HSNV104 - Thermoplasticity and metallurgy in plane deformations with restoration of work hardening

Summary:

One treats the determination of the mechanical evolution of a right-angled parallelepiped in plane deformations subjected to evolutions thermics $T_{(t)}$ and metallurgical $Z_{(t)}$ known and uniform (the metallurgical transformation is of bainitic type).

The elements used are two-dimensional elements in plane deformations and the relation of behavior is the plasticity of von Mises with linear isotropic work hardening. One takes account of the restoration of work hardening, but not of the plasticity of transformation.

The dilation coefficient α depends on the metallurgical composition.

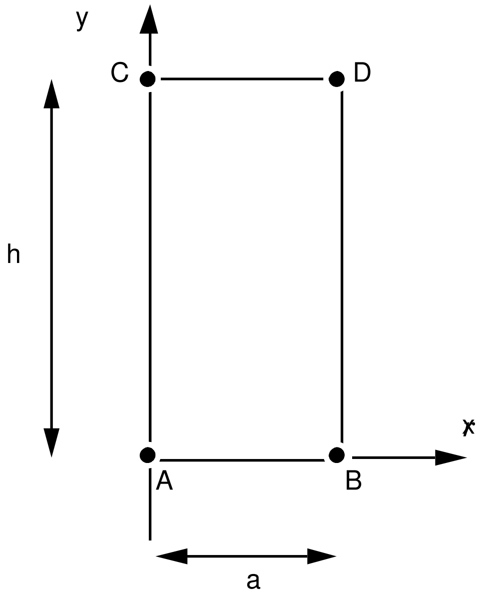
The reference solution is obtained by the analytical resolution of the problem.

Results provided by *Code_Aster* are very satisfactory with errors lower than $0{,}8\%$.

1 Problem of reference

1.1 Geometry

Largeur : a = 0.05 m.
Hauteur : h = 0.2 m.



1.2 Properties of materials

Following convention is adopted in order to distinguish the parameters from the hot phase (austenitic) parameters of the cold phases (ferrito-perlitic, bainitic and martensitic):

$*^{aust}$ = characteristics relating to the austenitic phase
$*^{fbm}$ = characteristics relating to the phases ferrito-perlitic, bainitic and martensitic

Metallurgical parameters:
TRC to model a metallurgical evolution of bainitic type, on all the structure, of the form:

$$Z_{fbm}=\begin{cases} 0. & si \quad t\leq\tau_1 & \tau_1=60\,s \\ \dfrac{t-\tau_1}{\tau_2-\tau_1} & si \quad \tau_1\leq t<\tau_2 & \tau_2=112\,s \\ 1. & si \quad t\geq\tau_2 & \end{cases}$$

Thermal parameters:
Heat-storage capacity: $\rho C_p=2.\,10^6\,J.m^{-3}.\,°C^{-1}$
Conductivity: $\lambda=9999.9\,W.m^{-1}.\,°C^{-1}$

Thermomechanical parameters:
- Thermoelastic parameters:

Young modulus $E=200000\,10^6\,Pa$
Poisson's ratio $\nu=0.3$
Dilation coefficients thermal $\alpha_{fbm}=\alpha_{aust}=20.\,10^{-6}\,°C^{-1}$

Temperature of definition of the dilation coefficient: $T_{ref}=900\,°C$

Thermal state of deformation of reference: $\Delta \varepsilon_{f\gamma}^{T_{ref}} = 2.52\,10^{-3}$
Elastic limit:

$\sigma_y^{fbm} = 1200.\,10^6\,Pa$

$\sigma_y^{aust} = 400.10^6\,Pa$

- Thermoplastic parameters (law with linear work hardening)
  Tangent modules: $E_T^{fbm} = E_T^{aust} = 2000.10^6\,Pa$
  One has then: $H^{fbm} = H^{aust} = \dfrac{EE_T}{(E - E_T)} = 2{,}04.10^9\,Pa$

- Parameters for the restoration of work hardening (complete restoration): $\theta_{\gamma,3} = 0$
  $\theta_{\gamma,3}$ is the rate of work hardening transmitted of austenite to the ferritic phase 3 (bainite).

