

## HSNS102 - Reinforced concrete plate with thermal loading

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### Summary:

The purpose of this test is principal to check the modeling of the elements reinforced concrete under thermal loading according to three techniques:

- Modeling a: Models hull: DKT + GRILLE\_EXCENTREE
- Modeling b: Models solid: Voluminal + GRILLE\_MEMBRANE
- Modeling C: Solid model: Voluminal + MEMBRANE

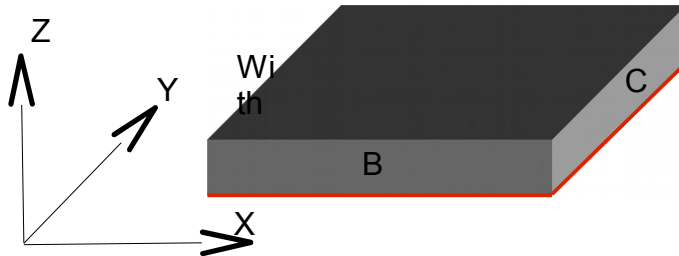
The objective is to check the mechanical answer by comparison with a reference solution obtained analytically.

Moreover, it tests, in modeling B, the functionality MATRICE=ELASTIQUE of STAT\_NON\_LINE for the elements GRILLE\_MEMBRANE, as well as the options EPVC\_ELGA, EPME\_ELGA and EPSP\_ELGA.

In modeling C, one also tests the calculation of the options EFGE\_ELGA and EFGE\_ELNO on the elements of MEMBRANE (into linear and non-linear).

## 1 Problem of reference

### 1.1 Geometry



Square plate:

Length:  $l=1.0\text{ m}$

Thickness:  $ep=0.2\text{ m}$

Reinforcements:

Section:  $S_a=0.01\text{ m}^2$

Offsetting:  $e=-0.01\text{ m}$

### 1.2 Properties of materials

**Concrete:**

Young modulus,  $E=3\ 10^{10}\text{ Pa}$

Poisson's ratio,  $\nu=0.0$

Thermal dilation coefficient,  $\alpha=10^{-5}\text{ K}^{-1}$

**Steel:**

Young modulus,  $E=2\ 10^{11}\text{ Pa}$

Poisson's ratio,  $\nu=0.0$

Thermal dilation coefficient,  $\alpha=10^{-5}\text{ K}^{-1}$

### 1.3 Boundary conditions and loadings

On with dimensions one  $A$  one blocks displacements according to  $X$  and  $Z$  and rotation around  $Y$  :

$$U_x=0.0; U_z=0.0; R_y=0.0$$

On the edge  $B$  one blocks displacement according to  $Y$  :

$$U_y=0.0$$

The initial temperature is  $20^\circ\text{C}$  for steel and the concrete.

One increases the temperature of steel to reach  $120^\circ\text{C}$ .

## 2 Reference solution

### 2.1 Method of calculating

The total deflections in steel and the concrete are:

$$\epsilon_a^t = \epsilon_a^m + \epsilon_a^{th} \quad \text{and} \quad \epsilon_b^t = \epsilon_b^m \quad \text{with} \quad \epsilon_a^{th} = \alpha \Delta T$$

Are  $\epsilon$  deformation of the average plan of the plate and  $\chi$  curve of the plate, two unknown factors to be found. By respecting kinematics (the sections remain plane), steel being perfectly related to the concrete, one a:

$$\epsilon_a^t = \epsilon - e\chi \quad \text{and} \quad \epsilon_b^t = \epsilon - y\chi$$

The normal effort imposed on the plate is null:

$$N = N_a + N_b = E_a S_a \epsilon_a^m + E_b \int \epsilon_b^m = E_a S_a (\epsilon - e\chi - \epsilon_a^{th}) + E_b S_b \epsilon = 0$$

In the same way, the bending moment imposed on the plate is null:

$$M = M_a + M_b = e E_a S_a \epsilon_a^m + E_b \int y \epsilon_b^m = E_a S_a (e\epsilon - e^2\chi - e\epsilon_a^{th}) + E_b I_b \chi = 0$$

