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## HSNV124 - Element of volume in traction and temperature variables

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### Summary:

This test, suggested by the IPSI for the Phi2As day of March 30th, 2000 on the nonlinear behaviors makes it possible to validate the good taking into account of the variation of the coefficients with the temperature for four models of behavior (nonlinear isotropic work hardening, linear kinematic work hardening, and two types of nonlinear kinematic work hardening) in 3D .

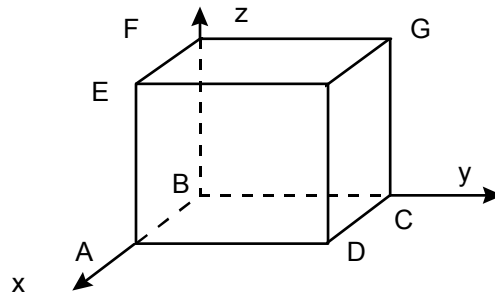
Four modelings make it possible to validate each one of these behaviors.

The reference solution is analytical for the first three behaviors, and the results will be compared with those of the other participants in the Phi2as day for the fourth.

## 1 Problem of reference

### 1.1 Geometry

Element of volume materialized by a unit cube on side:



### 1.2 Properties of materials

$$E = 2.10^5 \text{ MPa}, \quad \nu = 0.3, \quad \alpha = 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

The material is elastoplastic with various types of behaviors:

*C1* : Isotropic work hardening: the traction diagram is form:

$$\begin{aligned} \sigma(\varepsilon^P, T) &= \sigma_y(T) + Q(T)(1 - e^{-b(T)\varepsilon^P}) \\ SIGY &= 200. - 1.7.T \quad (\text{in MPa}) \\ Q(T) &= 100. + 1.7.T \quad (\text{in MPa}) \\ b(T) &= 50. + 2.T \end{aligned}$$

*C2* : Linear kinematic work hardening:

$$\begin{aligned} \sigma(\varepsilon^P, T) &= \pm \sigma_y(T) + C(T)\varepsilon^P \\ SIGY &= 200. - 1.7.T \quad (\text{in MPa}) \\ C(T) &= 1000 + 2990.T \quad (\text{in MPa}) \end{aligned}$$

*C3* : Nonlinear kinematic work hardening ( *I* ):

$$\begin{aligned} \sigma(\varepsilon^P, \dot{\varepsilon}^P, T) &= \pm \sigma_y(T) + Q(T)\alpha \\ \dot{\alpha} &= \dot{\varepsilon}^P - D(T)\alpha |\dot{\varepsilon}^P| \\ SIGY &= 200. - 1.7.T \quad (\text{in MPa}) \\ C(T) &= (100 + 1.7.T)(50 + \dots) \quad (\text{in MPa}) \\ D(T) &= 50 \end{aligned}$$

*C4* : Nonlinear kinematic work hardening ( *II* ):

$$\begin{aligned} \sigma(\varepsilon^P, \dot{\varepsilon}^P, T) &= \pm \sigma_y(T) + Q(T)\alpha \\ \dot{\alpha} &= \dot{\varepsilon}^P - D(T)\alpha |\dot{\varepsilon}^P| \\ &\text{same characteristics as for the behavior } C3, \text{ except } D(T) = 50 + 2T \end{aligned}$$

## 1.3 Boundary conditions and loadings

Such that the stress and strain state are uniform in the element of volume:

Not  $B$  blocked in  $x$ ,  $y$  and  $z$ . Not  $A$  blocked in  $z$ ,  $DY=0$  on the face  $ABFE$

Force distributed on the face  $CDHG$ :  $Fy$

Uniform temperature  $T(t)$  on the cube. The temperature of reference is worth  $0^\circ C$ .

$Fy$  and  $T$  vary according to time in the following way:

moment $t$	0	1	2
$Fy(t)$	0	210 MPa	210 MPa
$T(t)$	0	0	100°C

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution [bib1]

#### 2.1.1 Isotropic work hardening: analytical

$$t=1s \quad (T=0^\circ C): \quad \varepsilon^P = \frac{1}{b} \ln \left[ \frac{Q + \sigma_y - \sigma}{Q} \right] \quad \text{with } \sigma = 210 MPa = cste \text{ that is to say}$$

$$\varepsilon^P = 0.21072\%$$

**Heating:** Maximum plastic deformation with  $t=1.45222s$  ( $T=45.222^\circ C$ ):

$$\varepsilon^P = \frac{1}{b(T)} \ln \left[ \frac{Q(T) + \sigma_y(T) - \sigma}{Q(T)} \right] \quad \text{that is to say}$$

$$\varepsilon^P = 0.48108\%$$

Then: the plastic deformation does not evolve any more.

