HSNV121 - Traction in great deformations plastics of a bar under loading thermics

Summary:

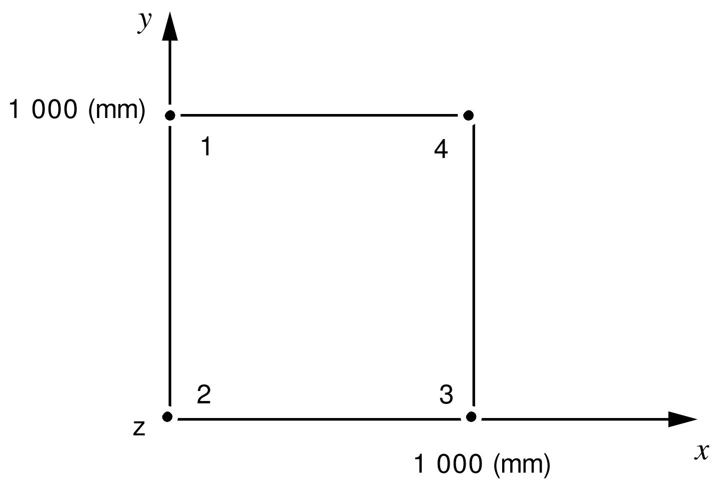
This quasi-static thermomechanical test consists in heating a bar of rectangular section uniformly ($3D$) or cylindrical ($2D$ axisymmetric) then to subject it to a traction. One thus validates the kinematics of the great deformations in plasticity (order STAT_NON_LINE, keyword DEFORMATION: 'SIMO_MIEHE' or 'PETIT_REAC') for a relation of behavior in great deformations with linear isotropic work hardening (order STAT_NON_LINE, keyword RELATION: 'VMIS_ISOT_LINE' and 'VMIS_ISOT_TRAC') with thermomechanical loading. With modelings hull or plate, the great deformations in plasticity are accessible thanks to the keyword DEFORMATION: 'PETIT_REAC' provided that rotations remain weak.

The bar is modelled by a voluminal element (HEXA20, modeling A) or quadrangular (QUAD4, for an axisymmetric modeling, modeling B) or by elements of plate or hull (DKT for modeling C and COQUE_3D for modeling D).

The solution is analytical.

1 Problem of reference

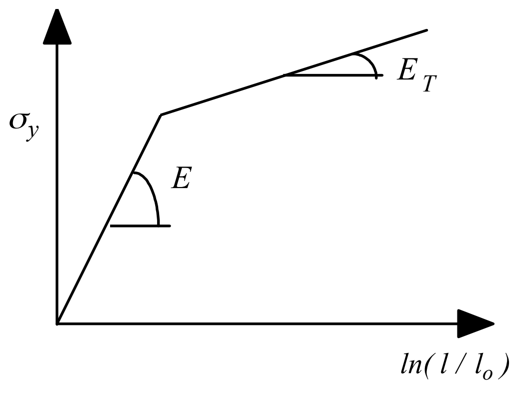
1.1 Geometry



1.2 Properties of material

The material obeys a law of behavior in great deformations figure with linear isotropic work hardening, whose characteristics depend on the temperature.
The traction diagram is given in the plan deformation logarithmic curve - rational constraint.

$$\sigma = \frac{F}{S} = \frac{F}{S_o} \cdot \frac{l}{l_o}$$



$\nu = 0.3$

$\alpha = 10^{-4} K^{-1}$

$\sigma_y = 1000$ MPa

à $T = 20°$ C

$E = 250000$MPa

$E_T = 2500$ MPa

à $T = 120°$ C

$E = 200000$MPa

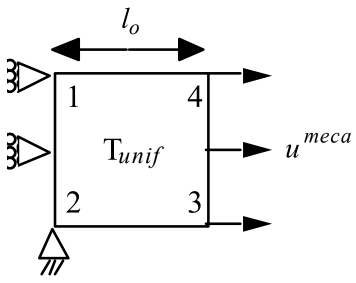
$E_T = 2000$ MPa

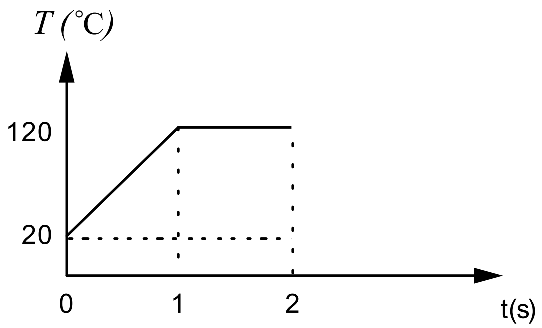
l_o and l are, respectively, the initial length and the current length of the useful part of the test-tube.