One thus obtains two equations to determine the two unknown factors:

$$\begin{aligned} (E_a S_a + E_b S_b)\epsilon - E_a S_a e\chi &= E_a S_a \epsilon_a^{th} \\ E_a S_a e\epsilon + (E_b I_b - E_a S_a e^2)\chi &= E_a S_a e\epsilon_a^{th} \end{aligned}$$

One obtains:

$$\epsilon = \frac{\alpha \Delta T}{A} \quad \text{and} \quad \chi = \frac{-\alpha \Delta T}{B}$$

with:

$$A = 1 + \frac{E_b S_b}{E_a S_a} + \frac{S_b e^2}{I_b} \quad \text{and} \quad B = e + \left( \frac{1}{E_a S_a} + \frac{1}{E_b S_b} \right) + \frac{E_b I_b}{e}$$

From these values one can calculate:

- the lengthening of the plate:  $\Delta L = \epsilon L$
- the rotation of the plate:  $R_y = \chi L$
- the arrow at the end of the plate:  $f = -\chi \frac{L^2}{2}$  or in the middle of the plate  $f = -\chi \frac{L^2}{8}$
- the normal constraint in steel:  $\sigma_a = E_a (\epsilon - e\chi - \epsilon_a^{th})$
- normal effort in the concrete:  $N_b = E_b S_b \epsilon$

### 2.2 Sizes and results of reference

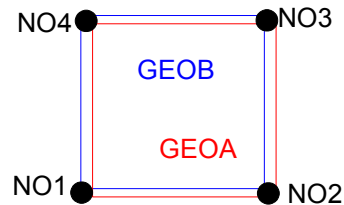
One calculates the lengthening and the arrow of the plate (displacement  $U_x$  and  $U_z$  of a node of the edge  $C$  plate), rotation ( $R_y$  constant on the length), the normal effort in the reinforcements, the normal effort in the concrete  $N_b$ .

### 2.3 Uncertainties on the solution

Exact solution.

### 3 Modeling A

#### 3.1 Characteristics of modeling



**Modeling :**

1 element DKT (GEOB) superimposed to 1 element GRILLE\_EXCENTRE (GEOA) supported on the same nodes.

**Boundary conditions:**

Nodes *NO1* and *NO4* :  $DX=0.$ ,  $DZ=0.$ ,  $DRY=0.$

Nodes *NO1* and *NO2* :  $DY=0$

**Thermal loading:**

The concrete remains with  $20^{\circ}C$ .

Steel passes from  $20^{\circ}C$  with  $120^{\circ}C$  between the moments 0 and 1,

The temperature of reference is in both cases of  $20^{\circ}C$ .

#### 3.2 Characteristics of the grid

Nodes: 4

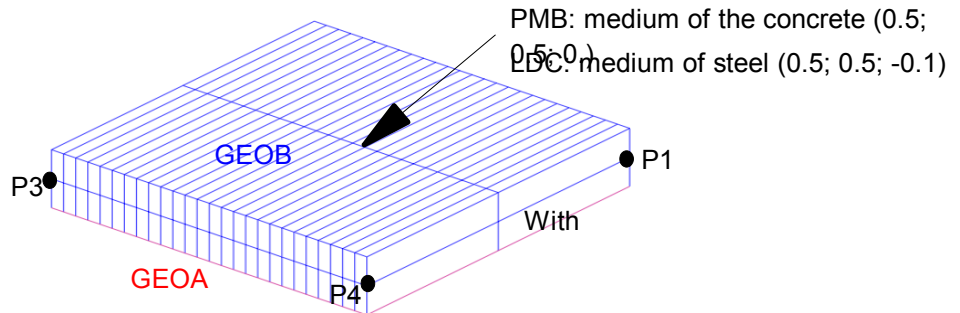
Meshes: 1 QUAD4 for the concrete and 1 QUAD4 for steel

#### 3.3 Sizes tested and results

Identification	Type of Reference	Value of reference	Tolerance
Displacement DX in NO2	'ANALYTICAL'	1.4285714E-04	0.0001%
Displacement DZ in NO2	'ANALYTICAL'	2.1428571E-03	0.0001%
Rotation RY in NO2	'ANALYTICAL'	-4.2857143E-03	0.0001%
Constraint SIXX in the mesh steel AMA1	'ANALYTICAL'	-8.571429E+07	0.0001%
Effort NXX in the mesh concrete MA1	'ANALYTICAL'	8.571429E+05	0.0001%

## 4 Modeling B

### 4.1 Characteristics of modeling



#### Modeling :

Elements 3D linear (GEOB) and elements **GRILLE\_MEMBRANE** (GEOA) supported on the nodes of the lower face.

