WTNV100 - Triaxial compression test not drained with model CJS (level 1)

Summary

This test makes it possible to validate level 1 of model CJS. It is about a triaxial compression test in not drained condition.
In the first two modelings, calculations are carried out only on the solid part of the ground, the aspect not drained being modelled by a worthless voluminal deformation of the skeleton, they are modelings $3D$ who differ one from the other only by the grid.

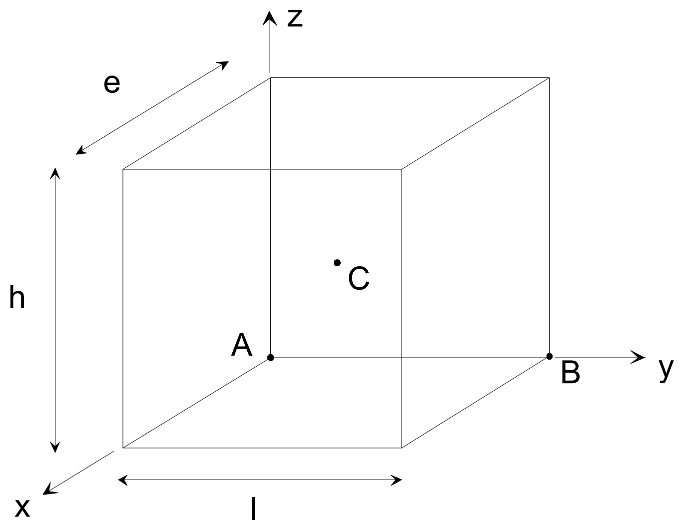
In the third modeling, the hydraulic coupling is taken into account, the sample is completely saturated, the skeleton and the fluid is supposed to be incompressible.

By reason of symmetry, one is interested only in the eighth of a sample subjected to a triaxial compression test. The level of containment is of $100 kPa$.

The results got with model CJS1 are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



hauteur : h = 1 m
largeur : l = 1 m
épaisseur : e = 1 m

Coordinates of the points (in meters):

	A	B	C
x	0.	0.	0.5
y	0.	1.	0.5
z	0.	0.	0.5

1.2 Material property

$E=22{,}4 10^3\,kPa$

$\nu=0{,}3$

Coefficient of biot $b=1$

Water is supposed to be incompressible: $\mathrm{UN\_SUR\_K}=0$

Parameters CJS1: $\beta=-0{,}03$ $\gamma=0{,}82$ $R_m=0{,}289$ $P_a=-100\,kPa$

1.3 Initial conditions, boundary conditions, and loading

1.3.1 Pure mechanical modeling

Phase 1:

One brings the sample in a homogeneous state: $\sigma_{xx}^0=\sigma_{yy}^0=\sigma_{zz}^0$, by imposing the corresponding confining pressure on the front, side right-hand side and higher faces. Displacements are blocked on the faces postpones ($u_x=0$), side left ($u_y=0$) and lower ($u_z=0$).

Phase 2:

One maintains displacements blocked on the faces postpones ($u_x=0$), side left ($u_y=0$) and lower ($u_z=0$). One applies a displacement imposed to the higher face: $u_z(t)$, in order to obtain a deformation $\varepsilon_{zz}=-20\,\%$ (counted starting from the beginning of phase 2). On the front faces and side right-hand side, one imposes displacements respectively $u_x(t)$ and $u_y(t)$, in order to have a worthless voluminal deformation for the sample, i.e. finally that one imposes $\varepsilon_{xx}=\varepsilon_{yy}=-\frac{\varepsilon_{zz}}{2}$. It is the manner of reproducing the behavior of the solid phase during a triaxial compression test not drained.

1.3.2 Modeling coupled with hydraulics

Phase 1:

One brings the sample in a homogeneous state of effective stresses: $\sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0$, by imposing the corresponding total pressure on the front, side right-hand side and higher faces and by imposing worthless water pressures everywhere. Displacements are blocked on the faces postpones ($u_x = 0$), side left ($u_y = 0$) and lower ($u_z = 0$).

