WTNL102 - Dimensional mono problem of forced convection

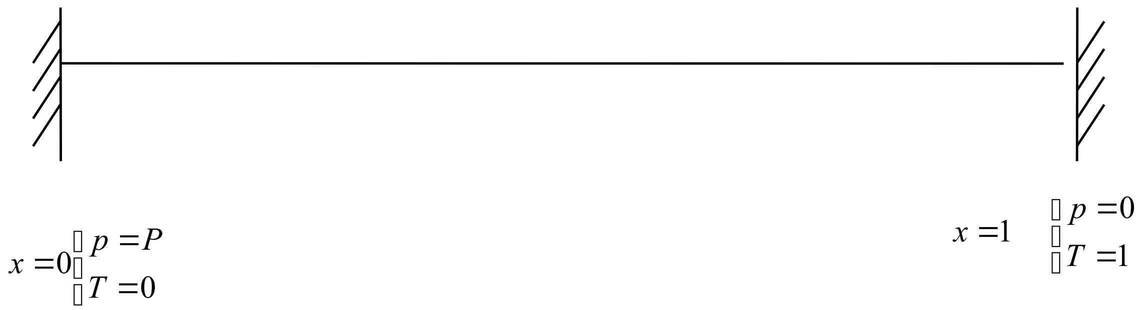
Summary:

It is the dimensional mono transport of heat by a flow constant speed. The hydraulic mode is characterized by a linear pressure in space. The reference solution is analytical.

1 Problem of reference

1.1 Geometry

One places oneself within the framework of a dimensional mono problem in Cartesian coordinates. The “structure” considered, is finally a segment length 1



1.2 Boundary conditions and loadings

One imposes a pressure varying linearly P in $x=0$ with 0 in $x=1$: $p(x)=P(1-x)$

In $x=0$: the temperature is imposed worthless
In $x=1$: the temperature is imposed on 1.

1.3 Initial conditions

$T(x)=0$ everywhere
One is interested in the permanent mode

2 Reference solution

2.1 Method of calculating

One leaves the equation of the energy [éq 3.1.3-1] of the document [R7.01.11], which in this case gives:

$$h\dot{m}+\dot{Q}'+Div(h\mathbf{M})+Div(\mathbf{q})=0 \qquad \text{éq 2.1-1}$$

In which h indicate the enthalpy of water, $\mathbf{M}$ its mass flow, m the mass water contribution and $\mathbf{q}$ heat flow.

Taking into account the made assumptions, one sees easily that:

$$\mathbf{M}=M_x=\rho_w\lambda_h P \qquad \text{éq 2.1-2}$$

$$h=C_w^p T \qquad \text{éq 2.1-3}$$

$$\mathbf{q}=q_x=-\lambda_T\frac{\partial T}{\partial x} \qquad \text{éq 2.1-4}$$

$$\dot{Q}'=\rho_w C_w^p \dot{T} \qquad \text{éq 2.1-5}$$

λ_T is the thermal coefficient of diffusion process, $\lambda_h=\frac{K_{\text{int}}}{\mu_w}$ is the hydraulic coefficient of diffusion, K_{int} the intrinsic permeability, ρ_w, μ_w, C_w^p are respectively the density, viscosity and calorific heat with constant pressure of water.

While deferring [éq 2.1-2], [éq 2.1-3], [éq 2.1-4] and [éq 2.1-5] in [éq 2.1-1] one finds:

$$\frac{\rho_w C_w^p}{\lambda_T}\dot{T}+\rho_w C_w^p\frac{\lambda_h}{\lambda_T}P\frac{\partial T}{\partial x}-\frac{\partial^2 T}{\partial x^2}=0 \qquad \text{éq 2.1-6}$$

One poses:

$$R=\rho_w C_w^p\frac{\lambda_h}{\lambda_T}P$$

and

$$S=\frac{\rho_w C_w^p}{\lambda_T}$$

One obtains

$$S\dot{T}+R\frac{\partial T}{\partial x}-\frac{\partial^2 T}{\partial x^2}=0 \qquad \text{éq 2.1-7}$$

2.2 Results of reference

In order to obtain the permanent mode more quickly, one chooses coefficients such as:

$$\frac{S}{R}=\frac{1}{\lambda_h P}<<1$$

The solution of [éq 2.1-7] is then:

$$T=\frac{e^{Rx}-1}{e^R-1}$$

3 Modeling A

3.1 Characteristics of modeling A

One makes a modeling with 500 elements, each element thus has a length $h=\frac{1}{500}$.

The coefficients are chosen:

ρ_w	1
C_w^p	1
μ_w	1
K_{int}	100
λ_T	10
P	1

These values lead to a Peclet number $R=10$ and with a Peclet number local $Rh=\frac{1}{50}$.

3.2 Results

X	Temperature of reference	Temperature *Aster*	Relative error ($\%$)
6,00E-01	0.0182710686	0.0182567	0.079%
7,00E-01	0.0497439270	0.0497269	0.034%
8,00E-01	0.1352960260	0.1352760	0.015%
9,00E-01	0.3678507400	0.3678309	0.005%
1,00E+00	1.0000000000	1.0000000	0.0%

4 Summary of the results

A good agreement is obtained between the temperatures calculated by *Code_Aster* and values of reference.