#### Boundary conditions :

Face *A* : blocked by  $DX = 0$   
Line *P1-P4* : blocked by  $DZ = 0$   
Not *P1* : blocked by  $DY = 0$

#### Thermal loading:

The concrete remains with  $20^{\circ}C$ .  
Steel passes from  $20^{\circ}C$  with  $120^{\circ}C$  between the moments 0 and 1,  
The temperature of reference is in both cases of  $20^{\circ}C$ .

#### Note:

Compared to other modelings, a non-linear calculation with the behavior **GRILLE\_ISOT\_LINE** is added to validate the option **EPSP\_ELGA**.  
Parameters of material **ECRO\_LINE** used are the following:  $D\_SIGM\_EPSI = 0.5E11$ ,  
 $SY = 1E7$ .

### 4.2 Characteristics of the grid

Nodes: 243  
Meshes: 104 CUB8 for the concrete and 52 QUAD4 for steel

### 4.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
Displacement DX at the point PMB	'ANALYTICAL'	7.19892100E-05	1.00%
Displacement DZ at the point PMB	'ANALYTICAL'	5.35714274E-04	1.05%
Constraint SIXX in steel at the point LDC	'ANALYTICAL'	-8.571429E+07	1.00%
Constraint SIXX in the concrete at the point PMB	'ANALYTICAL'	4.52857145E+06	1.00%

#### Note:

The values of displacements and constraints are evaluated in the center of the plate because in modeling 3D there are effects edge free to the end, not considered by the analytical solution.

Validation of EPVC\_ELGA, EPME\_ELGA and EPSP\_ELGA on the elastic design (tests on EPSI\_ELGA and SIEF\_ELGA are used for obtaining of the values of reference) :

Moment	Mesh/Not	Field/Component	Type of reference	Value of reference	Tolerance (%)
1.0	M107/1	EPIF/EXX	'NON_REGRESSION'	-	
1.0	M107/1	EPVC/EP_THER_L	'ANALYTICAL'	1.0E-03	0.1
1.0	M107/1	EPME/EXX	'NON_REGRESSION'	-	-
1.0	M107/1	EPSP/EXX	'ANALYTICAL'	0.	-

Complement of validation of EPME\_ELGA and EPSP\_ELGA on an unelastic calculation:

Moment	Mesh/Not	Field/Component	Type of reference	Value of reference	Tolerance (%)
1.0	M107/1	SIEF/SIXX	'NON_REGRESSION'	-	-
1.0	M107/1	EPME/EPXX	'NON_REGRESSION'	-	-
1.0	M107/1	EPSP/EPXX	'NON_REGRESSION'	-	-

## 5 Modeling C

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### 5.1 Characteristics of modeling

**Modeling :**

Elements 3D linear (GEOB) and elements MEMBRANE (GEOA) supported on the nodes of the lower face.

**Boundary conditions and thermal loading :**

Identical to modeling B.

### 5.2 Characteristics of the grid

Grid identical to modeling B.

### 5.3 Values tested and results

Values identical to modeling B.

**Note:**

Validation of the options *EFGE\_ELGA* and *EFGE\_ELNO* is made by inter comparison with the fields *SIEF\_ELGA* and *SIEF\_ELNO* : these fields have the same contents.

## 6 Summary of the results

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This test in the case of compares the solutions obtained with three types of modeling with an analytical solution a reinforced concrete plate subjected to a thermal loading.

- Model hull: DKT + GRILLE\_EXCENTREE
- Solid model: Voluminal + GRILLE\_MEMBRANE
- Solid model: Voluminal + MEMBRANE

The assessment of the comparisons indicates a negligible difference between the results. Only one finite element is sufficient to find the analytical solution in the case DKT + GRILLE\_EXCENTREE.

In the voluminal cases with GRILLE\_MEMBRANE or MEMBRANE , the error is lower than 1% for a sufficiently fine grid (26 elements in the length).