Phase 2:

One maintains displacements blocked on the faces postpones ($u_x = 0$), side left ($u_y = 0$) and lower ($u_z = 0$).
On all the faces, hydraulic flows are worthless.
One applies a displacement forced to the higher face in order to obtain a deformation $\varepsilon_{zz} = -20\%$ (counted starting from the beginning of phase 2). On the front faces and side right-hand side, one imposes boundary conditions in total constraint:

$$\sigma.n = \sigma^0 \,(= 100 kPa)$$

2 Reference solution

2.1 Reference solution for the water pressure into linear

σ_0, ε_0 p_0 indicating the constraints, deformations and pressures of water obtained in the phase one a:

$$\sigma-\sigma_0=\lambda tr(\varepsilon-\varepsilon_0)+2\mu(\varepsilon-\varepsilon_0)-b(p-p_0)$$
$$\frac{m}{\rho^{fl}}=\frac{p-p_0}{M}+btr(\varepsilon-\varepsilon_0)$$

In this writing, M indicate the module of biot and $N=\frac{1}{M}$.

The boundary conditions of null flow and the conservation of the water mass give $m=0$
The boundary conditions on the side walls and the fact that the state of stress is homogeneous give:

$$\sigma_{xx}-\sigma_{xx0}=0$$

One has thus finally to solve the two equations:

$$\begin{cases}\lambda(2\Delta\varepsilon_{xx}+\Delta\varepsilon_{zz})+2\mu\Delta\varepsilon_{xx}=bp\\ b(2\Delta\varepsilon_{xx}+\Delta\varepsilon_{zz})=-\frac{p}{M}=-Np\end{cases}$$

And one obtains:

$$\begin{cases}\Delta\varepsilon_{xx}=-\frac{\Delta\varepsilon_{zz}}{2}\frac{b^2+\lambda N}{b^2+(\lambda+\mu)N}\\ p=-\frac{\mu b\Delta\varepsilon_{zz}}{b^2+(\lambda+\mu)N}\end{cases}$$

In our case,

$$\Delta\varepsilon_{xx}=-\frac{\Delta\varepsilon_{zz}}{2}\quad ;\quad p=-\mu\Delta\varepsilon_{zz}$$

2.2 Development of analytical solution CJS

One has permanently:

for the deformations: $\varepsilon_{xx}=\varepsilon_{yy}=-\frac{\varepsilon_{zz}}{2}$

for the constraints: $\sigma_{xx}=\sigma_{yy}$

Elastic phase:
While writing the elastic law simply, it comes:

$$\sigma_{xx}=\sigma_{xx}^0-\mu\,\varepsilon_{zz}$$
$$\sigma_{zz}=\sigma_{zz}^0+2\,\mu\,\varepsilon_{zz}$$

In addition, one also knows that during this phase I_1 ($=trace(\sigma)$) remain constant because $\varepsilon_v=0$. One from of deduced for the components from the diverter:

$$s_{xx}=\sigma_{xx}-\frac{I_1}{3}=\sigma_{xx}^0-\frac{I_1^0}{3}-\mu\,\varepsilon_{zz}=-\mu\,\varepsilon_{zz} \text{ and } s_{zz}=\sigma_{zz}-\frac{I_1}{3}=\sigma_{zz}^0-\frac{I_1^0}{3}+2\,\mu\,\varepsilon_{zz}=2\,\mu\,\varepsilon_{zz}$$

that is to say: $s_{II}=-\sqrt{6}\,\mu\,\varepsilon_{zz}$ and $\det(\underline{\underline{s}})=2\,\mu^3\,\varepsilon_{zz}^3$

Consequently: $h(\theta_s)=(1-\gamma)^{1/6}$

Thus when the criterion is reached $f^d=0$, one a:

$$s_{II}\,(1-\gamma)^{1/6}+R_m\,I_1^0=-\sqrt{6}\,\mu\,\varepsilon_{zz}\,(1-\gamma)^{1/6}+R_m\,I_1^0=0$$

I.e. the transition enters the states rubber band and perfectly plastic is done for an axial deformation equalizes with:

$$\varepsilon_{zz}^{trans}=\frac{R_m\,I_1^0}{\sqrt{6}\,\mu\,(1-\gamma)^{1/6}}$$

The corresponding state of stresses is noted:

$$\sigma_{xx}^{trans}=\sigma_{xx}^0-\mu\,\frac{R_m\,I_1^0}{\sqrt{6}\,\mu\,(1-\gamma)^{1/6}} \quad\text{and}\quad \sigma_{zz}^{trans}=\sigma_{zz}^0+2\,\mu\,\frac{R_m\,I_1^0}{\sqrt{6}\,\mu\,(1-\gamma)^{1/6}}$$

