DST`,
- 312 elements `DKT`.

The elements are squares of which the length is equal to $0.05m$.

Edges AB , BC , CD and DA are embedded.

The plate is subjected to a variation in temperature of $100°C$ in the thickness. This gradient is uniform on the plate.

3.2 Sizes tested and results

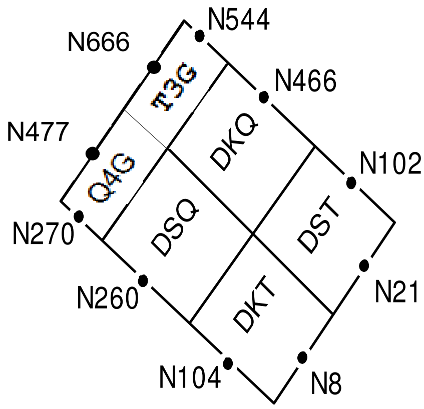
Rx = reaction according to Ox
Ry = reaction according to Oy

Identification	Type of reference	Reference	Tolerance
$N104$ (on the edge AD in the part with a grid in `DKT`)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}
$N260$ (on the edge AD in the part with a grid in `DSQ`)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}
$N270$ (on the edge AD in the part with a grid in `Q4G`)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}
$N8$ (on the edge AB in the part with a grid in `DKT`)	'ANALYTICAL'	$DRx=95.2381$ $DRy=71.4286$	3.10^{-5} 3.10^{-5}
$N21$ (on the edge AB in the part with a grid in `DST`)	'ANALYTICAL'	$DRx=95.2381$ $DRy=71.4286$	3.10^{-5} 3.10^{-5}
$N102$ (on the edge BC in the part with a grid in `DST`)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}
$N466$ (on the edge BC in the part with a grid in `DKQ`)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}

$N544$ (on the edge BC in the part with a grid in T3G)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}
$N477$ (on the edge CD in the part with a grid in Q4G)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}
$N666$ (knownr le edge CD in the part with a grid in T3G)	'ANALYTICAL'	$DRx=71.4286$ $DRy=95.2381$	3.10^{-5} 3.10^{-5}

3.3 Remarks

The nodes tested are about placed as follows:



4 Modeling B

4.1 Characteristics of modeling

Modeling is `COQUE_3D`.

The model consists of:

- 662 elements,
- 2267 nodes,

of which:

- 462 triangles with 7 nodes,
- 200 quadrilaterals with 9 nodes.

Edges AB , BC , CD and DA are embedded.

The plate is subjected to a variation in temperature of $100\,°C$ in the thickness. This gradient is uniform on the plate.

4.2 Sizes tested and results

One tests the moments MXX and MYY . These values are given in the local reference mark to the plate, chosen parallel at the sides.

One thus has: $MXX = MYY = M = -2{,}38095\,10^3\,N$ as the moment is uniform in the plate, it is enough to test the values maximum and minimum of the moments and to check that they are both equal to M :

	Identification	Type of reference	Reference	Tolerance (%)
Efforts obtained by `EFGE_ELNO` :	MXX Maximum	'ANALYTICAL'	– 2.38095 10^3	0.1
	MXX Minimum	'ANALYTICAL'	– 2.38095 10^3	0.1
	MYY Maximum	'ANALYTICAL'	– 2.38095 10^3	0.1
	MYY Minimum	'ANALYTICAL'	– 2.38095 10^3	0.1

5 Summary of the results

The perfect adequacy of the results with the analytical reference shows the good taking into account of the variation in the temperature.