1.3 Boundary conditions and loadings

- $u_Y = 0$ on the side AB ; $u_X = 0$ in A .
- $T = T^0 + \mu t$, $\mu = -5\,°C.s^{-1}$ on all the structure.
- The loading on the structure is due with the phenomena of thermal and metallurgical dilation constrained in the direction z by the condition of plane deformations.

1.4 Initial conditions

$T^0 = 900\,°C = T^{ref}$

2 Reference solution

2.1 The shape of the field solution

The stress field solution $\boldsymbol{\sigma}(t)$ is form:

$$\boldsymbol{\sigma}(t)=\sigma_o(t)\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

One from of deduced the following form from the tensor of the elastic strain:

$$\boldsymbol{\varepsilon}^e(t)=\frac{\sigma_o(t)}{E}\begin{pmatrix}-\nu & 0 & 0\\ 0 & -\nu & 0\\ 0 & 0 & 1\end{pmatrix}$$

Moreover, since $\boldsymbol{\sigma}(t)$ keep a constant direction, one a:

$$\boldsymbol{\varepsilon}^p(t)=\varepsilon_o^p(t)\begin{pmatrix}\frac{-1}{2} & 0 & 0\\ 0 & \frac{-1}{2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

where $\boldsymbol{\varepsilon}^p$ is the tensor of the plastic deformations.

2.2 Method of calculating used for the reference solution

Notation: thereafter, one will note ε_α^{eff} (resp. ε_γ^{eff}) the effective variable of work hardening of the cold phases (resp. hot phase).

Before transformation, thermoelastic solution for $t<t_1$.

$$\begin{cases}\varepsilon_{zz}(t)=\varepsilon_{zz}^e(t)+\varepsilon_{zz}^{th}(t)=0\\ \varepsilon_{zz}^{th}(t)=\alpha_{aust}(T-T^0)\\ \sigma_{zz}(t)=-E\,\varepsilon_{zz}^{th}(t)\end{cases}$$

The yield stress is reached for $t=t_1$ such as:

$$\sigma_{zz}(t_1)=-E\,\varepsilon_{zz}^{th}(t_1)=\sigma_y^{aust} \quad\Leftrightarrow\quad T(t_1)-T^0=\frac{-\sigma_y^{aust}}{E\,\alpha_{aust}}=-100.\,°C$$

$$\Leftrightarrow\quad t_1=\frac{T(t_1)-T^0}{\mu}=20\,s$$

Before transformation, thermoelastoplastic solution, $t_1\le t\le\tau_1$.

$$\begin{cases}\varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t)=0\\ \varepsilon_{zz}^{th}(t)=\alpha_{aust}(T-T^0)\\ \varepsilon_{zz}^{p}(t)=\dfrac{-\sigma_y^{aust}-E\,\alpha_{aust}(T-T^0)}{E+H^{aust}}\\ \sigma_{zz}(t)=-E\left(\varepsilon_{zz}^{p}(t)+\varepsilon_{zz}^{th}(t)\right)\\ \varepsilon_{\gamma}^{eff}(t)=p(t)=\varepsilon_{zz}^{p}(t)\\ \varepsilon_{\alpha}^{eff}(t)=0\end{cases}$$

During the transformation, for $\tau_1<t\le\tau_2$, one is in elastic mode, one thus has an elastic solution thermo - with phase shift.

$$\begin{cases}\varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(\tau_1)=0\\ \varepsilon_{zz}^{th}(t)=Z_{aust}\,\alpha_{aust}(T-T^0)+Z_{fbm}\left(\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}}\right)\\ \sigma_{zz}(t)=-E\left(\varepsilon_{zz}^{p}(\tau_1)+\varepsilon_{zz}^{th}(t)\right)\\ \varepsilon_{\gamma}^{eff}(t)=\varepsilon_{zz}^{p}(\tau_1)\\ \varepsilon_{\alpha}^{eff}(t)=0\end{cases}$$