Plastic phase:

One notes s^{-d} the diverter of the reverse of the tensor S

Generally, there are the following sizes:

$$s_{xx}=-\frac{1}{3}(\sigma_{zz}-\sigma_{xx})=s_{yy} \quad s_{xx}^{-1}=\frac{-3}{\sigma_{zz}-\sigma_{xx}} \quad s_{xx}^{-d}=\frac{-3}{2(\sigma_{zz}-\sigma_{xx})}$$

$$s_{zz}=\frac{2}{3}(\sigma_{zz}-\sigma_{xx}) \quad s_{zz}^{-1}=\frac{3}{2(\sigma_{zz}-\sigma_{xx})} \quad s_{zz}^{-d}=\frac{3}{\sigma_{zz}-\sigma_{xx}}$$

that is to say: $s_{II}=-\sqrt{\frac{2}{3}}(\sigma_{zz}-\sigma_{xx})$ and $\det(\underline{\underline{s}})=\frac{2}{3^3}(\sigma_{zz}-\sigma_{xx})^3$

Consequently: $h(\theta_s)=(1-\gamma)^{1/6}$

One from of deduced:

$$Q_{xx}=\frac{1}{\sqrt{6}}(1-\gamma)^{1/6} \text{ and } Q_{zz}=-\sqrt{\frac{2}{3}}(1-\gamma)^{1/6}$$

moreover: $\frac{\partial f^d}{\partial\sigma_{xx}}=\frac{1}{\sqrt{6}}(1-\gamma)^{1/6}+R_m$ and $\frac{\partial f^d}{\partial\sigma_{zz}}=-\sqrt{\frac{2}{3}}(1-\gamma)^{1/6}+R_m$

Like one a: $\beta'=signe(s_{ij}\,\dot{\varepsilon}_{ij})\,\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)=\beta\left(\frac{R_m}{R_c}-1\right)=\beta$

then the tensor $\underline{\underline{n}}$ is written:

$$n_{xx} = \frac{1}{\sqrt{\beta^2+3}}\left(\frac{1}{\sqrt{6}}\beta+1\right) \text{ and } n_{zz} = \frac{1}{\sqrt{\beta^2+3}}\left(-\sqrt{\frac{2}{3}}\beta+1\right)$$

It comes then for $\underline{\underline{G}}^d$:

$$G_{xx}^d = \frac{1}{\sqrt{6}}(1-\gamma)^{1/6} + R_m - \frac{\beta(1-\gamma)^{1/6}+3R_m}{\beta^2+3}\left(\frac{1}{\sqrt{6}}\beta+1\right)$$

$$G_{zz}^d = -\sqrt{\frac{2}{3}}(1-\gamma)^{1/6} + R_m - \frac{\beta(1-\gamma)^{1/6}+3R_m}{\beta^2+3}\left(-\sqrt{\frac{2}{3}}\beta+1\right)$$

One also has according to the elastic law:

$$\sigma_{xx} = \sigma_{xx}^{trans} + \Delta\sigma_{xx} \text{ and } \sigma_{zz} = \sigma_{zz}^{trans} + \Delta\sigma_{zz}$$

where:

$$\Delta\sigma_{xx} = 2\mu\left(\Delta\varepsilon_{xx} - \Delta\lambda^d G_{xx}^d\right) + \lambda\left(\Delta\varepsilon_v - \Delta\lambda^d tr\left(\underline{\underline{G}}^d\right)\right) = -\mu\Delta\varepsilon_{zz} - 2\mu\Delta\lambda^d G_{xx}^d - \lambda\Delta\lambda^d\left(2G_{xx}^d + G_{zz}^d\right)$$