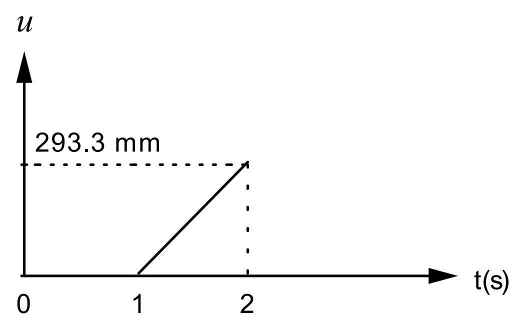
S_o and S are, respectively, initial and current surface. Between the temperatures $20 °C$ and $120 °C$, the characteristics are interpolated linearly.

1.3 Boundary conditions and loadings

The bar, initial length l_o, blocked in the direction $O\mathbf{x}$ on the face [1.2] is subjected to a uniform temperature T and with a mechanical displacement of traction u^{meca} on the face $[3,4]$. The sequences of loading are the following ones:







Temperature of reference: $T_{réf}$ = 20°C.

Note:

Mechanical displacement is measured starting from the configuration deformed by the thermal loading ($t=1\text{s}$). To have total displacement, it is thus necessary to add the thermal displacement obtained at time $t=1\text{s}$.

2 Reference solution

2.1 Result of the reference solution

For a tensile test according to the direction x, the tensor of Kirchhoff τ is form:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Tensor gradients of the transformation $\mathbf{F}$ and $\overline{\mathbf{F}}$ and the isochoric tensor of plastic deformations $\mathbf{G}^p$ are form:

$$\mathbf{F} = \begin{bmatrix} F & 0 & 0 \\ 0 & F_{yy} & 0 \\ 0 & 0 & F_{yy} \end{bmatrix} \quad \text{et} \quad J = \det \mathbf{F} = F F_{yy}^2 \quad \Rightarrow \quad F_{yy} = \sqrt{J/F}$$

$$\overline{\mathbf{F}} = J^{-1/3}\mathbf{F} = \begin{bmatrix} \overline{F} & 0 & 0 \\ 0 & \overline{F}_{yy} & 0 \\ 0 & 0 & \overline{F}_{yy} \end{bmatrix} \quad \text{et} \quad \det \overline{\mathbf{F}} = 1 \quad \Rightarrow \quad \begin{cases} \overline{F} = J^{-1/3} F \\ \overline{F}_{yy} = \overline{F}^{-1/2} \end{cases}$$

$$\mathbf{G}^p = \begin{bmatrix} G^p & 0 & 0 \\ 0 & G^p_{yy} & 0 \\ 0 & 0 & G^p_{yy} \end{bmatrix} \quad \text{et} \quad \det \mathbf{G}^p = 1 \quad \Rightarrow \quad G^p_{yy} = (G^p)^{-1/2}$$

By the law of behavior, one obtains the following relation:

$$\tau = \frac{3K}{2}(J^2 - 1) - \frac{9K\alpha(T - T_{ref})}{2}\left(J + \frac{1}{J}\right)$$

that is to say

$$J^3 - 3\alpha(T - T_{ref})J^2 - J\left(1 + \frac{2\tau}{3K}\right) - 3\alpha(T - T_{ref}) = 0$$

The constraint of Cauchy is written:

$$J\sigma = \tau$$

In plastic load for an isotropic work hardening R linear, such as:

$$R(p) = \frac{E E_T}{E - E_T} p$$

one a:

$$p = \frac{E - E_T}{E\, E_T}(\tau - \sigma_y)$$

The integration of the law of flow of the plastic deformation G^P give (knowing that $G^P(p=0)=1$):

$$G^P=e^{-2p}$$

The component $\bar{F}$ gradient of the transformation is given by the resolution of:

$$\bar{F}^3-\frac{\tau}{\mu G^p}\bar{F}-\frac{1}{(G^p)^{3/2}}=0$$

The field of displacement $\boldsymbol{u}$ (in the initial configuration) is form $\boldsymbol{u}=u_x\boldsymbol{X}+u_y\boldsymbol{Y}+u_z\boldsymbol{Z}$. The components are given by:

$$u_x=\frac{\tilde{u}}{l_o}X \quad \text{avec } \tilde{u}=(F-1).l_o$$

$$u_y=\frac{\tilde{v}}{l_o}Y \quad \text{avec } \tilde{v}=\left(\sqrt{\frac{J}{F}}-1\right)l_o$$

$$u_z=\frac{\tilde{v}}{l_o}Z$$

2.2 Results of reference

One will adopt like results of reference displacements, the constraint of Cauchy σ and cumulated plastic deformation p.