#### 2.1.2 Linear kinematic work hardening: analytical

$$t=1s \quad (T=0^\circ C): \quad \varepsilon^P = 1\%$$

**Heating:** Constant plastic deformation until  $t=356/316=1.12658s$  ( $T=12.658^\circ C$ ):

Then, the plastic deformation decreases to reach with  $t=2s$  :  $\varepsilon^P = 0.08\%$

#### 2.1.3 Kinematic work hardening nonlinear I: analytical

$$t=1s \quad (T=0^\circ C): \quad \varepsilon^P = \frac{1}{D} \ln \left[ \frac{A + \sigma_y - \sigma}{A} \right] \quad \text{with } A = \frac{C}{D} = 100 \quad \text{that is to say}$$

$$\varepsilon^P = 0.21072\%$$

**Heating:** Maximum plastic deformation with  $t=1.26011s$  ( $T=26.011^\circ C$ ):

$$\varepsilon^P = \frac{1}{D} \ln \left[ \frac{A(T) + \sigma_y(T) - \sigma}{A(T)} \right] \quad \text{that is to say}$$

$$\varepsilon^P = 0.40729\% = \varepsilon_0^P$$

The plastic deformation does not evolve any more until  $t_1=1.98332s$  ( $T_1=98.332^\circ C$ ) where one meets the other end of the field of elasticity.

Then: the plastic deformation decreases to reach with  $t=2s$  :

$$\varepsilon^P = \varepsilon_0^P + \frac{1}{D} \ln \left[ \frac{A(T) + \sigma_y(T) - \sigma}{X_0(T_1) + A(T)} \right] \quad \text{with } X_0(T_1) = A(T_1)(1 - e^{-D\varepsilon_0^P}) \quad \text{that is to say}$$

$$\varepsilon^P = 0.4037229\%$$

## 2.1.4 Nonlinear kinematic work hardening II

Comparison with the reference solution suggested to the day  $\Phi^2_{As}$ . (digital result got with 10 pas de time), and comparison with the results got with *Code\_Aster* with a very fine discretization in time

Plastic deformation $YY$	Calculation fine Aster: 100 pas until 1.26s , 100 pas between 1.98 and 2s	Reference $\Phi^2_{As}$ : Result for 10 pas
$t = 1s$	$2.1072 \cdot 10^{-03}$	$2.1072 \cdot 10^{-03}$
$1.26s < t < 1.98s$	$4.18947 \cdot 10^{-03}$	$4.38 \cdot 10^{-03}$
$t = 2s$	$4.12131 \cdot 10^{-03}$	$4.32 \cdot 10^{-03}$

## 2.2 Precision on the results of reference

There is an analytical solution for the first three behaviors, uncertainty is thus worthless. It is estimated at 4% for the fourth (variation enters the result for 10 pas and that for 200 pas, the solution strongly depending on the temporal discretization).

## 2.3 References bibliographical

- 1) IPSI: day of Phi2AS study of March 30th, 2000 on the nonlinear behaviors of materials.

## 3 Modeling A

### 3.1 Characteristics of modeling

Behavior *CI* : isotropic work hardening, in 3D . It is modelled in two ways:

- maybe using the behavior *VMIS\_ISOT\_TRAC*, with traction diagrams given all them  $10^{\circ}C$ , and interpolated for each temperature, which can be vague,
- maybe using the behavior *VMIS\_CIN1\_CHAB*, by cancelling nonlinear kinematic work hardening, and by keeping only the isotropic work hardening which is precisely expressed in the form:

$$\sigma(\varepsilon^P, T) = \sigma_y(T) + Q(T)(1 - e^{-b(T)\varepsilon^P})$$

It is enough to take then:  $R_0 = SIGY = 200. -1.7.T$   
 $R_I = SIGY + Q(T) = 200. -1.7.T + 100. + 1.7.T = 300$   
 $C_I = G_0 = 0$

1 pas de time enters  $t=0s$  and  $t=1s$  and 20 pas de time enters  $t=1s$  and  $t=2s$  .

### 3.2 Characteristics of the grid

The grid comprises a mesh *HEXA8*.