$$\Delta\sigma_{zz} = 2\mu\left(\Delta\varepsilon_{zz} - \Delta\lambda^d G_{zz}^d\right) + \lambda\left(\Delta\varepsilon_v - \Delta\lambda^d tr\left(\underline{\underline{G}}^d\right)\right) = 2\mu\Delta\varepsilon_{zz} - 2\mu\Delta\lambda^d G_{zz}^d - \lambda\Delta\lambda^d\left(2G_{xx}^d + G_{zz}^d\right)$$

and with: $\Delta\varepsilon_{xx} = \varepsilon_{xx} - \varepsilon_{xx}^{trans}$ and $\Delta\varepsilon_{zz} = \varepsilon_{zz} - \varepsilon_{zz}^{trans}$

maybe, according to what precedes, one has for s_{II} :

$$s_{II} = -\sqrt{\frac{2}{3}}\left[\left(\sigma_{zz}^{trans} - \sigma_{xx}^{trans}\right) + 3\mu\left(\varepsilon_{zz} - \varepsilon_{zz}^{trans}\right) - 2\mu\Delta\lambda^d\left(G_{zz}^d - G_{xx}^d\right)\right]$$

$$= s_{II}^{trans} - \sqrt{\frac{2}{3}}\left[3\mu\left(\varepsilon_{zz} - \varepsilon_{zz}^{trans}\right) - 2\mu\Delta\lambda^d\left(G_{zz}^d - G_{xx}^d\right)\right]$$

and for I_1 :

$$I_1 = I_1^{trans} - (3\lambda + 2\mu)\Delta\lambda^d\left(2G_{xx}^d + G_{zz}^d\right)$$

One from of deduced that the function of load déviatoire is written:

$$f^d = s_{II}^{trans}(1-\gamma)^{1/6} - \sqrt{\frac{2}{3}}\left[3\mu\left(\varepsilon_{zz} - \varepsilon_{zz}^{trans}\right) - 2\mu\Delta\lambda^d\left(G_{zz}^d - G_{xx}^d\right)\right](1-\gamma)^{1/6}$$
$$+ R_m I_1^{trans} - R_m(3\lambda + 2\mu)\Delta\lambda^d\left(2G_{xx}^d + G_{zz}^d\right)$$

By taking account owing to the fact that $f^d\left(\underline{\underline{\sigma}}^{trans}\right) = 0$, one finds then for the plastic multiplier:

$$\Delta\lambda^d = \frac{3\mu(1-\gamma)^{1/6}}{2\mu\left(G_{zz}^d - G_{xx}^d\right) - \sqrt{\frac{3}{2}}R_m(3\lambda + 2\mu)\left(2G_{xx}^d + G_{zz}^d\right)}\left(\varepsilon_{zz} - \varepsilon_{zz}^{trans}\right)$$

what gives with the formulas of G_{xx}^{d} and G_{zz}^{d} the preceding ones:

$$\Delta\lambda^d = \frac{\sqrt{\frac{2}{3}}\,\mu\,(1-\gamma)^{1/6}\,(\beta^2+3)}{\left(R_m\,\beta-(1-\gamma)^{1/6}\right)\left(2\,\mu(1-\gamma)^{1/6}-(3\,\lambda+2\,\mu)\,R_m\,\beta\right)}\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

One concludes from it finally the analytical expression from the constraints:

While posing:

$$a=(1-\gamma)^{1/6}\quad;\qquad b=(\beta^2+3)$$

One a:

$$\sigma_{xx}-\sigma_{xx}^{trans}=$$

$$-\left(\mu+\frac{\sqrt{\frac{2}{3}}\,\mu\,a\,b\left(2\,\mu\left(\frac{1}{\sqrt{6}}\,a+R_m-\frac{\beta\,a+3\,R_m}{b}\left(\frac{1}{\sqrt{6}}\,\beta+1\right)\right)+3\,\lambda\,\beta\frac{(R_m\,\beta-a)}{b}\right)}{(R_m\,\beta-a)\left(2\,\mu a-(3\,\lambda+2\,\mu)\,R_m\,\beta\right)}\right)\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

$$\sigma_{zz}-\sigma_{zz}^{trans}=$$

$$\left(2\,\mu-\frac{\sqrt{\frac{2}{3}}\,\mu\,a\,b\left(2\,\mu\left(-\sqrt{\frac{2}{3}}\,a+R_m-\frac{\beta a+3\,R_m}{b}\left(-\sqrt{\frac{2}{3}}\,\beta+1\right)\right)+3\,\lambda\,\beta\frac{(R_m\,\beta-a)}{b}\right)}{(R_m\,\beta-a)\left(2\,\mu a-(3\,\lambda+2\,\mu)\,R_m\,\beta\right)}\right)\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

2.3 Results of reference

Constraints σ_{xx}, σ_{yy} and σ_{zz} at the points A, B and C.