At time $t=2\ s$ ($\Delta T=100°C$, traction u)

One seeks total displacement (thermal + mechanical) such as the constraint τ that is to say equalizes with:

$\tau=1500\,Mpa$ (with $T=120°C$)

- $3K=500\,000\,MPa$ $\mu=76923\,MPa$
- $J=1.03$
- $\sigma=1453\,MPa$
- $p=0{,}2475$
- $G^p=0{,}609$
- $\bar{F}=1{,}289$
- $F=1{,}303$
- $\tilde{u}=303\,mm$
- $\tilde{v}=-110\,mm$

With these sizes, it is possible to determine the elastic energy of the bar. Attention, the presence of thermics generates a high jacobien, requiring a specific correction as described in R5.03.21. With final, one obtains at the material point: $\Psi_{elas}=5{,}6\,MPa$

2.3 Uncertainty on the solution

The solution is analytical. With the rounding errors near, one can consider it exact.

2.4 Bibliographical references

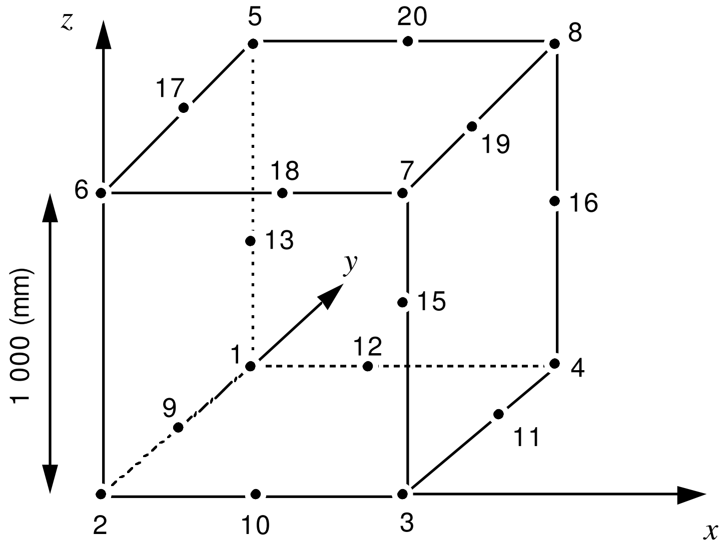
One will be able to refer to:

1) V. CANO, E. LORENTZ: Introduction into *Code_Aster* of a model of behavior in great deformations elastoplastic with isotropic work hardening - Note interns EDF DER HI - 74/98/006/0

3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling: 1 mesh HEXA20
1 mesh QUAD8



Boundary conditions:

$N2$: $U_x = U_y = U_z = 0$ $N9$, $N13$, $N14$, $N5$, $N17$: $U_x = 0$

$N1$: $U_x = U_z = 0$

$N6$: $U_x = U_y = 0$

Load: Traction on the face $[3\ 4\ 8\ 7\ 11\ 16\ 19\ 15]$ + assignment of the same temperature on all the nodes.

The full number of increments is of 21 (1 increment enters $t=0\text{s}$ and 1s, 20 increments enters $t=1\text{s}$ and 2s)

Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to 10^{-6}.

3.2 Characteristics of the grid

Many nodes: 20
Many meshs: 2

1 HEXA20
1 QUAD8

3.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N8$)	303	1,00%
$t=2$ Displacement DY ($N8$)	– 110	1,00%
$t=2$ Displacement DZ ($N8$)	– 110	1,00%
$t=2$ Constraints $SIGXX$ ($PG1$)	1453	1,00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1.50%
$t=2$ Variable $INDIPLAS$ ($PG1$)	1.	0.10%
$t=2$ ENER_ELAS, TOTAL	5,60E+009	5,00%

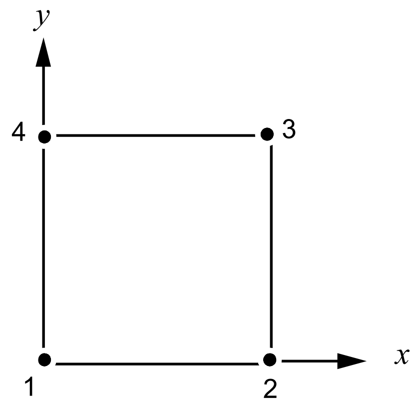
With $INDIPLAS$ the indicator of plasticity.