### 3.3 Sizes tested and results

Behavior *VMIS\_ISOT\_TRAC* :

Moment ( s )	Plastic deformation according to <i>Y</i>	Reference	Aster	% difference
<i>tI</i> = 1.	<i>EPYY</i>	2.1072 10 <sup>-03</sup>	2.1096 10 <sup>-03</sup>	0.1
<i>tI</i> = 1.45	<i>EPYY</i>	4.8108 10 <sup>-03</sup>	4.8135E-03	0,056
2	<i>EPYY</i>	4.8108 10 <sup>-03</sup>	4.8135E-03	0,056

Behavior *VMIS\_CIN1\_CHAB* :

Moment ( s )	Plastic deformation according to <i>Y</i>	Reference	Aster	% Difference
<i>tI</i> = 1.	<i>EPYY</i>	2.1072 10 <sup>-03</sup>	2.1096 10 <sup>-03</sup>	0.1
<i>tI</i> = 1.45	<i>EPYY</i>	4.8108 10 <sup>-03</sup>	4.81079 10 <sup>-03</sup>	0.0001
2	<i>EPYY</i>	4.8108 10 <sup>-03</sup>	4.81079 10 <sup>-03</sup>	0.0001

## 4 Modeling B

### 4.1 Characteristics of modeling

Behavior  $C2$  : linear kinematic work hardening, in 3D . It is modelled in three ways:

- maybe using the behavior  $VMIS\_CINE\_LINE$ , while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E + C(T))$  with  $C(T) = (1000 + 2990.T)$
- maybe using the behavior  $VMIS\_ECMI\_LINE$ , while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E + C(T))$  and the constant of Prager  $PRAG = 2/3 C(T)$
- maybe using the behavior  $VMIS\_CIN1\_CHAB$ , by keeping only linear kinematic work hardening: it is enough to take then:  $R_0 = R_1 = SIGY$  ,  $b = 0$  ,  $C_1 = C(T)$  ,  $G_0 = 0$

Temporal discretization: 1 pas de time enters  $t = 0s$  and  $t = 1s$  and 20 pas de time enters  $t = 1s$  and  $t = 2s$  .

### 4.2 Characteristics of the grid

The grid comprises a mesh  $HEXA8$ .

### 4.3 Sizes tested and results

Behavior  $VMIS\_CINE\_LINE$  :

Moment ( s )	Plastic deformation according to $Y$	Reference	Aster	% difference
$tI = 1.1$	$EPYY$	0.01	0.01	0.
2	$EPYY$	8.E-4	8.E-4	0

Behavior  $VMIS\_ECMI\_LINE$  :

Moment ( s )	Plastic deformation according to $Y$	Reference	Aster	% difference
$tI = 1.1$	$EPYY$	0.01	0.01	0.
2	$EPYY$	8.E-4	8.E-4	0

Behavior  $VMIS\_CIN1\_CHAB$  :

Moment ( s )	Plastic deformation according to $Y$	Reference	Aster	% difference
$tI = 1.1$	$EPYY$	0.01	0.01	0.
2	$EPYY$	8.E-4	8.E-4	0



## 5 Modeling C

### 5.1 Characteristics of modeling

Behavior *C3* : nonlinear kinematic work hardening (*I*) in 3D . It is modelled in two ways:

- maybe using the behavior *VMIS\_CIN1\_CHAB*. It is enough to take then:  
 $R_0 = R_I = SIGY$  ,  $b = 0$  ,  $C_I = C(T) = (100 + 1.7.T)(50 + 2.T)$  ,  $G_0 = 50$
- maybe using the behavior *VMIS\_CIN2\_CHAB*, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:  
 $R_0 = R_I = SIGY$  ,  $b = 0$  ,  $C_{I1} = C_{I2} = C(T)/2$  ,  $G_{I0} = G_{I2} = 50$

Temporal discretization: 20 pas de time enters  $t = 0s$  and  $t = 1s$  and 60 pas de time enters  $t = 1s$  and  $t = 2s$ .

### 5.2 Characteristics of the grid

The grid comprises a mesh *HEXA8*.