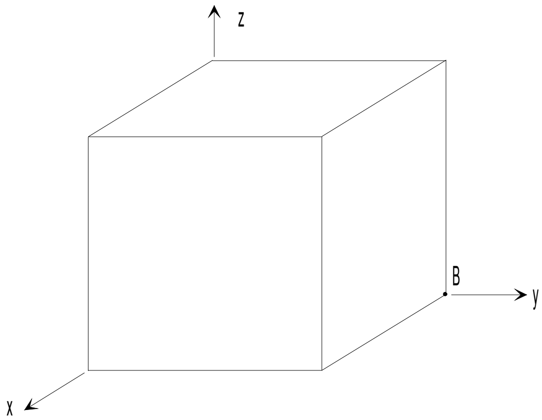
2.4 Uncertainty on the solution

Exact analytical solution for CJS1.

3 Modeling A

3.1 Characteristics of modeling

3D :



Cutting: 1 in height, in width and thickness.

Loading of phase 1:

Confining pressure: $\sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 : -100\,kPa$.

Level 1 of model CJS

3.2 Characteristic of the grid

Many nodes: 8

Many meshs and types: 1 HEXA8 and 6 QUA4

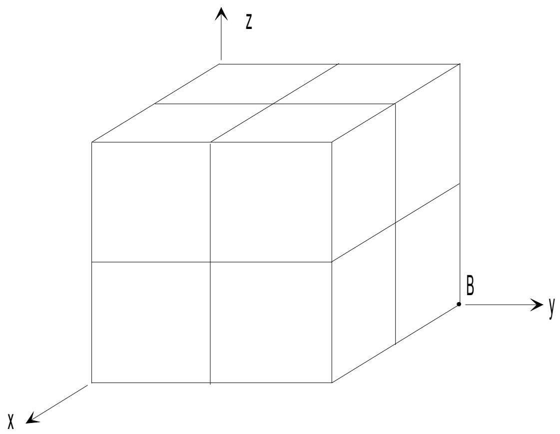
3.3 Sizes tested and results

Localization	Sequence number	Axial deformation ε_{zz} ($\%$)	Constraint (kPa)	Reference
Points A and B	1	– 0.25	σ_{xx}	– 78.461538
	2	– 0.50	σ_{xx}	– 56.923077
	3	– 0.75	σ_{xx}	– 53,606
	4	– 1.0	σ_{xx}	– 54,480
	8	– 5.0	σ_{xx}	– 68,467
	23	– 20.0	σ_{xx}	– 120,918
	1	– 0.25	σ_{yy}	– 78.461538
	2	– 0.50	σ_{yy}	– 56.923077
	3	– 0.75	σ_{yy}	– 53,606
	4	– 1.0	σ_{yy}	– 54,480
	8	– 5.0	σ_{yy}	– 68,467
	23	– 20.0	σ_{yy}	– 120,918
	1	– 0.25	σ_{zz}	– 143.07692
	2	– 0.50	σ_{zz}	– 186.153846
	3	– 0.75	σ_{zz}	– 196,818
	4	– 1.0	σ_{zz}	– 200,028
	8	– 5.0	σ_{zz}	– 251,383
	23	– 20.0	σ_{zz}	– 443,961

4 Modeling B

4.1 Characteristics of modeling

This modeling differs from the preceding one by the smoothness of the grid `3D`:



Cutting: 2 in height, in width and thickness.

Loading of phase 1:

Confining pressure: $\sigma_{xx}^{0} = \sigma_{yy}^{0} = \sigma_{zz}^{0} : -100\,kPa$.