After the transformation, thermoelastic solution for $\tau_2<t<t_2$.

$$\begin{cases}\varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(\tau_1)=0\\ \varepsilon_{zz}^{th}(t)=\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}}\\ \sigma_{zz}(t)=-E\left(\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(\tau_1)\right)\\ \varepsilon_{\gamma}^{eff}(t)=\varepsilon_{zz}^{p}(\tau_1)\\ \varepsilon_{\alpha}^{eff}(t)=0\end{cases}$$

The yield stress is reached for $t=t_2$ such as:

$$\sigma_{zz}(t_2)=R(T,Z,\varepsilon^{eff})+\sigma_y(T,Z)$$

Because of the restoration of work hardening and owing to the fact that one was in elastic mode during all the transformation: $R=0$ before replastification.

One thus has in t_2 :

$$\sigma_{zz}(t_2)=-E\left(\varepsilon_{zz}^{th}(t_2)+\varepsilon_{zz}^{p}(\tau_1)\right)=\sigma_y^{fbm}\quad\Leftrightarrow\quad T(t_2)-T^0=-\frac{\sigma_y^{fbm}+E\left(\Delta\varepsilon_{f\gamma}^{T_{ref}}+\varepsilon_{zz}^{p}(\tau_1)\right)}{E\,\alpha_{fbm}}\simeq-624\,°C$$

$$\Rightarrow\quad t_2=\frac{T(t_2)-T^0}{\mu}\simeq125\,s$$

After the transformation, thermoelastoplastic solution for $t\ge t_2$.

$$
\begin{cases}
\varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t)=0 \\
\varepsilon_{zz}^{th}(t)=\alpha_{fbm}(T-T^{0})+\Delta\varepsilon_{f\gamma}^{T_{ref}} \\
\varepsilon_{zz}^{p}(t)=\dfrac{-\sigma_{y}^{fbm}-E(\alpha_{fbm}(T-T^{0})+\Delta\varepsilon_{f\gamma}^{T_{ref}})+H^{fbm}\varepsilon^{p}(\tau_{1})}{E+H^{fbm}} \\
\sigma_{zz}(t)=-E(\varepsilon_{zz}^{p}(t)+\varepsilon_{zz}^{th}(t)) \\
\varepsilon_{\gamma}^{eff}(t)=0 \\
\varepsilon_{\alpha}^{eff}(t)=\varepsilon_{zz}^{p}(t)-\varepsilon_{zz}^{p}(\tau_{1})
\end{cases}
$$

2.3 Results of reference

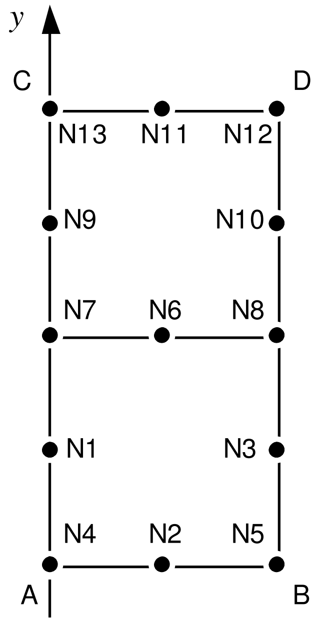
With $t=60\,s$:	σ_{zz}	$\varepsilon_{\gamma}^{eff}$	$\varepsilon_{\alpha}^{eff}$	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}
With $t=89\,s$:	σ_{zz}	$\varepsilon_{\gamma}^{eff}$	$\varepsilon_{\alpha}^{eff}$	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}
With $t=112\,s$:	σ_{zz}	$\varepsilon_{\gamma}^{eff}$	$\varepsilon_{\alpha}^{eff}$	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}
With $t=176\,s$:	σ_{zz}	$\varepsilon_{\gamma}^{eff}$	$\varepsilon_{\alpha}^{eff}$	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}

2.4 Bibliographical references

1. DONORE A.M. - WAECKEL F. - Influence of structure transformations in the elastoplastic laws of behavior Notes HI-74/93/024.
2. DONORE.A.M. - WAECKEL.F. - RAZAKANAIVO.A. - Doc. Aster [R4.04.02].