4 Modeling B

4.1 Characteristics of modeling

Modeling $2D$ axisymmetric:

1 mesh QUAD4
1 mesh SEG2



Boundary conditions:

$N1$: $U_y = 0$

$N2$: $U_y = 0$

Loading:

Traction on the face $[3\ 4]$ (mesh SEG2) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters $t = 0\mathrm{s}$ and $1\mathrm{s}$, 20 increments enters $t = 1\mathrm{s}$ and $2\mathrm{s}$)

Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to 10^{-6}.

4.2 Characteristics of the grid

Many nodes: 4
Many meshs: 2

1 QUAD4
1 SEG2

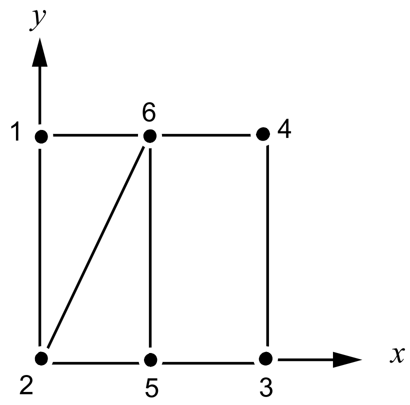
4.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	– 110	1,00%
$t=2$ Displacement DY ($N3$)	303	1,00%
$t=2$ Constraints $SIGYY$ ($PG1$)	1453	1,00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1,00%
$t=2$ ENER_ELAS, TOTAL	5,60E+009	5,00%

5 Modeling C

5.1 Characteristics of modeling

Modeling plates DKT of thickness $1000\,mm$: 1 mesh QUAD4, 2 meshs TRIA3
1 mesh SEG2



Boundary conditions:

$N2$: $U_x=0 \quad U_y=0 \quad U_z=0 \quad \theta_x=0 \quad \theta_y=0 \quad \theta_z=0$

$N1$: $U_x=0 \quad U_z=0$

Loading:

Traction on the face $[3\,4]$ (mesh SEG2) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters $t=0\text{s}$ and 1s, 20 increments enters $t=1\text{s}$ and 2s)

Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to 10^{-6}.

5.2 Characteristics of the grid

Many nodes: 8
Many meshs: 4

1 QUAD4
2 TRIA3
1 SEG2

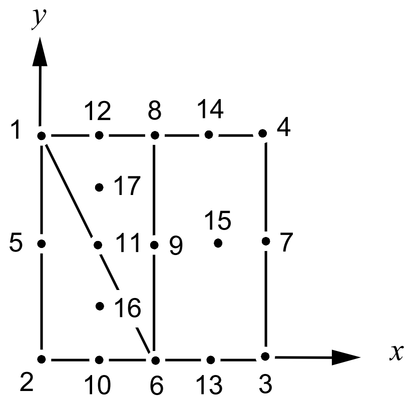
5.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	– 110	1,50%
$t=2$ Displacement DY ($N3$)	303	1,00%
$t=2$ Effort NXX ($PG1$)	1497	1,00%
$t=2$ Effort NYY ($PG1$)	0,79	1,00%
$t=2$ Effort NXY ($PG1$)	0,61	1,00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1,50%

6 Modeling D

6.1 Characteristics of modeling

Modeling `COQUE_3D` of thickness $1000\,mm$: 1 mesh `QUAD9`, 2 meshs `TRIA7`
1 mesh `SEG3`



Boundary conditions:

$N2$: $U_x=0 \quad U_y=0 \quad U_z=0 \quad \theta_x=0 \quad \theta_y=0 \quad \theta_z=0$

$N5$: $U_x=0 \quad U_z=0$

$N1$: $U_x=0 \quad U_z=0$

Loading:

Traction on the face $[3\ 4]$ (mesh `SEG3`) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters $t=0\mathrm{s}$ and $1\mathrm{s}$, 20 increments enters $t=1\mathrm{s}$ and $2\mathrm{s}$)

Convergence is carried out if the residue `RESI_GLOB_RELA` is lower or equal to 10^{-6}.

6.2 Characteristics of the grid

Many nodes: 17
Many meshs: 4

1 `QUAD9`
2 `TRIA7`
1 `SEG3`

6.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	– 110	1,50%
$t=2$ Displacement DY ($N3$)	303	1,00%
$t=2$ Constraint $SIXX$ ($PG1$)	1453	3,10%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1,50%

7 Summary of the results

Results found with *Code_Aster* and `DEFORMATION: 'SIMO_MIEHE'` are very satisfactory with percentages of error lower than 0.4% on the constraint and with 1.2% on the variable of work hardening. For elements of plate and hull the use of `DEFORMATION: 'PETIT_REAC'` give satisfactory results with percentages of error of 3% on the effort or the constraint and lower than 0.7% on the variable of work hardening.