### 5.3 Sizes tested and results

Behavior *VMIS\_CIN1\_CHAB* :

Moment (S)	Plastic deformation according to <i>Y</i>	Reference	Aster	% difference
1	<i>EPYY</i>	2.1072 10 <sup>-03</sup>	2,113 10 <sup>-03</sup>	0.27
1.26	<i>EPYY</i>	4.0729 10 <sup>-03</sup>	4.0875 10 <sup>-03</sup>	0.36
1.98	<i>EPYY</i>	4.0729 10 <sup>-03</sup>	4.0875 10 <sup>-03</sup>	0.36
2	<i>EPYY</i>	4.0372 10 <sup>-03</sup>	3,978 10 <sup>-03</sup>	1.46

Behavior *VMIS\_CIN2\_CHAB* :

Moment (S)	Plastic deformation according to <i>Y</i>	Reference	Aster	% difference
1	<i>EPYY</i>	2.1072 10 <sup>-03</sup>	2,113 10 <sup>-03</sup>	0.27
1.26	<i>EPYY</i>	4.0729 10 <sup>-03</sup>	4.0875 10 <sup>-03</sup>	0.36
1.98	<i>EPYY</i>	4.0729 10 <sup>-03</sup>	4.0875 10 <sup>-03</sup>	0.36
2	<i>EPYY</i>	4.0372 10 <sup>-03</sup>	3,978 10 <sup>-03</sup>	1.46

### 5.4 Notice

The variation with the reference solution comes from the temporal discretization. While refining more, the solution approaches the analytical solution. One chose a compromise between a reasonable temporal discretization in time calculation and nevertheless rather precise.

## 6 Modeling D

### 6.1 Characteristics of modeling

Behavior *C4* : nonlinear kinematic work hardening (*II*) in 3D . It is modelled in two ways:

- maybe using the behavior VMIS\_CIN1\_CHAB. It is enough to take then:  

$$R_0 = R_I = SIGY, b = 0, C_I = C(T) = (100 + 1.7.T)(50 + 2.T), G_0 = D(T)$$
- maybe using the behavior VMIS\_CIN2\_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:

$$R_0 = R_I = SIGY, b = 0, C_{I1} = C_{I2} = C(T)/2, G_{I0} = G_{I2} = D(T)$$

Temporal discretization: 40 pas de time enters  $t=0s$  and  $t=1s$  and 30 pas de time enters  $t=1s$  and  $t=2s$ .

### 6.2 Characteristics of the grid

The grid comprises a mesh HEXA8.

### 6.3 Sizes tested and results

Behavior VMIS\_CIN1\_CHAB :

Moment (S)	Plastic deformation according to <i>Y</i>	Reference	Aster	% difference
1	EPYY	2.1072 10 <sup>-03</sup>	2.11 10 <sup>-03</sup>	0.14
1.26	EPYY	4.18947 10 <sup>-03</sup>	4,231 10 <sup>-03</sup>	0.99
1.98	EPYY	4.18947 10 <sup>-03</sup>	4,231 10 <sup>-03</sup>	0.99
2	EPYY	4.12131 10 <sup>-03</sup>	4,163 10 <sup>-03</sup>	1.00

Behavior VMIS\_CIN2\_CHAB :

Moment (S)	Plastic deformation according to <i>Y</i>	Reference	Aster	% difference
1	EPYY	2.1072 10 <sup>-03</sup>	2.11 10 <sup>-03</sup>	0.14
1.26	EPYY	4.18947 10 <sup>-03</sup>	4,231 10 <sup>-03</sup>	0.99
1.98	EPYY	4.18947 10 <sup>-03</sup>	4,231 10 <sup>-03</sup>	0.99
2	EPYY	4.12131 10 <sup>-03</sup>	4,163 10 <sup>-03</sup>	1.00

### 6.4 Notice

The variation with the reference solution (obtained for a very fine temporal discretization) comes from the temporal discretization. One chose here a compromise between a reasonable temporal discretization in time calculation and nevertheless rather precise.

## 7 Summary of the results

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This test makes it possible to highlight the effects of variation of the coefficients of the elastoplastic behaviors with the temperature.

The results are identical to the analytical solution for the linear kinematic work hardening (where the solution does not depend on the temporal discretization). For the other behaviors, the precision is less good (lower deviation than 1.5% ) because the solution strongly depends on the selected temporal discretization.

This test thus makes it possible to validate the integration of the behaviors VMIS\_ISOT\_TRAC, VMIS\_CINE\_LINE, VMIS\_ECMI\_LINE, VMIS\_CIN1\_CHAB, and VMIS\_CIN2\_CHAB compared to the variation of the coefficients with the temperature.