Level 1 of model CJS

4.2 Characteristic of the grid

Many nodes: 27

Many meshs and types: 8 `HEXA8` and 24 `QUA4`

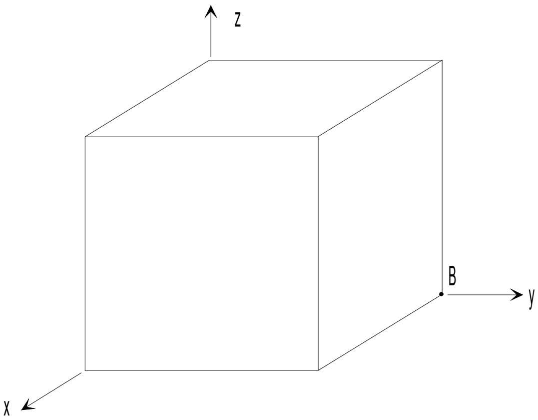
4.3 Sizes tested and results

Localization	Sequence number	Axial deformation ε_{zz} (%)	Constraint (kPa)	Reference
Points A , B and C	5	– 0.2	σ_{xx}	– 82.76923
	10	– 0.4	σ_{xx}	– 65.53846
	20	– 0.8	σ_{xx}	– 53.78079
	40	– 1.6	σ_{xx}	– 56.578176
	60	– 5.6	σ_{xx}	– 70.565109
	100	– 20.0	σ_{xx}	– 120.918065
	5	– 0.2	σ_{yy}	– 82.76923
	10	– 0.4	σ_{yy}	– 65.53846
	20	– 0.8	σ_{yy}	– 53.78079
	40	– 1.6	σ_{yy}	– 56.578176
	60	– 5.6	σ_{yy}	– 70.565109
	100	– 20.0	σ_{yy}	– 120.918065
	5	– 0.2	σ_{zz}	– 134.46154
	10	– 0.4	σ_{zz}	– 168.92308
	20	– 0.8	σ_{zz}	– 197.460849
	40	– 1.6	σ_{zz}	– 207.731697
	60	– 5.6	σ_{zz}	– 259.085935
	100	– 20.0	σ_{zz}	– 443.961194

5 Modeling C

5.1 Characteristics of modeling

3D_HM :



Cutting: 1 in height, in width and thickness.

Loading of phase 1:

Confining pressure: $\sigma_{xx}^{0} = \sigma_{yy}^{0} = \sigma_{zz}^{0} : -100\,kPa$.

Level 1 of model CJS
Coefficient of biot: 1
UN_SUR_K water: 0

5.2 Characteristic of the grid

Many nodes: 20
Many meshs and types: 1 HEXA20 and 6 QUA8

5.3 Sizes tested and results

Localization	**Sequence number**	**Axial deformation ε_{zz} (%)**	**Constraint (kPa)**	**Reference**
Points A and B	1	– 0.25	σ_{xx}	– 78.461538
	2	– 0.50	σ_{xx}	– 56.923077
	3	– 0.75	σ_{xx}	– 53,606
	4	– 1.0	σ_{xx}	– 54,480
	8	– 5.0	σ_{xx}	– 68,467
	23	– 20.0	σ_{xx}	– 120,918
	1	– 0.25	σ_{yy}	– 78.461538
	2	– 0.50	σ_{yy}	– 56.923077
	3	– 0.75	σ_{yy}	– 53,606
	4	– 1.0	σ_{yy}	– 54,480
	8	– 5.0	σ_{yy}	– 68,467
	23	– 20.0	σ_{yy}	– 120,918
	1	– 0.25	σ_{zz}	– 143.07692
	2	– 0.50	σ_{zz}	– 186.153846
	3	– 0.75	σ_{zz}	– 196,818
	4	– 1.0	σ_{zz}	– 200,028
	8	– 5.0	σ_{zz}	– 251,383
	23	– 20.0	σ_{zz}	– 443,961
	1	– 0.25	pressure water	2,1538E+04
	2	– 0.50	pressure water	4,3077E+04

For the water pressure, there is the reference as long as the behavior is elastic linear

6 Summary of the results

Values of *Code_Aster* are in perfect agreement with the values of reference. Concerning the coupling with hydraulics, this test proves that by means of computer, coupling CJS/THM functions and that the equations of hydraulics are at least able to give again the worthless variation of volume when water is incompressible.