3 Modeling A

3.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

3.2 Characteristics of the grid

Many nodes: 13.
Many meshs and types: 2 meshs `QUAD8`, 6 meshs `SEG3`.

3.3 Sizes tested and results

Identification	Type of Reference	Reference	Tolerance (%)
σ_{zz} $t=60\text{s}$	`ANALYTICAL`	4.0792E8	0.1
$\varepsilon_{\gamma}^{eff}$ $t=60\text{s}$	`ANALYTICAL`	3.9604E-3	0.1
$\varepsilon_{\alpha}^{eff}$ $t=60\text{s}$	`ANALYTICAL`	0.	0.1
ε_{xx}^{th} $t=60\,s$	`ANALYTICAL`	-6.0E-3	0.1
ε_{xx}^{meca} $t=60\,s$	`ANALYTICAL`	-2.59208E-3	0.1
ε_{xx}^{plas} $t=60\,s$	`ANALYTICAL`	-1.9802E-3	0.1
σ_{zz} $t=89\text{s}$	`ANALYTICAL`	7.0684E8	0.80
$\varepsilon_{\gamma}^{eff}$ $t=89\text{s}$	`ANALYTICAL`	3.9604E-3	0.1
$\varepsilon_{\alpha}^{eff}$ $t=89\text{s}$	`ANALYTICAL`	0.	0.1
ε_{xx}^{th} $t=89\,s$	`ANALYTICAL`	-7.49460E-3	0.5
ε_{xx}^{meca} $t=89\,s$	`ANALYTICAL`	-3.04046E-3	0.1
ε_{xx}^{plas} $t=89\,s$	`ANALYTICAL`	-1.9802E-3	0.3
σ_{zz} $t=112\text{s}$	`ANALYTICAL`	9.4392E8	0.1
$\varepsilon_{\gamma}^{eff}$ $t=112\text{s}$	`ANALYTICAL`	0.	0.1

$\varepsilon_{\alpha}^{eff}$ $t=112\text{s}$	ANALYTICAL	0.	0.1
ε_{xx}^{th} $t=112\,s$	ANALYTICAL	-8.68E-3	0.1
ε_{xx}^{meca} $t=112\,s$	ANALYTICAL	-3.39608E-3	0.1
ε_{xx}^{plas} $t=112\,s$	ANALYTICAL	-1.9802E-3	0.1
σ_{zz} $t=176\text{s}$	ANALYTICAL	12.101E8	0.1
$\varepsilon_{\gamma}^{eff}$ $t=176\text{s}$	ANALYTICAL	0.	0.1
$\varepsilon_{\alpha}^{eff}$ $t=176\text{s}$	ANALYTICAL	5.068921E-3	0.1
ε_{xx}^{th} $t=176\,s$	ANALYTICAL	-1.508E-2	0.1
ε_{xx}^{meca} $t=176\,s$	ANALYTICAL	-6.3298E-3	0.1
ε_{xx}^{plas} $t=176\,s$	ANALYTICAL	-4.51465E-3	0.1

3.4 Remarks

In this modeling:

$$\varepsilon_{zz}^{pt}(T,Z)=0$$

The error on σ_{zz} at 89 seconds comes by way of the mistake made on the digital description of the metallurgical transformation which is, at this moment, of approximately 56%.

4 Summary of the results

Results found with *Code_Aster* are very satisfactory, with percentages of error lower than 0.8%.